Manufacturing Process Performance

2.810
T. Gutowski
See Refs and extra slides at end
State of the Teams:

• Please form teams with 4-6 members

• We can help if you are unsure how to proceed
Comparing Mfg Processes

Ref University Loughborough, UK

Conventional casting

3D Printed

Guidelines: redesign for process, design for same objectives
Manufacturing process performance attributes

• **Cost** - mostly direct costs, DFM, software

• **Rate and Time** – process rate & system rate

• **Quality/Variation** – physics & statistical control

• **Energy/ CO₂** – processes & systems
Manufacturing process performance attributes

- **Cost** - DFM Boothroyd, Oct 2 Lewandowski
- **Rate and Time** – by process & for systems
  
  Oct 30  Gershwin
- **Quality/Variation** – Oct 28  Hardt
- **Energy/ CO₂** – Oct 2, Nov 13
Profit = Sales - Costs

Our focus: the cost to make a product

Indirect costs: common activities that support many products

Direct costs: “touch” labor, direct materials & tooling...

Ostwald
Components of Direct Cost:

Direct Recurring Costs (Variable $C = VN$):
• Materials, including Buy to Fly, auxiliary…
• Labor, usually “touch”, but…
• Energy, usually small for parts
• Equipment (Prices as a “rental”)

Direct Non-recurring costs (Fixed $C = F$):
• Tooling, special equipment..
Unit cost: \( C/N = F/N + V \)

Serial processes take longer, larger variable costs
Specialty mat’l add to variable costs

Parallel processes require tooling, larger fixed costs, but short cycle time
Estimate breakeven

\[ N^* = \frac{F_E - F_W}{V_W - V_E} \]
Direct Cost comparison; breakeven part volume

**Extrusion**
- **Fixed** = extrusion die
- **Variable** = (30grams/part)
- Material = aluminum billet, scrap
- Labor = set-up & run
- Machine = set-up, run, maintenance

**Water jet**
- **Fixed** = programing
- **Variable** = (30g part, 90 secs/part)
- Materials = aluminum bar, waste, abrasive
- Labor = some monitoring needed
- Machine = includes machine, financing, installation, maintenance
Aluminum Extrusion

- Extrusion die = $5,000
- Extrusion rate, less than 1 second to extrude one part
- Equipment rate ~ $40/hr
- Labor rate ~ $50/hr
- Materials billet aluminum,
  some start up loss, kerf, say 30g + 1g waste => $0.31
- Setup/cleanup times dominates Equip and Labor
- Sawing, post processing
- V = $0.31 + $0.09
- Cost = $5,000 + $0.40N
Nest Parts for Waterjet, waste
~30%
Waterjet cutting

• **Variable Costs:**
  • Aluminum bar stock ~ $ 5/lb (~$10/kg)
  • 30 gram part + waste = 40g/part = $0.40/part
  • Run time ~ 90 sec
  • Abrasive ~ $0.30/part
  • Machine cost (~$100k/5years/1shift (1000h/y) + maintenance) ~ $25/hr
  • Labor cost ~ $35/hr

• **Cost = $2.2N**
Waterjet cutting

• **Variable Cost Breakdown:**

<table>
<thead>
<tr>
<th>Variable Cost</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$0.4</td>
</tr>
<tr>
<td>Abrasive</td>
<td>$0.3</td>
</tr>
<tr>
<td>Labor</td>
<td>$0.875</td>
</tr>
<tr>
<td>Machine</td>
<td>$0.625</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$2.2</strong></td>
</tr>
</tbody>
</table>
Estimated breakeven

N* ~ 3,000 parts
Breakeven cost $2.20/part
Big Blue Saw quote

Material: Aluminum 6061, 0.375 inches thick. Overall dimensions: 1.063 inches × 6.000 inches (152.40 mm × 27.01 mm). See part details.

<table>
<thead>
<tr>
<th>Parts made by waterjet machining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>----------</td>
</tr>
</tbody>
</table>

Cost/Part: $11/part

Our estimate: $2.20/part

Number of Parts

Figure 2: Composition of cost element for a product cost estimate.

Fig. 75 Cost break down per part for injection molding, Stereolithography, FDM and SLS [139]
Additive Mfg Vs Injection Mold

Figure 8: Total energy use per part versus production volume for SLS and IM of the paintball handle

Ref. Cassandra Telenko et al
Components of Indirect Cost:

Indirect Costs
• Support activities that can be charged “evenly” to all products i.e. “overhead” may include things like; design, programming, maintenance, quality control, purchasing, marketing, sales, general and administrative expenses…

e.g. Composites parts may require more inspection than metal parts
Indirect costs..

- Become more important for higher levels of automation,
- Become more difficult to allocate as the number of products and variation grows.
- Use “Activity Based Costing” and other tools
Cincinnati MAXIM FMS Cell

(5) Machines
(24) Pallets
(3) Shifts
Part Types/ Total Produced
System H; (2,000/35,000)

System 2; (20/1,500)

System 3; (200/ 10,600)
Cost Models

- **Time & Motion**: F.W. Taylor, Ostwald, Polgar
- **DFM and DFA**: Boothroyd, Dewhurst & Knight (Parametric Models)
- **Software & On-Line Quotes**
DFM guidelines are helpful, they identify important features that drive cost:

Examples:
- Handling and insertion difficulty for manual assembly
- Injection mold tooling complexity
Example: Time Estimation for Manual Assembly

- Handling
  - pick up
  - orient

- Insertion
  - location (obstructed view? Self locating?)
  - hold down and resistance
  - securing method
Handling Issues

Symmetry

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>180</th>
<th>180</th>
<th>90</th>
<th>360</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>180</td>
<td>180</td>
<td>90</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>180</td>
<td>0</td>
<td>360</td>
</tr>
</tbody>
</table>

Size

Fragile/Sharp

Nest/Tangle

Slippery/Flexible
B-D Manual handling chart

Handling difficulties: nest, tangle slippery, sharp...

<table>
<thead>
<tr>
<th>Manual Handling — Estimated Times (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>parts are easy to grasp and manipulate</td>
</tr>
<tr>
<td>thickness &gt; 2 mm</td>
</tr>
<tr>
<td>size &gt; 15 mm</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
Insertion Issues

Alignment

self-locating

holding down and alignment required for subsequent operation

Obstructed Access/View

Insertion Force

difficult to insert

easy to insert

part can hang-up

part falls into place

part must be released before it is located

part located before release
## B-D Manual Insertion Chart

**Manual Insertion — Estimated Times (seconds)**

<table>
<thead>
<tr>
<th>Description</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy to align and position during assembly (4)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Not easy to align or position during assembly</td>
<td>5.5</td>
<td>6.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

**Key:**
- **PART ADDED but NOT SECURED**
- **PART SECURED IMMEDIATELY**

---

**Screws**

**Plastic Deformation Immediately After Insertion**

<table>
<thead>
<tr>
<th>Description</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy to align and position during assembly (4)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Not easy to align or position during assembly</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

**Screw Tightening Immediately After Insertion (6)**

<table>
<thead>
<tr>
<th>Description</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
<th>Resistance to Insertion (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy to align and position during assembly (4)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Not easy to align or position during assembly</td>
<td>5.5</td>
<td>6.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

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**Obstructed View**

**Part and Associated Tool (Including Hands):**
- Can easily reach the desired location
- Due to obstructed access or restricted vision (2)
1 – screw(2) (steel) not easy to align

2 – cover(steel) not easy to align – assembly worker’s fingers must be used to align edges

3 – spring(steel) (closed ends) subject to continuous cycling and must be spring steel

4 – piston stop(plastic) edge is chamfered for ease of alignment

5 – piston(aluminum) obstructed access for insertion of spindle into bottom of bore

6 – main block(plastic) depth of bore is 28mm with small through hole for piston spindle
B-D Manual handling chart

### Manual Handling - Estimated Times (seconds)

<table>
<thead>
<tr>
<th>Parts are easy to grasp and manipulate</th>
<th>Parts present handling difficulties (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness &gt; 2 mm</td>
<td>thickness ≤ 2 mm</td>
</tr>
<tr>
<td>size &gt; 15 mm</td>
<td>size ≤ 15 mm</td>
</tr>
<tr>
<td>size &lt; 6 mm</td>
<td>size ≥ 6 mm</td>
</tr>
<tr>
<td>size ≤ 6 mm</td>
<td>size &gt; 15 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key:</th>
<th>ONE HAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>(α + β) &lt; 360°</td>
<td></td>
</tr>
<tr>
<td>360° ≤ (α + β) &lt; 540°</td>
<td></td>
</tr>
<tr>
<td>540° ≤ (α + β) &lt; 720°</td>
<td></td>
</tr>
<tr>
<td>(α + β) = 720°</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston screws</td>
<td>1.13</td>
<td>1.43</td>
<td>1.88</td>
<td>1.69</td>
<td>2.18</td>
<td>1.84</td>
<td>2.17</td>
<td>2.65</td>
<td>2.45</td>
<td>2.98</td>
</tr>
<tr>
<td>Piston cover</td>
<td>1.5</td>
<td>1.8</td>
<td>2.25</td>
<td>2.06</td>
<td>2.55</td>
<td>2.25</td>
<td>2.57</td>
<td>3.06</td>
<td>3</td>
<td>3.38</td>
</tr>
<tr>
<td>Piston spring</td>
<td>1.8</td>
<td>2.1</td>
<td>2.55</td>
<td>2.36</td>
<td>2.85</td>
<td>2.57</td>
<td>2.9</td>
<td>3.38</td>
<td>3.18</td>
<td>3.7</td>
</tr>
<tr>
<td>Piston screws</td>
<td>1.95</td>
<td>2.25</td>
<td>2.7</td>
<td>2.51</td>
<td>3</td>
<td>2.73</td>
<td>3.06</td>
<td>3.55</td>
<td>3.34</td>
<td>4</td>
</tr>
</tbody>
</table>
B-D Manual insertion chart A

MANUAL INSERTION — ESTIMATED TIMES (seconds)

<table>
<thead>
<tr>
<th>Spring</th>
<th>Piston</th>
<th>Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key:</th>
<th>PART ADDED but NOT SECURED</th>
<th>PART ADDED but NOT SECURED</th>
<th>PART ADDED but NOT SECURED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
<td>2.5</td>
<td>6.5</td>
</tr>
<tr>
<td>1</td>
<td>4.0</td>
<td>5.0</td>
<td>6.5</td>
</tr>
<tr>
<td>2</td>
<td>5.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Addition of any part (1) where neither the part itself, nor any other part, is finally secured immediately.

Part and associated tool (including hands) cannot easily reach the desired location.

Due to obstructed access or restricted vision (2).
B-D Manual insertion chart B

<table>
<thead>
<tr>
<th>螺丝</th>
<th>塑料变形立即后插入 (无螺丝或塑料变形)</th>
<th>塑料弯曲或扭</th>
<th>铆钉或其他类似操作</th>
<th>拧紧螺母紧后插入 (6)</th>
</tr>
</thead>
</table>
|     | 否 | 是 | 否 | 是 | 是 | 否 | 是 | 是 | 100%
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
| 3 | 2 | 5 | 4 | 5 | 6 | 7 | 8 | 9 | 6
| 4 | 4.5 | 7.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 8.5
| 5 | 6 | 9 | 8 | 9 | 10 | 11 | 12 | 13 | 10

part and associated tool (including hands) cannot easily reach the desired location and tool can be operated easily due to obstructed access or restricted vision (2)

part and associated tool (including hands) cannot easily reach the desired location or tool cannot be operated easily due to obstructed access or restricted vision (2)
# Pneumatic Piston Sub-Assembly

<table>
<thead>
<tr>
<th>Part I.D. No.</th>
<th>number of times the operation is carried out consecutively</th>
<th>two-digit manual handling code</th>
<th>manual handling time per part</th>
<th>two-digit manual insertion code</th>
<th>manual insertion time per part</th>
<th>operation time, seconds</th>
<th>operation cost, cents 0.4*(7)</th>
<th>figures for estimation of theoretical minimum parts</th>
<th>Name of Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>30</td>
<td>1.95</td>
<td>00</td>
<td>1.50</td>
<td>3.45</td>
<td>1.38</td>
<td>1</td>
<td>MAIN BLOCK</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10</td>
<td>1.50</td>
<td>10</td>
<td>4.00</td>
<td>5.50</td>
<td>2.20</td>
<td>1</td>
<td>PISTON</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>10</td>
<td>1.50</td>
<td>00</td>
<td>1.50</td>
<td>3.00</td>
<td>1.20</td>
<td>1</td>
<td>PISTON STOP</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>05</td>
<td>1.84</td>
<td>00</td>
<td>1.50</td>
<td>3.34</td>
<td>1.34</td>
<td>1</td>
<td>SPRING</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>23</td>
<td>2.36</td>
<td>08</td>
<td>6.50</td>
<td>8.86</td>
<td>3.54</td>
<td>0</td>
<td>COVER</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>11</td>
<td>1.80</td>
<td>39</td>
<td>8.00</td>
<td>19.60</td>
<td>7.84</td>
<td>0</td>
<td>SCREW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TM</th>
<th>CM</th>
<th>NM</th>
<th>design efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.75</td>
<td>17.50</td>
<td>4</td>
<td>((3\times NM)/TM)</td>
</tr>
</tbody>
</table>


Boothroyd/Dewhurst Design Rules

1. Reduce part count and part types
2. Strive to eliminate adjustments
3. Design parts to be self-aligning and self-locating
4. Ensure adequate access and unrestricted vision
5. Ensure the ease of handling of parts from bulk
6. Minimize the need for reorientations during assembly
7. Design parts that cannot be installed incorrectly
8. Maximize part symmetry if possible or make parts obviously asymmetrical
Rules to reduce part count

1. During operation of the product, does the part move relative to all other parts already assembled?
   Only gross motion should be considered – small motions that can be accommodated by elastic hinges, for example, are not sufficient for a positive answer

2. Must the part be of a different material than or be isolated from all other parts already assembled?
   Only fundamental reasons concerned with material properties are acceptable

3. Must the part be separate from all other parts already assembled because otherwise necessary assembly or disassembly of other separate parts would be impossible?
Redesign:
Pneumatic Piston Sub-Assembly

1 - snap on cover and stop (plastic)
2 - spring (steel)
3 - piston (aluminum)
4 - main block (plastic)
Re-design

Redesigns need to be thoroughly checked that they do not compromise the functioning of the device!
Process Rate Limits

• Machining — limits on (MRR), large forces…

• Injection Molding — die development, heat transfer

• Sheet metal forming — die development, part complexity of CNC machines, multiple presses

• Assembly — time between steps, out of tolerance
Nominal Mfg Process Rates Vary by more than 8 orders

More on this later
\[ P = P_{aux} + k \dot{m} \]

\[ \frac{P}{\dot{m}} = \frac{E}{m} = \frac{P_{aux}}{\dot{m}} + k \]
The consequences of slow machines

- Bottlenecks
- Large capital investment

String ribbon process for PV

Metal Additive Part
- AM-machine utilization: 4500 h/year
- Depreciation time: 5 years
- Investment costs: 500,000€
- Costs for maintenance: 21,666 €/Year
- Build rate: 6.3 cm³/h
- Build Material: Stainless Steel 316L
- Material Price: 89 €/kg
- Part Volume: 1 cm³
- Layer thickness: 0.3 μm

Analyzing Product Lifecycle Costs for a Better Understanding of Cost Drivers in Additive Manufacturing
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*Direct Manufacturing Research Center (DMRC) and Chair of Computer Application and Integration in Design and Planning (C.I.K.), The University of Paderborn, Mersenweg 3, 33098 Paderborn, Germany
REVIEWED, Accepted August 20, 2012

Abstract

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**Figure 4:** Sample Part from Augsburg and the given cost breakdown structure by a batch size of 190 parts per build

<table>
<thead>
<tr>
<th>Costs</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs for Preparation</td>
<td>4.25%</td>
</tr>
<tr>
<td>Building process_fix</td>
<td>2.62%</td>
</tr>
<tr>
<td>Oven</td>
<td>1.37%</td>
</tr>
<tr>
<td>Material Costs</td>
<td>11.37%</td>
</tr>
<tr>
<td>Postprocessing</td>
<td>6.65%</td>
</tr>
<tr>
<td>Machine Costs</td>
<td>73.73%</td>
</tr>
</tbody>
</table>
Production Rate

- Process rate - physics
- System rate – waiting for needed resource

Flow efficiency = process time/total time in the system
System Time & Rate

• Oct 30, Dr. Stan Gershwin
  – Little’s Law,
  – unreliable machines,
  – Buffers (zero, infinite and finite)
  – M/M/1 queue
  – Simulations
Quality Issues at the Process

Satisfied customer and meeting target specification, later TPS and hidden problems
Where you find the problem matters

- Product recalls
- Customer complaints
- Warranty claims
- Dealer returns
- In-house inspection
- Assembly
- Observed at the process

Fix problems close to home!
Quality Loss Function

After G Taguchi

Deviations from
The target value
should be penalized
Inspecting in Quality

Distribution of color density in television sets.

Ref M. Phadke
Nominal Process Variation
(process in control)

Surface Roughness

Dimensional Tolerance
Process variation/tolerance

What are the most important variables?

**FIGURE 35.20** Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Process variation/tolerance

**FIGURE 35.20** Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Process variation

**temperature**

**size**

**FIGURE 35.20** Dimensional tolerances as a function of part size for various manufacturing processes; note that because many factors are involved, there is a broad range for tolerances.
Quality Control Actions

\[ Y = \Phi(\alpha + \Delta \alpha, U) \]

Ref Hardt, 2001
How to control variation

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]
Real Time Control

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

- output variation
- sensitivity
- disturbance
- control action

Covered in 2.830, D. Hardt
Designed Experiments

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

- output variation
- sensitivity
- disturbance
- control action

![Diagram showing output feature vs controllable parameter]
Statistical Process Control

\[ \Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \]

- **output variation**
- **sensitivity**
- **disturbance**
- **control action**
Mean drift, and variance of the output $Y$

Mean on target, but large variation due to many random effects

Mean drift has assignable cause, tight grouping means small variation
Comparing the variation with the specifications

Goals: $6\sigma < (\text{USL}-\text{LSL})$ and mean centered
If UCL-LCL = 6σ and the process mean is in the center, then
The out of compliance parts are given by
2(0.500 - φ(3σ)) =
2(0.500 - 0.4987) =
0.0026 or 0.26% or 2600ppm
Some propose a process capability index $C_p$ that compares the tolerance interval USL-LSL vs the process variation $6\sigma$.

$$C_p = \frac{USL - LSL}{6\sigma}$$

<table>
<thead>
<tr>
<th>$C_p$</th>
<th>% out</th>
<th>ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3}$</td>
<td>4.55</td>
<td>45,500</td>
</tr>
<tr>
<td>1</td>
<td>0.26</td>
<td>2600</td>
</tr>
<tr>
<td>$1 \frac{1}{3}$</td>
<td>.0063</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>.00034</td>
<td>3.4</td>
</tr>
</tbody>
</table>
Mean Drift

Normal distribution $N(0, 1)$

<table>
<thead>
<tr>
<th>Specification Limit</th>
<th>Target</th>
<th>Upper Specification Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.135%</td>
<td>13.6</td>
<td>+2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+0.135</td>
</tr>
<tr>
<td></td>
<td>15.835</td>
<td>+0.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.97%</td>
</tr>
</tbody>
</table>
“Not In-Control”

The Parent Distribution Changes with Time

2.810 Manufacturing Processes and Systems - Process Control

Slide taken from D. Hardt
Statistical Control Methods

**Strategy:**

1. Monitor process when under control
2. Determine **Centerline & Control Limits**
3. Data outside of UCL/LCL indicates change
4. Investigate and eliminate causes of change
SPC chart showing “Not-in-Control” condition

- X bar chart

Average value \( x \)

Sampling period

Upper Control Limit

Centerline

Lower Control Limit

Read 1.5-1 Hogg & Ledolter, “Control Charts”
Histogram for CNC Turning

From Dave Hardt
Sampling Frequency

Factors that determine the appropriate sampling frequency:

– Stability of process

– Potential loss

– Cost of sampling inspection
What causes variation in dimensions?

• Machine variation
  – e.g. change in settings, environment, equipment
• Material variation
  – e.g. suppliers, substitutes, mixtures..
• Operator variation
  – Jim instead of Joe, or Alice instead of Mary
• Method variation
  – Late Friday afternoon, Mary always does it this way…
“x-bar charts”
Mean of the means

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}
\]

\[
\bar{x} = \frac{1}{k} \sum_{j=1}^{k} \bar{x}_j
\]

\[
UCL, LCL = \bar{x} \pm A_2 \bar{R}
\]

Also see alternative method
Presented by Dave Hardt
"R-charts" Range = high - low

- Standard Deviation can be estimated from R

\[ R = \max(x_i, K, x_n) - \min(x_i, K, x_n) \]

\[ \bar{R} = \frac{1}{k} \sum_{j=1}^{k} R_j \]

\[ LCL = D_3 \bar{R} \]

\[ UCL = D_4 \bar{R} \]

Where,  
- \( n \) = sample size  
- \( k \) = number of samples  
- \( D_3, D_4 \) = constants from Table C.1
Estimate of standard deviation from range ref. P. Lyonnet

Estimate for \( m \), and if \( W \) is the range or spread of values in the sample, i.e. the difference between the greatest and least values, an estimate for \( \sigma \) is \( W/d_n \), where \( n \) is the number of items in the sample and \( d_n \) is a known function (Table 3.3 gives values of \( d_n \)).

<table>
<thead>
<tr>
<th>Size of each sample</th>
<th>1/( d_n )</th>
<th>( d_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.886</td>
<td>1.128</td>
</tr>
<tr>
<td>3</td>
<td>0.591</td>
<td>1.693</td>
</tr>
<tr>
<td>4</td>
<td>0.486</td>
<td>2.059</td>
</tr>
<tr>
<td>5</td>
<td>0.430</td>
<td>2.326</td>
</tr>
<tr>
<td>6</td>
<td>0.395</td>
<td>2.534</td>
</tr>
<tr>
<td>7</td>
<td>0.370</td>
<td>2.704</td>
</tr>
<tr>
<td>8</td>
<td>0.351</td>
<td>2.847</td>
</tr>
<tr>
<td>9</td>
<td>0.337</td>
<td>2.970</td>
</tr>
<tr>
<td>10</td>
<td>0.325</td>
<td>3.078</td>
</tr>
<tr>
<td>11</td>
<td>0.315</td>
<td>3.173</td>
</tr>
<tr>
<td>12</td>
<td>0.307</td>
<td>3.258</td>
</tr>
</tbody>
</table>

Here he used \( W \) for range \( R \), but in general, \( \bar{s} \) is a better estimate of std dev. \( \sigma \). This leads to \( UCL, LCL = \bar{x} \pm A_3 \bar{s} \).
<table>
<thead>
<tr>
<th>Number of Observations in Sample, $n$</th>
<th>$A_3$</th>
<th>$A_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.66</td>
<td>1.88</td>
</tr>
<tr>
<td>3</td>
<td>1.95</td>
<td>1.02</td>
</tr>
<tr>
<td>4</td>
<td>1.63</td>
<td>0.73</td>
</tr>
<tr>
<td>5</td>
<td>1.43</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>1.29</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>1.18</td>
<td>0.37</td>
</tr>
<tr>
<td>8</td>
<td>1.10</td>
<td>0.34</td>
</tr>
<tr>
<td>9</td>
<td>1.03</td>
<td>0.31</td>
</tr>
<tr>
<td>10</td>
<td>0.98</td>
<td>0.29</td>
</tr>
<tr>
<td>11</td>
<td>0.93</td>
<td>0.27</td>
</tr>
<tr>
<td>12</td>
<td>0.85</td>
<td>0.25</td>
</tr>
<tr>
<td>13</td>
<td>0.82</td>
<td>0.24</td>
</tr>
<tr>
<td>14</td>
<td>0.79</td>
<td>0.22</td>
</tr>
<tr>
<td>15</td>
<td>0.76</td>
<td>0.21</td>
</tr>
<tr>
<td>16</td>
<td>0.74</td>
<td>0.20</td>
</tr>
<tr>
<td>17</td>
<td>0.72</td>
<td>0.19</td>
</tr>
<tr>
<td>18</td>
<td>0.70</td>
<td>0.19</td>
</tr>
<tr>
<td>19</td>
<td>0.68</td>
<td>0.18</td>
</tr>
<tr>
<td>20</td>
<td>0.66</td>
<td>0.18</td>
</tr>
</tbody>
</table>

$D_4$ and $D_3$ values are shown in the table:

- $D_4$ values: 3.27, 2.57, 2.28, 2.11, 2.00, 1.92, 1.86, 1.78, 1.74, 1.67, 1.65, 1.64, 1.62, 1.61, 1.60, 1.59
- $D_3$ values: 0, 0, 0, 0, 0, 0.08, 0.14, 0.22, 0.26, 0.31, 0.35, 0.38, 0.40, 0.41, 0.41

### Tables of Constants for Control charts

**Table 8A - Variable Data**  
ref: AIAG manual for SPC

<table>
<thead>
<tr>
<th>Subgroup size (n)</th>
<th>Control Limits Factor</th>
<th>Chart for Averages</th>
<th>Chart for Ranges (R)</th>
<th>Control Limits Factor</th>
<th>Chart for Averages</th>
<th>Chart for Standard Deviation (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d_2</td>
<td>D_3</td>
<td>D_4</td>
<td>A_2</td>
<td>D_4</td>
<td>A_3</td>
</tr>
<tr>
<td>2</td>
<td>1.880</td>
<td>1.128</td>
<td>-</td>
<td>3.267</td>
<td>2.659</td>
<td>0.7979</td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td>1.693</td>
<td>-</td>
<td>2.574</td>
<td>1.954</td>
<td>0.8862</td>
</tr>
<tr>
<td>4</td>
<td>0.729</td>
<td>2.059</td>
<td>-</td>
<td>2.282</td>
<td>1.628</td>
<td>0.9213</td>
</tr>
<tr>
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<td>0.577</td>
<td>2.326</td>
<td>-</td>
<td>2.114</td>
<td>1.427</td>
<td>0.9400</td>
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<tr>
<td>6</td>
<td>0.483</td>
<td>2.534</td>
<td>-</td>
<td>2.004</td>
<td>1.287</td>
<td>0.9515</td>
</tr>
<tr>
<td>7</td>
<td>0.419</td>
<td>2.704</td>
<td>0.076</td>
<td>1.924</td>
<td>1.182</td>
<td>0.9594</td>
</tr>
<tr>
<td>8</td>
<td>0.373</td>
<td>2.847</td>
<td>0.136</td>
<td>1.864</td>
<td>1.099</td>
<td>0.9650</td>
</tr>
<tr>
<td>9</td>
<td>0.337</td>
<td>2.970</td>
<td>0.184</td>
<td>1.816</td>
<td>1.032</td>
<td>0.9693</td>
</tr>
<tr>
<td>10</td>
<td>0.308</td>
<td>3.078</td>
<td>0.223</td>
<td>1.777</td>
<td>0.975</td>
<td>0.9727</td>
</tr>
<tr>
<td>15</td>
<td>0.223</td>
<td>3.472</td>
<td>0.347</td>
<td>1.653</td>
<td>0.789</td>
<td>0.9823</td>
</tr>
<tr>
<td>25</td>
<td>0.153</td>
<td>3.931</td>
<td>0.459</td>
<td>1.541</td>
<td>0.606</td>
<td>0.9896</td>
</tr>
</tbody>
</table>

### Control Chart Equations

**X bar and R Charts**

- Centerline: $CL_X = \bar{X}$
- Control Limits:
  - $UCL_X = \bar{X} + A_2 \bar{R}$
  - $LCL_X = \bar{X} - A_2 \bar{R}$

- $CL_R = \bar{R}$
- $UCL_R = D_4 \bar{R}$
- $LCL_R = D_3 \bar{R}$

**X bar and s Charts**

- Centerline: $CL_s = \bar{s}$
- Control Limits:
  - $UCL_s = B_4 \bar{s}$
  - $LCL_s = B_3 \bar{s}$

- $CL_X = \bar{X}$
- $UCL_X = \bar{X} + A_3 \bar{s}$
- $LCL_X = \bar{X} - A_3 \bar{s}$

### Equations for Control Limits

- $\sigma_X = \frac{\bar{R}}{d_2}$
- $\sigma_s = \frac{\bar{s}}{c_4}$
Robustness Vs Sensitivity

• Mean shift by $1\sigma$ requires on average 44 data points to observe

• Alternatives
  – $\pm 2\sigma$; two consecutive observations
  – $\pm 1\sigma$; three consecutive observations
  – other charts (e.g. “Cusum”)
Finding the cause of a disturbance is the most difficult part of quality control. There are only aids to help you with this problem solving exercise like the Ishikawa Diagrams which helps you cover all categories, and the “5 Whys” which helps you go to the root cause.
5.3.1 Cause Listing Diagram

In a cause listing diagram, the problem is put at the far right side of the page, as shown in Fig. 5.1.

![Cause Listing Diagram](image)

**Figure 5.1** Example of a cause listing diagram for voids in a product

![Variation Analysis Diagram](image)

**Figure 5.2** Example of variation analysis diagram for dimensional variation
References

• Boothroyd & Dewhurst DFA (manual)
• Control Charts (sect 1.5-1), Hogg & Ledolter
• also see your text, topics are listed in the index. For the 7th edition: SPC section 36.8, Cost section 40.10, and Energy sections 40.5 &40.6 these topics are also listed in other places in the text
• “hockey stick diagram” for energy and rate, Gutowski et al, See “electrical energy used in manufacturing processes” p 1586-1588 your text