Manufacturing Process Performance

2.810
T. Gutowski

See Refs and extra slides at end
## State of the Teams:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>Z</th>
<th>Requested Help</th>
<th>Free Agents</th>
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</table>
Comparing Mfg Processes

Ref University Loughborough, UK

Conventional casting 3D Printed

Two steps: comparing for same part, redesign for process
Manufacturing process performance attributes

- **Cost*** - direct costs, DFM
- **Rate and Time*** – process rate & system rate
- **Quality/Variation** – physics & statistics
- **Energy*/ CO$_2$ – processes & systems

*Assuming alternatives meet part performance requirements
Manufacturing process performance attributes

• Cost* - see Boothroyd et al handouts
• Rate and Time* – by process & for systems
  Oct 31 Dr. Stan Gershwin
• Quality/Variation – Oct 3, Prof. Dave Hardt
• Energy*/ CO₂ – Lecture 11/28

*Assuming alternatives meet part performance requirements
Profit = Sales - Costs

Our focus: the cost to make a product

Indirect costs: common activities that support many products

Direct costs: “touch” labor, direct materials & tooling…

Ostwald
Components of Direct Cost:

Direct Recurring Costs (Variable $C = VN$):
- Materials, including Buy to Fly, auxiliary…
- Labor, usually “touch”, but…
- Energy, usually small for parts
- Equipment (Prices as a “rental”)

Direct Non-recurring costs (Fixed $C = F$):
- Tooling, special equipment..
Unit cost: $C/N = F/N + V$

Serial processes take longer, larger variable costs. Specialty materiel add to variable costs.

Parallel processes require tooling, larger fixed costs, but short cycle time.
Estimate breakeven

\[ \frac{C}{N} \]

Extrusion

Waterjet

\[ N^* \]
Cost comparison; breakeven part volume

Extrusion
- **Fixed** = extrusion die
- **Variable** = (30grams/part)
- Material = aluminum billet
- Labor = mostly set-up
- Machine = mostly set-up, maintenance

Water jet
- **Fixed** = ???,
- **Variable** = (30g part, 90 secs/part)
- Materials = aluminum bar, waste, abrasive
- Labor = some monitoring needed
- Machine = includes machine, financing, installation, maintenance
Aluminum Extrusion

- Extrusion die = $5,000
- Extrusion rate, less than 1 second to extrude one part
- Equipment rate ~ $40/hr
- Labor rate ~ $50/hr
- Materials billet aluminum, some start up loss, kerf loss
- Setup/cleanup times dominates Equip and Labor
- Sawing, post processing
- Cost = $5,000 + $0.40N
Nest Parts for Waterjet, waste
~30\%
Waterjet cutting

- **Variable Costs:**
  - Aluminum bar stock ~ $5/lb (~$10/kg)
  - 30 gram part + waste = 40g/part = $0.40/part
  - Run time ~ 90 sec
  - Abrasive ~ $0.30/part
  - Machine cost (~$100k/5years/1shift (1000h/y) + maintenance) ~ $25/hr
  - Labor cost ~ $35/hr
  - Cost = $2.2N
Estimated breakeven

$N^* \sim 3,000$ parts
Breakeven cost $2.20$/part
Big Blue Saw quote

Material: Aluminum 6061, 0.375 inches thick. 1 part in the file. Overall dimensions: 1.063 inches X 6.000 inches (152.40 mm X 27.01 mm). See part details.

<table>
<thead>
<tr>
<th>Parts made by waterjet machining</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td><strong>Price, Original Raw Finish</strong></td>
</tr>
<tr>
<td>Add Basic Finish, Each Part</td>
</tr>
</tbody>
</table>

See more 3D views...
Big Blue Saw quote

Material: Aluminum 6061, 0.375 inches thick. Overall dimensions: 1.063 inches X 6.000 inches (152.40 mm X 27.01 mm). See part details.

<table>
<thead>
<tr>
<th>Parts made by waterjet machining</th>
<th>1-4</th>
<th>5-9</th>
<th>10-49</th>
<th>50-99</th>
<th>100-999</th>
<th>1000+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 75 Cost break down per part for injection molding, Stereolithography, FDM and SLS [139]
Additive Mfg Vs Injection Mold

Figure 8: Total energy use per part versus production volume for SLS and IM of the paintball handle

Ref. Cassandra Telenko et al
Components of Indirect Cost:

**Indirect Costs**

- Support activities that can be charged “evenly” to all products i.e. “overhead” includes things like; design, programming, maintenance, quality control, purchasing, marketing, sales, general and administrative expenses…

  e.g. Composites parts may require more inspection than metal parts
Indirect costs..

• Become more important for higher levels of automation,
• Become more difficult to allocate as the number of products and variation grows.
• Use “Activity Based Costing” and other tools
Cincinnati MAXIM FMS Cell

(5) Machines
(24) Pallets
(3) Shifts
Part Types/ Total Produced

System H; (2,000/35,000)

System 2; (20/1,500)

System 3; (200/10,600)
Cost Models

- **Time & Motion:** F.W. Taylor, Ostwald, Polgar
- **Machine Parameters:** e.g. Mastercam
- **DFM and DFA:** Boothroyd, Dewhurst & Knight (Parametric Models)
- **Software**
  - On-line
- **Quotes:** see for example, “Big Blue Saw” for water jet quotes, on-line metals...
Pros & Cons of Parametric Models

- Hint at the problem
- Become dated, and need updating

Handouts for
1. Machining
2. Injection molding
3. Sheet Metal
4. Assembly
Example: Time Estimation for Manual Assembly

- Handling
  - pick up
  - orient
- Insertion
  - location (obstructed view? Self locating?)
  - hold down and resistance
  - securing method
Handling Issues

Symmetry

<table>
<thead>
<tr>
<th>α</th>
<th>0</th>
<th>180</th>
<th>180</th>
<th>90</th>
<th>360</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>180</td>
<td>0</td>
<td>360</td>
</tr>
</tbody>
</table>

Size

Fragile/Sharp

Nest/Tangle

Slippery/Flexible
B-D Manual handling chart

Handling difficulties: nest, tangle slippery, sharp...

<table>
<thead>
<tr>
<th>Key: ONE HAND</th>
<th>(α + β) &lt; 360°</th>
<th>360° ≤ (α + β) &lt; 540°</th>
<th>540° ≤ (α + β) &lt; 720°</th>
<th>(α + β) = 720°</th>
</tr>
</thead>
<tbody>
<tr>
<td>parts can be grasped and manipulated by one hand without the aid of grasping tools</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manual Handling—Estimated Times (seconds)</td>
<td>parts are easy to grasp and manipulate</td>
<td>parts present handling difficulties (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>thickness &gt; 2 mm</td>
<td>thickness ≤ 2 mm</td>
<td>thickness &gt; 2 mm</td>
<td>thickness ≤ 2 mm</td>
</tr>
<tr>
<td></td>
<td>size &gt; 15 mm</td>
<td>6 mm ≤ size ≤ 15 mm</td>
<td>size &gt; 6 mm</td>
<td>size ≤ 6 mm</td>
</tr>
<tr>
<td>0</td>
<td>1.13</td>
<td>1.43</td>
<td>1.88</td>
<td>1.69</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>1.8</td>
<td>2.25</td>
<td>2.06</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>2.1</td>
<td>2.55</td>
<td>2.36</td>
</tr>
<tr>
<td>3</td>
<td>1.95</td>
<td>2.25</td>
<td>2.7</td>
<td>2.51</td>
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</table>
Insertion Issues

Alignment

- Self-locating
- Holding down and alignment required for subsequent operation

Obstructed Access/View

- Restricted access for assembly of screw
- Part must be released before it is located
- Part located before release

Insertion Force

- Difficult to insert
- Part can hang up
- Easy to insert
- Part falls into place
### B-D Manual insertion chart

**Manual Insertion — Estimated Times (seconds)**

<table>
<thead>
<tr>
<th></th>
<th>after assembly no holding down required to maintain orientation and location (3)</th>
<th>holding down required during subsequent processes to maintain orientation or location (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>easy to align and position during assembly</td>
<td>not easy to align or position during assembly</td>
</tr>
<tr>
<td></td>
<td>easy to align and position during assembly</td>
<td>not easy to align or position during assembly</td>
</tr>
<tr>
<td>no</td>
<td>resistance to insertion (5)</td>
<td>resistance to insertion (5)</td>
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<tr>
<td>resistance to insertion (4)</td>
<td>resistance to insertion (4)</td>
<td>resistance to insertion (4)</td>
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<td>0</td>
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<td>1</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>8.5</td>
<td>9.5</td>
</tr>
</tbody>
</table>

**Key:**
- **PART ADDED but NOT SECURED**
- **PART SECURED IMMEDIATELY**

**Part and Associated Tool:**
- (including hands) can easily reach the desired location.
- (including hands) cannot easily reach the desired location.
- Due to obstructed access or restricted vision.

**Additional Notes:**
- Screws

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**Plastic Deformation Immediately after Insertion**
- Plastic bending or torsion
- Rivetting or similar operation

**Screw Tightening Immediately after Insertion (6)**
1 – screw(2) (steel) not easy to align

2 – cover(steel) not easy to align – assembly worker’s fingers must be used to align edges

3 – spring(steel) (closed ends) subject to continuous cycling and must be spring steel

4 – piston stop(plastic) edge is chamfered for ease of alignment

5 – piston(aluminum) obstructed access for insertion of spindle into bottom of bore

6 – main block(plastic) depth of bore is 28mm with small through hole for piston spindle
B-D Manual handling chart

**Manual Handling—Estimated Times (seconds)**

<table>
<thead>
<tr>
<th>Parts are easy to grasp and manipulate</th>
<th>Parts present handling difficulties</th>
<th>Thickness &gt; 2 mm</th>
<th>Thickness ≤ 2 mm</th>
<th>Thickness &gt; 2 mm</th>
<th>Thickness ≤ 2 mm</th>
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</thead>
<tbody>
<tr>
<td>Thickness &gt; 2 mm</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Size &gt; 15 mm</td>
<td>6 mm ≤ size ≤ 15 mm</td>
<td>Size &gt; 6 mm</td>
<td>Size ≤ 6 mm</td>
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</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1.13</td>
<td>1.43</td>
<td>1.88</td>
<td>1.69</td>
<td>2.18</td>
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<td>2.98</td>
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<td></td>
</tr>
</tbody>
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**Key:**

- **ONE HAND**
- **piston**
- **screws**
- **cover**
- **spring**
## B-D Manual insertion chart A

### MANUAL INSERTION — ESTIMATED TIMES (seconds)

<table>
<thead>
<tr>
<th></th>
<th>after assembly no holding down required to maintain orientation and location (3)</th>
<th>holding down required during subsequent processes to maintain orientation or location (3)</th>
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<tbody>
<tr>
<td></td>
<td>easy to align and position during assembly (4)</td>
<td>easy to align and position during assembly (4)</td>
</tr>
<tr>
<td></td>
<td>not easy to align or position during assembly</td>
<td>not easy to align or position during assembly</td>
</tr>
<tr>
<td></td>
<td>resistance to insertion (5)</td>
<td>resistance to insertion (5)</td>
</tr>
<tr>
<td></td>
<td>resistance to insertion (5)</td>
<td>resistance to insertion (5)</td>
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<td>resistance to insertion (5)</td>
<td>resistance to insertion (5)</td>
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<tr>
<td>Key:</td>
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<tr>
<td>PART ADDED but NOT SECURED</td>
<td>0 1 2 3 6 7 8 9</td>
<td>0 1 2 3 6 7 8 9</td>
</tr>
</tbody>
</table>

- Spring: 6.5 seconds
- Piston: 5.5 seconds
- Cover: 6.5 seconds
## B-D Manual Insertion Chart B

<table>
<thead>
<tr>
<th>Screws</th>
<th>Plastic Deformation Immediately After Insertion</th>
<th>Rivetting or Similar Operation</th>
<th>Screw Tightening Immediately After Insertion (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Screwing Operation or Plastic Deformation Immediately After Insertion (Snap/Press Fits, Circlips, Spire Nuts, etc.)</td>
<td>Plastic Bending or Torsion</td>
<td>Not Easy to Align or Position During Assembly</td>
<td>Not Easy to Align or Position During Assembly</td>
</tr>
<tr>
<td>0</td>
<td>Easy to Align and Position During Assembly (4)</td>
<td>Easy to Align and Position During Assembly (4)</td>
<td>Easy to Align and Position During Assembly (4)</td>
</tr>
<tr>
<td>1</td>
<td>Not Easy to Align or Position During Assembly (5)</td>
<td>Not Easy to Align or Position During Assembly (5)</td>
<td>Not Easy to Align or Position During Assembly (5)</td>
</tr>
<tr>
<td>2</td>
<td>No Resistance to Insertion</td>
<td>Resistance to Insertion (5)</td>
<td>Resistance to Insertion (5)</td>
</tr>
<tr>
<td>3</td>
<td>Resistance to Insertion</td>
<td>Resistance to Insertion (5)</td>
<td>Resistance to Insertion (5)</td>
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<td>4</td>
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<td>Resistance to Insertion (5)</td>
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<td>Resistance to Insertion (5)</td>
<td>Resistance to Insertion (5)</td>
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<tr>
<td>9</td>
<td>Resistance to Insertion</td>
<td>Resistance to Insertion (5)</td>
<td>Resistance to Insertion (5)</td>
</tr>
</tbody>
</table>

### Screws
- 8

Part and associated tool (including hands) can easily reach the desired location and the tool can be operated easily.
- 4.5

Part and associated tool (including hands) cannot easily reach desired location or tool cannot be operated easily due to obstructed access or restricted vision.
- 6

Part and associated tool (including hands) cannot easily reach desired location or tool cannot be operated easily due to obstructed access and restricted vision.
Pneumatic Piston Sub-Assembly

<table>
<thead>
<tr>
<th>Part I.D. No.</th>
<th>number of times the operation is carried out consecutively</th>
<th>two-digit manual handling code</th>
<th>manual handling time per part</th>
<th>two-digit manual insertion code</th>
<th>manual insertion time per part</th>
<th>operation time, seconds</th>
<th>operation cost, cents</th>
<th>0.4 * (2 + (4 * (6 / (5.5 + 1)) + 1))</th>
<th>figures for estimation of theoretical minimum parts</th>
<th>Name of Assembly</th>
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<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>30</td>
<td>1.95</td>
<td>00</td>
<td>1.50</td>
<td>3.45</td>
<td>1.38</td>
<td>1</td>
<td>1</td>
<td>MAIN BLOCK</td>
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<td>5</td>
<td>1</td>
<td>10</td>
<td>1.50</td>
<td>10</td>
<td>4.00</td>
<td>5.50</td>
<td>2.20</td>
<td>1</td>
<td>1</td>
<td>PISTON</td>
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<td>1</td>
<td>10</td>
<td>1.50</td>
<td>00</td>
<td>1.50</td>
<td>3.00</td>
<td>1.20</td>
<td>1</td>
<td>1</td>
<td>PISTON STOP</td>
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<td>3</td>
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<td>1</td>
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<td>23</td>
<td>2.36</td>
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<td>6.50</td>
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<td>39</td>
<td>8.00</td>
<td>19.60</td>
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<td>SCREW</td>
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</table>

<table>
<thead>
<tr>
<th>TM</th>
<th>CM</th>
<th>NM</th>
<th>4</th>
<th>(3 * NM) / TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.75</td>
<td>17.50</td>
<td>0.27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Design efficiency
Boothroyd/Dewhurst Design Rules

1. Reduce part count and part types
2. Strive to eliminate adjustments
3. Design parts to be self-aligning and self-locating
4. Ensure adequate access and unrestricted vision
5. Ensure the ease of handling of parts from bulk
6. Minimize the need for reorientations during assembly
7. Design parts that cannot be installed incorrectly
8. Maximize part symmetry if possible or make parts obviously asymmetrical
Rules to reduce part count

1. During operation of the product, does the part move relative to all other parts already assembled?
   Only gross motion should be considered – small motions that can be accommodated by elastic hinges, for example, are not sufficient for a positive answer

2. Must the part be of a different material than or be isolated from all other parts already assembled?
   Only fundamental reasons concerned with material properties are acceptable

3. Must the part be separate from all other parts already assembled because otherwise necessary assembly or disassembly of other separate parts would be impossible?
Redesign:
Pneumatic Piston
Sub-Assembly

1 – snap on cover and stop (plastic)

2 – spring (steel)

3 – piston (aluminum)

4 – main block (plastic)
# Re-design

<table>
<thead>
<tr>
<th>Part I.D. No.</th>
<th>number of times the operation is carried out consecutively</th>
<th>two-digit manual handling code</th>
<th>manual handling time per part</th>
<th>two-digit manual insertion code</th>
<th>manual insertion time per part</th>
<th>operation time, seconds</th>
<th>operation cost, cents</th>
<th>figures for estimation of theoretical minimum parts</th>
<th>Name of Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>30</td>
<td>1.95</td>
<td>00</td>
<td>1.50</td>
<td>3.45</td>
<td>1.38</td>
<td>1</td>
<td>MAIN BLOCK</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>10</td>
<td>1.50</td>
<td>00</td>
<td>1.50</td>
<td>3.00</td>
<td>1.20</td>
<td>1</td>
<td>PISTON</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>05</td>
<td>1.84</td>
<td>00</td>
<td>1.50</td>
<td>3.34</td>
<td>1.34</td>
<td>1</td>
<td>SPRING</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>10</td>
<td>1.50</td>
<td>30</td>
<td>2.00</td>
<td>3.50</td>
<td>1.40</td>
<td>1</td>
<td>COVER &amp; STOP</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TM</th>
<th>CM</th>
<th>NM</th>
<th>design efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.29</td>
<td>5.32</td>
<td>4</td>
<td>(3*NM)/TM = 0.90</td>
</tr>
</tbody>
</table>


Process Rate Limits

• Machining — limits on (MRR), large forces…

• Injection Molding — heat transfer

• Sheet metal forming — lead time for dies, die changes

• Assembly — shimming, unavailable resources
Nominal Mfg Process Rates Vary by more than 8 orders

See extra slides at end
\[ P = P_{aux} + k \dot{m} \]

\[ \frac{P}{\dot{m}} = \frac{E}{m} = \frac{P_{aux}}{\dot{m}} + k \]
Life Cycle Perspective

Mfg. Process Equipment

- Mat’l Prod
- Mfg.
- Use
- EOL

Raw Mat’ls
Life Cycle Perspective

Product

Mat'l Prod

Mfg.

Use

EOL

EOL
Production Rate

- **Process rate** – slow machines = large capital investment

- **System rate**

String ribbon process for PV
• AM-machine utilization: 4500 h/year
• Depreciation time: 5 years
• Investment costs: 500,000€
• Costs for maintenance 21,666 €/ Year
• Build rate: 6.3 cm³/h
• Build Material: Stainless Steel 316L
• Material Price 89 €/kg
• Part Volume: 1cm³
• Layer thickness 0.3 μm

Analyzing Product Lifecycle Costs for a Better Understanding of Cost Drivers in Additive Manufacturing
C. Lindemann*, U. Jahnke*, M. Moi*, R. Koch*

*Direct Manufacturing Research Center (DMRC) and Chair of Computer Application and Integration in Design and Planning (C.I.K.), The University of Paderborn, Meretinweg 3, 33098 Paderborn, Germany

REVIEWS, Accepted August 20, 2012

Abstract

figure 4: Sample Part from Augsburg and the given cost breakdown structure by a batch size of 190 parts per build
Production Rate

- Process rate - physics
- System rate – waiting for needed resource
System Time & Rate

• Oct 31, Dr. Stan Gershwin
  – Little’s Law,
  – unreliable machines,
  – Buffers (zero, infinite and finite)
  – M/M/1 queue
  – Simulations
Quality Issues at the Process

Satisfied customer and meeting target specification
Where you find the problem matters

Increasing Cost

- Product recalls
- Customer complaints
- Warranty claims
- Dealer returns
- In-house inspection
- Assembly
- Observed at the process

Repair and upgrade of the Hubble Telescope

Fix problems close to home!
Inspecting in Quality

Distribution of color density in television sets.

Ref M. Phadke
Quality Loss Function

Figure 2.4 Variations of the quadratic loss function.

Ref M.Phadke

After G Taguchi
Nominal Process Variation
(process in control)

Surface Roughness

Dimensional Tolerance
Process variation/tolerance

What are the most important variables?

**FIGURE 35.20** Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Process variation/tolerance

FIGURE 35.20 Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
FIGURE 35.20 Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Quality Control Actions

\[ Y = \Phi(\alpha + \Delta \alpha, U) \]

Ref Hardt, 2001
How to control variation

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

output variation  sensitivity  disturbance  control action

ref. D. Hardt
Real Time Control

\[ \Delta Y = \left( \frac{\partial Y}{\partial \alpha} \right) \Delta \alpha + \left( \frac{\partial Y}{\partial u} \right) \Delta u \]

output variation  sensitivity  disturbance  control action
Designed Experiments

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

- output variation
- sensitivity
- disturbance
- control action

Output feature

Controllable parameter
Statistical Process Control

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

- output variation
- sensitivity
- disturbance
- control action
Mean drift, and variance of the output Y

Mean on target, but large variation due to many random effects

Mean drift has assignable cause, tight grouping means small variation
Comparing the variation with the specifications

Goals: $6\sigma < (USL-LSL)$
and mean centered
If UCL - LCL = 6σ and the process mean is in the center, then The out of compliance parts are given by

\[ 2(0.500 - \phi(3\sigma)) = 2(0.500 - 0.4987) = 0.0026 \text{ or } 0.26\% \text{ or } 2600 \text{ppm} \]
Some propose a process capability index $C_p$ that compares the tolerance interval USL-LSL vs the process variation $6\sigma$.

$$C_p = \frac{USL - LSL}{6\sigma}$$

<table>
<thead>
<tr>
<th>$C_p$</th>
<th>% out</th>
<th>ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3}$</td>
<td>4.55</td>
<td>45,500</td>
</tr>
<tr>
<td>1</td>
<td>0.26</td>
<td>2600</td>
</tr>
<tr>
<td>$1 \frac{1}{3}$</td>
<td>.0063</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>.00034</td>
<td>3.4</td>
</tr>
</tbody>
</table>
Mean Drift

Normal distribution $N(0, 1)$

- Lower Specification Limit: 0.135%
- Target: 0.4
- Upper Specification Limit: 15.97

Key values:
- 13.6
- +2.1
- +0.135
- 15.835
- +0.135
- 15.97
“Not In-Control”

The Parent Distribution Changes with Time

Slide taken from D. Hardt
Statistical Control Methods

**Strategy:**

1. Monitor process when under control
2. Determine **Centerline & Control Limits**
3. Data outside of UCL/LCL indicates change
4. Investigate and eliminate causes of shift
Mean drift Vs Normal Variance

• Statistical Process Control

How likely is this to occur?

“Shewhart Control Charts”
Histogram for CNC Turning

From Dave Hardt
Sampling Frequency

Factors that determine the appropriate sampling frequency:

– Stability of process
– Potential loss
– Cost of sampling inspection
What causes variation in dimensions?

• Machine variation
  – e.g. change in settings, environment, equipment

• Material variation
  – e.g. suppliers, substitutes, mixtures..

• Operator variation
  – Jim instead of Joe, or Alice instead of Mary

• Method variation
  – Late Friday afternoon, Mary always does it this way…
“x-bar charts”
Mean of the means

\[ \bar{x} = \frac{1}{k} \sum_{j=1}^{k} \bar{x}_j \]

\[ UCL, LCL = \bar{x} \pm A_2 \bar{R} \]

Where,
- \( n \) = sample size
- \( k \) = number of samples
- \( A_2 \) = constant from Table C.1
- \( R \) = defined next slide

Also see alternative method
Presented by Dave Hardt
“R-charts” Range = high - low

- Standard Deviation can be estimated from R

\[ R = \max(x_i, K, x_n) - \min(x_i, K, x_n) \]

\[ \overline{R} = \frac{1}{k} \sum_{j=1}^{k} R_j \]

\[ LCL = D_3 \overline{R} \]

\[ UCL = D_4 \overline{R} \]

Where,  
- \( n \) = sample size  
- \( k \) = number of samples  
- \( D_3, D_4 \) = constants from Table C.1
Estimate of standard deviation from range ref. P. Lyonnet

estimate for $m$, and if $W$ is the range or spread of values in the sample, i.e. the difference between the greatest and least values, an estimate for $\sigma$ is $W/d_n$, where $n$ is the number of items in the sample and $d_n$ is a known function (Table 3.3 gives values of $d_n$).

<table>
<thead>
<tr>
<th>Size of each sample</th>
<th>$1/d_n$</th>
<th>$d_n$</th>
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<tr>
<td>2</td>
<td>0.886</td>
<td>1.128</td>
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<tr>
<td>3</td>
<td>0.591</td>
<td>1.693</td>
</tr>
<tr>
<td>4</td>
<td>0.486</td>
<td>2.059</td>
</tr>
<tr>
<td>5</td>
<td>0.430</td>
<td>2.326</td>
</tr>
<tr>
<td>6</td>
<td>0.395</td>
<td>2.534</td>
</tr>
<tr>
<td>7</td>
<td>0.370</td>
<td>2.704</td>
</tr>
<tr>
<td>8</td>
<td>0.351</td>
<td>2.847</td>
</tr>
<tr>
<td>9</td>
<td>0.337</td>
<td>2.970</td>
</tr>
<tr>
<td>10</td>
<td>0.325</td>
<td>3.078</td>
</tr>
<tr>
<td>11</td>
<td>0.315</td>
<td>3.173</td>
</tr>
<tr>
<td>12</td>
<td>0.307</td>
<td>3.258</td>
</tr>
<tr>
<td>Number of Observations in Sample, ( n )</td>
<td>Factors for ( \bar{x} )-Charts</td>
<td>Factors for ( R )-Charts</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>2.66</td>
<td>3.27</td>
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<td>2.57</td>
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<td>1.63</td>
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<td>0.76</td>
<td>1.64</td>
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<td>16</td>
<td>0.74</td>
<td>1.62</td>
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<tr>
<td>17</td>
<td>0.70</td>
<td>1.61</td>
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<tr>
<td>18</td>
<td>0.70</td>
<td>1.60</td>
</tr>
<tr>
<td>19</td>
<td>0.68</td>
<td>1.59</td>
</tr>
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</table>

# Tables of Constants for Control charts

## Table 8A - Variable Data

<table>
<thead>
<tr>
<th>Subgroup size (n)</th>
<th>A₂</th>
<th>d₂</th>
<th>D₃</th>
<th>D₄</th>
<th>A₃</th>
<th>c₄</th>
<th>B₃</th>
<th>B₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.880</td>
<td>1.128</td>
<td>-</td>
<td>3.267</td>
<td>2.659</td>
<td>0.7979</td>
<td>-</td>
<td>3.267</td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td>1.693</td>
<td>-</td>
<td>2.574</td>
<td>1.954</td>
<td>0.8862</td>
<td>-</td>
<td>2.568</td>
</tr>
<tr>
<td>4</td>
<td>0.729</td>
<td>2.059</td>
<td>-</td>
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<td>0.9213</td>
<td>-</td>
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<tr>
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<td>-</td>
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<td>0.076</td>
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<td>0.9650</td>
<td>0.185</td>
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<tr>
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<td>0.337</td>
<td>2.970</td>
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<td>0.9693</td>
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<td>0.9727</td>
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<td>15</td>
<td>0.223</td>
<td>3.472</td>
<td>0.347</td>
<td>1.653</td>
<td>0.789</td>
<td>0.9823</td>
<td>0.428</td>
<td>1.572</td>
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<tr>
<td>25</td>
<td>0.153</td>
<td>3.931</td>
<td>0.459</td>
<td>1.541</td>
<td>0.606</td>
<td>0.9896</td>
<td>0.565</td>
<td>1.435</td>
</tr>
</tbody>
</table>

## Control Limits

**X bar and R Charts**

- Centerline: 
  \[
  CL_{\bar{X}} = \bar{X}
  \]
- Control Limits: 
  \[
  UCL_{\bar{X}} = \bar{X} + A_2 \bar{R} \quad LCL_{\bar{X}} = \bar{X} - A_2 \bar{R}
  \]
  \[
  CL_{R} = \bar{R} \quad UCL_{R} = D_4 \bar{R} \quad LCL_{R} = D_3 \bar{R}
  \]

**X bar and s Charts**

- Centerline: 
  \[
  CL_{\bar{X}} = \bar{X} 
  \]
- Control Limits: 
  \[
  UCL_{\bar{X}} = \bar{X} + A_3 \bar{s} \quad LCL_{\bar{X}} = \bar{X} - A_3 \bar{s}
  \]
  \[
  CL_{s} = \bar{s} \quad UCL_{s} = B_4 \bar{s} \quad LCL_{s} = B_3 \bar{s}
  \]

## Notes

- The divisor to estimate \( \sigma_x \) is taken from Table 1A. The factors for control limits are taken from Table 1B.

---

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Robustness Vs Sensitivity

• Mean shift by $1\sigma$ requires on average 44 data points to observe

• Alternatives
  – $\pm 2\sigma$; two consecutive observations
  – $\pm 1\sigma$; three consecutive observations
  – other charts (e.g. “Cusum”)
Finding the cause of a disturbance is the most difficult part of quality control. There are only aids to help you with this problem solving exercise like the Ishikawa Diagrams which helps you cover all categories, and the “5 Whys” which helps you go to the root cause.
5.3.1 Cause Listing Diagram

In a cause listing diagram, the problem is put at the far right side of the page, as shown in Fig. 5.1.

Figure 5.1 Example of a cause listing diagram for voids in a product

Figure 5.2 Example of variation analysis diagram for dimensional variation
Summary

• Cost – direct cost comparisons
• Time and Rate – process & system
• Quality/Variance – random/assignable
• Energy – process overview
Extra slides

1. Energy Intensity of Mfg Processes

2. Math Tools: Expectation and Variation Operators (Composites Example)

3. Propagation of Errors
Energy Used by Manufacturing Processes

- Fixed power usage during processing – auxiliary equipment
- Variable power usage – scales with the rate of material processing e.g. machining spindle power ~ MRR
Energy intensity of Mfg Processes

1. Machining
2. Grinding
3. Casting
4. Injection Molding
5. Abrasive Waterjet
6. EDM
7. Laser DMD
8. CVD
9. Sputtering
10. Thermal Oxidation

Electricity requirements for manufacturing processes
MJ electricity/kg processed
Energy Requirements at the Machine Tool

Energy Use Breakdown by Type

Production Machining Center

Automated Milling Machine

Electric Energy Intensity for Manufacturing Processes

\[ P = P_o + k_v \dot{V}_{\text{processed}} \]

\[ \frac{P}{\dot{V}} = \frac{P_o}{\dot{V}} + k_v = \frac{E}{V} \]
Injection Molding Machines

\[ \frac{P}{\dot{m}} = \frac{P_o}{\dot{m}} + k_m = \frac{E}{m} \]

Does not account for the electric grid.

Source: [Thiriez '06]
Thermal Oxidation, $\text{SiO}_2$

FIGURE 9. Energy consumption for growth of a 25-Å oxide layer as a function of equipment type (RTP vs vertical furnace), number of wafers processed per week, and total run time (production plus idle). The example shown is for 8-in. wafers.

Ref: Murphy et al, es&t 2003
## Power Requirements

Ref: Murphy et al es&t 2003

<table>
<thead>
<tr>
<th>Unit Operation</th>
<th>No. of Functions</th>
<th>Power (kW)</th>
<th>8-Layer Metal</th>
<th>6-Layer Metal</th>
<th>Wafers/Run</th>
<th>Wafers/H</th>
<th>Process</th>
<th>Idle</th>
</tr>
</thead>
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<td>16</td>
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<td></td>
</tr>
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<td>CVD</td>
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<td>16</td>
<td>11</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wafer Clean</td>
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<td>31</td>
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<td>Furnace (RTP)</td>
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<td>21</td>
<td>17</td>
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<td>35</td>
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<td>16</td>
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<td>Furnace (Coater)</td>
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<td>7</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Photo (Stepper)</td>
<td>27</td>
<td>48</td>
<td>23</td>
<td>60</td>
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<td>Photo (Coater)</td>
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<td>90</td>
<td>23</td>
<td>60</td>
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</tr>
<tr>
<td>Etch (Pattern)</td>
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<td>135</td>
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<td>35</td>
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<td></td>
</tr>
<tr>
<td>Etch (Ash)</td>
<td>27</td>
<td>135</td>
<td>23</td>
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<td>CMP</td>
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<td>Process Name</td>
<td>Power Required kW</td>
<td>Process Rate cm³/s</td>
<td>Electricity Required J/cm³</td>
<td>References</td>
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<td>Injection Molding</td>
<td>10.76 - 11.40</td>
<td>3.76 - 50.45</td>
<td>1.75E+08 - 3.41E+08</td>
<td>[Thiria 2006]</td>
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<td>Machining</td>
<td>2.80 - 19.80</td>
<td>0.35 - 20.00</td>
<td>3.50E+03 - 1.87E+05</td>
<td>[Dahmusi 2004], [Morrow, Q.] &amp; [Time Estimation Booklet 1996]</td>
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<td>CVD</td>
<td>14.78 - 25.00</td>
<td>6.54E-05 - 3.24E-03</td>
<td>4.63E+06 - 2.44E+08</td>
<td>[Wolf &amp; Tauber 1986], [Novellus Concept One 1999], [Krishnan Communication 2005]</td>
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<td>Sputtering</td>
<td>5.04 - 9.90</td>
<td>1.06E-05 - 6.70E-04</td>
<td>7.52E+06 - 6.45E+08</td>
<td>[Wolf &amp; Tauber 1986] &amp; [Holland Interview]</td>
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<td>Grinding</td>
<td>7.50 - 0.03</td>
<td>1.66E-02 - 2.85E-02</td>
<td>6.92E+04 - 3.08E+05</td>
<td>[Baniszewski 2005], [Chrysikouuris 1991]</td>
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<td>Waterjet</td>
<td>8.16 - 16.00</td>
<td>5.19E-03 - 8.01E-02</td>
<td>2.06E+05 - 3.66E+06</td>
<td>[Kurd 2004]</td>
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<td>Wire EDM</td>
<td>6.60 - 14.25</td>
<td>2.23E-03 - 2.71E-02</td>
<td>2.44E+06 - 6.39E+06</td>
<td>[Sodick], [Kalpakjian &amp; Schmid 2001], [AccuraX 2005]</td>
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<td>Drill EDM</td>
<td>2.63</td>
<td>1.70E-07</td>
<td>1.54E+10</td>
<td>[King EDM 2005] &amp; [McGeough, J.A. 1988]</td>
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<td>Laser DMD</td>
<td>80.00</td>
<td>1.28E-03</td>
<td>6.24E+07</td>
<td>[Morrow, Q.] &amp; [Skarkos 2004]</td>
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<td>Thermal Oxidation</td>
<td>21.00 - 48.00</td>
<td>4.36E-07 - 8.18E-07</td>
<td>2.57E+10 - 1.10E+11</td>
<td>[Murphy et al. 2003]</td>
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In General, over many manufacturing processes,

Idle Power

\[ 5kW \leq P_o \leq 50kW \]

and

Material Process Rates

\[ 10^{-7} \text{ cm}^3/\text{sec} \leq \dot{V} \leq 1 \text{ cm}^3/\text{sec} \]
Figure 1 Energy intensity, J/kg (electricity), and process rates, kg/hr for additive equipment (colored data points) compared to other manufacturing processes. Red symbols indicate additive metals, and blue symbols indicate additive polymers. Note: EDM is electrical discharge machining, FDM is fused deposition modeling, DMD is direct material deposition, BAAM is big area additive manufacturing, and CVD is chemical vapor deposition. J/kg = joules per kilogram; kg/hr = kilograms per hour.
Measuring Process Variation: Review
Process measurement reveals a distribution in output values.

Discrete probability distribution based upon measurements

![Discrete probability distribution](image1)

Continuous “Normal” distribution

![Continuous Normal distribution](image2)

In general if the randomness is due to many different factors, the distribution of the means will tend toward a “normal” distribution. (Central Limit Theorem)
If the dimension “X” is a random variable, the mean is given by

$$\mu = E(X)$$  \hspace{1cm} (1)$$

and the variation is given by

$$\text{Var}(x) = E[(x - \mu)^2] = \sigma^2$$  \hspace{1cm} (2)$$

both of these can be obtained from the probability density function \(p(x)\).

For a discrete pdf, the expectation operation is:

$$E(X) = \sum_i x_i p(x_i)$$  \hspace{1cm} (3)$$
Sample calculation of $E(x) = \mu$, and $Var(x) = \sigma^2$

\[
\begin{align*}
\sum p_i &= \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1 \\
\mu &= \sum x_ip = 1 \times \frac{1}{4} + 2 \times \frac{1}{2} + 3 \times \frac{1}{4} = 2 \\
Var &= \sum (x_i - \mu)^2 p_i = \frac{1}{4} + 0 + \frac{1}{4} = \frac{1}{2} \\
\sigma &= \frac{1}{\sqrt{2}}
\end{align*}
\]
Properties of the Expectation

1. If $Y = aX + b$;

where $Y$, $X$ are random variables; $a$, $b$ are constants,

$$E(Y) = aE(X) + b$$  \hfill (4)

2. If $X_1, \ldots, X_n$ are random variables,

$$E(X_1 + \ldots + X_n) = E(X_1) + \ldots + E(X_n)$$  \hfill (5)
Properties of the Variance

1. For a and b constants,

   \[ \text{Var}(aX + b) = a^2 \text{Var}(X) \]  \hspace{1cm} (6)

2. If \( X_1, \ldots, X_n \) are independent random variables

   \[ \text{Var}(X_1 + \ldots + X_n) = \text{Var}(X_1) + \text{Var}(X_2) + \ldots + \text{Var}(X_n) \]  \hspace{1cm} (7)
“Tolerance Stack up”, really about variance,

\[ E(X_1 + \ldots + X_n) = E(X_1) + \ldots + E(X_n) \]

but how about

\[ \text{Var}(X_1 + \ldots + X_n) = ? \]
If $X_1$ and $X_2$ are random variables and not necessarily independent, then

$$\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 2\text{Cov}(X_1 X_2)$$  \hspace{1cm} (8)

this can be written using the standard deviation “$\sigma$”, and the correlation “$\rho$” as

$$\sigma_L^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_1 \sigma_2 \rho$$  \hspace{1cm} (9)

where $L = X_1 + X_2$
If $X_1$ and $X_2$ are correlated ($\rho = 1$), then

$$\sigma_L^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2 = (\sigma_1 + \sigma_2)^2$$ \hspace{1cm} (14)$$

for $X_1 = X_2 = X_0$

$$\sigma_L^2 = 4\sigma_0^2$$ \hspace{1cm} (15)$$

for N

$$\sigma_L^2 = N^2 \sigma_0^2$$ \hspace{1cm} (16)$$

or

$$\sigma_L = N\sigma_0$$ \hspace{1cm} (17)$$
Now, if $X_1$ and $X_2$ are uncorrelated ($\rho = 0$) we get the result as in eq’n (7) or,

\[
\sigma_L^2 = \sigma_1^2 + \sigma_2^2
\]  

(10)

and for $N$

\[
\sigma_L^2 = \sum_{i=1}^{N} \sigma_i^2
\]  

(11)

If $X_1 = X = X_0$

\[
\sigma_L^2 = N\sigma_0^2
\]  

(12)

Or

\[
\sigma_L = \sqrt{N}\sigma_0
\]  

(13)
“Tolerance Stack-up”

As the number of variables grow so does the variation in the system; but when normalized…

\[
\frac{\sigma_L}{L} = \frac{N\sigma_0}{NL_0} = \frac{\sigma_0}{L_0}
\]

correlated

\[
\frac{\sigma_L}{L} = \frac{\sqrt{N}\sigma_0}{NL_0} = \frac{\sigma_0}{\sqrt{NL_0}}
\]

uncorrelated

Where \( L = NL_0 \)
Example: Variation in composites prepregs (±7%) and cured parts (±7%)
e.g. Thermal expansion 1-D

\[ \delta = \alpha L \Delta T \]

- \( \delta \): change in dimension
- \( \alpha \): coef. of thermal expansion
- \( L \): length of sample
- \( \Delta T \): change in temperature

Diagram showing a sample with length \( L \) and change in dimension \( \delta \).
Random variables

If the variables are independent:

\[ E(\delta) = E(\alpha)E(L)E(\Delta T) \]

..and if the variation is small, linearize to get:

\[
\left( \frac{\sigma_\delta}{\delta} \right)^2 = \left( \frac{\sigma_L}{L} \right)^2 + \left( \frac{\sigma_\alpha}{\alpha} \right)^2 + \left( \frac{\sigma_{\Delta T}}{\Delta T} \right)^2
\]

Ref: Lipschutz
Propagation of errors $y = \theta \cdot x$

\[
y = \bar{y} + \delta y = (\bar{\theta} + \delta \theta)(\bar{x} + \delta x)
\]

\[
\delta y \approx \bar{\theta} \delta x + \bar{x} \delta \theta
\]

\[
\text{Var}(y) = E[(\delta y)^2]
\]

\[
\delta y^2 \approx (\bar{\theta} \delta x)^2 + 2\bar{\theta} \delta x \cdot \bar{x} \delta \theta + (\bar{x} \delta \theta)^2
\]

recall \quad E(x) = \sum x_i p(x_i)

\[
\text{Var}(y) \approx \bar{\theta}^2 \text{Var}(x) + \bar{x}^2 \text{Var}(\theta)
\]

Ref. Young
This gives...

- this result is called “quadrature”,

in general, if $y = \theta x$, with $\theta$, $x$ independent random variables with small variation, then

with $\text{Var} (x) = \sigma_x^2$

\[
\left( \frac{\sigma_y}{\bar{y}} \right)^2 = \left( \frac{\sigma_\theta}{\bar{\theta}} \right)^2 + \left( \frac{\sigma_x}{\bar{x}} \right)^2
\]
A more general results is...

- for any relationship like $y = z^\alpha x^\beta$, with $z$, $x$ independent random variables with small variation, then

$$
\left( \frac{\sigma_y}{\bar{y}} \right)^2 = \alpha^2 \left( \frac{\sigma_z}{\bar{z}} \right)^2 + \beta^2 \left( \frac{\sigma_x}{\bar{x}} \right)^2
$$
Hence for Thermal Expansion...

If the variables are independent:

\[ E(\delta) = E(\alpha)E(L)E(\Delta T) \]

..and the variation is small:

\[
\left( \frac{\sigma_{\delta}}{\delta} \right)^2 = \left( \frac{\sigma_L}{L} \right)^2 + \left( \frac{\sigma_\alpha}{\alpha} \right)^2 + \left( \frac{\sigma_{\Delta T}}{\Delta T} \right)^2
\]
References

• Random Variables Ch 5 Seymour Lipschutz
• Statistical Process Control (SPC), Hogg & Ledolter
• Propagation of Errors, Statistical Treatment of Experimental Data, Hugh Young
• “hockey stick diagram” for energy and rate, Gutowski et al, See “electrical energy used in manufacturing processes” p 1586-1588 your text
• also see your text, topics are listed in the index. For the 7th edition: SPC section 36.8, Cost section 40.10, and Energy sections 40.5 & 40.6 these topics are also listed in other places in the text, see index in the back of the book.