Sheet Metal Forming

2.810
T. Gutowski

- “Sheet Metal Forming” Ch. 16 Kalpakjian
- “Design for Sheetmetal Working”, Ch. 9 Boothroyd, Dewhurst and Knight
Outline

• Examples
  • LMP Shop, Stamping, Stretch forming, hydro-forming, super plastic

• Basic Mechanics
  • Spring back
  • Forming limit diagrams

• Appendix
  • Plastic behavior of metals
  • Spring-back derivation
  • Developing forming technologies
LMP Shop

Brake press

Finger brake
Shearing

\[ F = 0.7 \times T \times L \text{ (UTS)} \]

\( T = \text{Sheet Thickness} \)
\( L = \text{Total length Sheared} \)
\( \text{UTS} = \text{Ultimate Tensile Strength of material} \)

Shear press - LMP Shop
Examples - sheet metal formed
Stamping Auto body panels

- 3 to 5 dies each
- Prototype dies ~ $50,000
- Production dies ~ $0.75-1M

- Forming dies
- Trimming station
- Flanging station
~ 90 million vehicles produced worldwide every year
Deep Drawing of beverage cans
Deep Drawing of drinks cans

Hosford and Duncan (can making): http://www.chymist.com/Aluminum%20can.pdf
Stretch forming: Forming force

\[ F = \frac{(Y_s + UTS)}{2} \times A \]

- \( F \): stretch forming force (lbs)
- \( Y_s \): material yield strength (psi)
- \( UTS \): ultimate tensile strength of the material (psi)
- \( A \): Cross-sectional area of the workpiece (in²)
**Stretch forming** – very cheap tooling, net thinning, slow, low formability, sheet metal up to 15mx9m

* source: http://www.cyrilbath.com/sheet_process.html

**Low volume batches**
Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

Low volume batches
Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

Low volume batches
Super-plastic forming – cheap tooling, net thinning, slow, expensive sheet metal, very high formability

Low volume batches, 0.5-0.75 melting temp
Bending & Spring back

\[ \Delta L = (L - L_0) = (\rho + y) \theta - \rho \theta = y \theta \]

\[ \varepsilon = \frac{\Delta L}{L_0} = \frac{y \theta}{\rho \theta} = \frac{y}{\rho} \]

\[ \varepsilon_{\text{max}} = \frac{w/2}{R + h/2} = \frac{1}{\frac{2R}{W} + 1} \]

Figure Coordinate system for analysis of bending.
Stress distribution through the thickness of the part

Elastic

Elastic-plastic

Fully plastic

Fully Plastic Moment, \( M = Y \left( bh/2 \right) \left( h/2 \right) = Yb h^2/4 \)
Balance external and internal moments

Fully plastic

\[ Ybh^2/4 = FL/4 = M_{\text{max}} \]

\[ F = bh^2Y/L \]
Strain:

\[ \varepsilon = \frac{l - l_0}{l_0} \]

\[ l_0 = \rho \theta \]

\[ l = (\rho + y) \theta \]

\[ \therefore \varepsilon = \frac{y}{\rho} \quad (1) \]
Stress

assumed

Elastic  Fully Plastic

\[ M = \int \sigma dA \cdot y \quad \text{(2)} \]

\[ N = 2 \int E \frac{y}{\rho} \cdot b \, dy \cdot y = E \frac{b h^3}{\rho 12} = \frac{E I}{\rho} \quad \text{(3)} \]
Moment - Curvature Diagram

For a fully plastic moment,

\[ M_{\text{max}} = 2 \int_{0}^{h/2} Y b y \, dy = \frac{Y b h^3}{4} \]
A rough estimate of the spring back:

\[ \frac{1}{R_o} - \frac{1}{R_i} = \frac{3Y}{Eh} \]

The full solution is:

\[ \frac{1}{R_o} - \frac{1}{R_i} = 3 \frac{Y}{Eh} - 4R_o^2 \left( \frac{Y}{Eh} \right)^3 \]

The details are at the end (8 slides)
Bending Moment – Curvature

\[
\frac{1}{R_0 - \frac{1}{R_1}} = 3 \frac{Y}{hE} - 4R_0^2 \left( \frac{Y}{hE} \right)^3
\]
\[ \left[ \frac{1}{R_0} - \frac{1}{R_1} \right] = 3 \frac{Y}{hE} - 4R_0^2 \left( \frac{Y}{hE} \right)^3 \]

<table>
<thead>
<tr>
<th>Ro = 3”</th>
<th>R1</th>
<th>Y/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=1/16”</td>
<td>Ti</td>
<td>~12”</td>
</tr>
<tr>
<td></td>
<td>AL 5052</td>
<td>~4”</td>
</tr>
<tr>
<td></td>
<td>Steel</td>
<td>~3.5”</td>
</tr>
</tbody>
</table>

Thick h = 0.0625”
Thin h = 0.03125”

Al-5052
Y = 23 ksi
E = 10200 ksi
Annealed Al
Y = 13 ksi
E = 10200 ksi
Steel
Y = 36 ksi
E = 29000 ksi
Titanium
Y = 120 ksi
E = 16500 ksi
### Aluminum Alloys

#### TABLE 6.3

<table>
<thead>
<tr>
<th>Alloy (UNS)</th>
<th>Temper</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Yield strength (MPa)</th>
<th>Elongation in 50 mm (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100 (A91100)</td>
<td>O</td>
<td>90</td>
<td>35</td>
<td>35–45</td>
</tr>
<tr>
<td></td>
<td>H14</td>
<td>125</td>
<td>120</td>
<td>9–20</td>
</tr>
<tr>
<td>2024 (A92024)</td>
<td>O</td>
<td>190</td>
<td>75</td>
<td>20–22</td>
</tr>
<tr>
<td></td>
<td>T4</td>
<td>470</td>
<td>325</td>
<td>19–20</td>
</tr>
<tr>
<td>3003 (A93003)</td>
<td>O</td>
<td>110</td>
<td>40</td>
<td>30–40</td>
</tr>
<tr>
<td></td>
<td>H14</td>
<td>150</td>
<td>145</td>
<td>8–16</td>
</tr>
<tr>
<td>5052 (A95052)</td>
<td>O</td>
<td>190</td>
<td>90</td>
<td>25–30</td>
</tr>
<tr>
<td></td>
<td>H34</td>
<td>260</td>
<td>215</td>
<td>10–14</td>
</tr>
<tr>
<td>6061 (A96061)</td>
<td>O</td>
<td>125</td>
<td>55</td>
<td>25–30</td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td>310</td>
<td>275</td>
<td>12–17</td>
</tr>
<tr>
<td>7075 (A97075)</td>
<td>O</td>
<td>230</td>
<td>105</td>
<td>16–17</td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td>570</td>
<td>500</td>
<td>11</td>
</tr>
</tbody>
</table>
Methods to reduce springback

- Smaller Y/E
- Larger thickness
- Over-bending
- Stretch forming
- “coining” or bottoming the punch
Methods to reduce springback

- Smaller Y/E

(a). Original Design (b). Modified Design for Springback Control

Figure 3. An example of springback control through part design
Apply Tension and Bending -

- This limits applicable part geometries

- Advantage of stretch forming
Blank holder force: forming window

Wrinkling

Tearing

Window for forming
Forming Limit Diagrams

Figure 15-8  Strips of varying width are stretched to obtain different $\varepsilon_1/\varepsilon_2$ ratios.

Excluded: $\varepsilon_1$ must be $> \varepsilon_2$
Forming Limit Diagrams

Figure 15-5 Distortion of printed circles near a localized neck and a plot of the strains in the circles. Solid points are for grid circles through which failures occurred, open points are for grid circles removed from failure, and partially filled points are for grid circles very near failure. From S. S. Hecker, Steel Met/Ind., 52 (1970), pp. 671-75.

Figure 15-6 Forming limit diagram for low-carbon steel determined from data like that in Fig. 15-5. The strains below the curve are acceptable while those above the curve correspond to regions affected by local necking. From S. S. Hecker, *ibid.*

Figures from Hosford & Caddell
Tensile instability - necking

\[ \sigma = k \varepsilon^n \]

True: \( \Delta \varepsilon = \frac{dL}{L} \)

Nominal:

\[ \sigma = \frac{F}{A_0}, \quad \varepsilon = \frac{L-L_0}{L_0} \]

neck forms at \( \varepsilon^* \)

Tensile instability (1-D)

\[ F = \sigma A \]

so \( dF = \sigma dA + A d\sigma = 0 \) at max load

\[ \frac{d\sigma}{\sigma} = -\frac{dA}{A} = d\varepsilon \]

\[ \frac{d\sigma}{d\varepsilon} = \sigma \]

With \( \sigma = k \varepsilon^n \):

\[ \frac{d\sigma}{d\varepsilon} = n k \varepsilon^{n-1} = \sigma = k \varepsilon^n \]

\[ \Rightarrow \quad \varepsilon^* = n \]
Process Performance

1. Cost – Dies, Material/ Waste
2. Quality – Spring-back, wrinkling, tearing
3. Rate – Lead time for dies
4. Flexibility - Dies
Sheet Metal Appendix

- Plastic behavior of metals
  - Power law behavior
  - 3D and Yield criterion

- Spring back derivation

- Developing technologies
  - Discrete Dies
  - Incremental Sheet Metal Forming
  - Flexible Spinning
Tensile test – the Stress-strain diagram

\[ \sigma_Y = Y \]

Grips for Holding Specimen Firmly

Fixed Head

Test Specimen Thickness 1/8"

Constant Rate of Motion

Force Measurement

Tensile strength at maximum load

UTS

Yield strength (0.2% offset)

Proportional limit

Load at fracture

Strain in inches/in. of gage length (elongation)
True stress & strain

\[ \varepsilon_{tr} = \ln(1 + \varepsilon_{en}) \]
\[ \sigma_{tr} = \sigma_{en}(1 + \varepsilon_{en}) \]

True stress can be expressed using a power law (Hollomon equation):

\[ \sigma_{tr} = K \varepsilon_{tr}^{n} \]
Power-Law Expression (Hollomon equation)

\[ \sigma_{tr} = K \varepsilon_{tr}^n \]

Can be re-written:

\[ \log(\sigma_{tr}) = n \log(\varepsilon_{tr}) + \log K \]
<table>
<thead>
<tr>
<th>Material</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1100-O</td>
<td>180</td>
<td>0.20</td>
</tr>
<tr>
<td>2024-T4</td>
<td>690</td>
<td>0.16</td>
</tr>
<tr>
<td>5052-O</td>
<td>202</td>
<td>0.13</td>
</tr>
<tr>
<td>6061-O</td>
<td>205</td>
<td>0.20</td>
</tr>
<tr>
<td>6061-T6</td>
<td>410</td>
<td>0.05</td>
</tr>
<tr>
<td>7075-O</td>
<td>400</td>
<td>0.17</td>
</tr>
<tr>
<td>Brass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-30, annealed</td>
<td>900</td>
<td>0.49</td>
</tr>
<tr>
<td>85-15, cold rolled</td>
<td>580</td>
<td>0.34</td>
</tr>
<tr>
<td>Cobalt-based alloy, heat treated</td>
<td>2070</td>
<td>0.50</td>
</tr>
<tr>
<td>Copper, annealed</td>
<td>315</td>
<td>0.54</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-C, annealed</td>
<td>530</td>
<td>0.26</td>
</tr>
<tr>
<td>1020, annealed</td>
<td>745</td>
<td>0.20</td>
</tr>
<tr>
<td>4135, annealed</td>
<td>1015</td>
<td>0.17</td>
</tr>
<tr>
<td>4135, cold rolled</td>
<td>1100</td>
<td>0.14</td>
</tr>
<tr>
<td>4340, annealed</td>
<td>640</td>
<td>0.15</td>
</tr>
<tr>
<td>304 stainless, annealed</td>
<td>1275</td>
<td>0.45</td>
</tr>
<tr>
<td>410 stainless, annealed</td>
<td>960</td>
<td>0.10</td>
</tr>
<tr>
<td>Titanium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4V, annealed, 20°C</td>
<td>1400</td>
<td>0.015</td>
</tr>
<tr>
<td>Ti-6Al-4V, annealed, 200°C</td>
<td>1040</td>
<td>0.026</td>
</tr>
<tr>
<td>Ti-6Al-4V, annealed, 600°C</td>
<td>650</td>
<td>0.064</td>
</tr>
<tr>
<td>Ti-6Al-4V, annealed, 800°C</td>
<td>350</td>
<td>0.146</td>
</tr>
</tbody>
</table>
For any general stress state we can find a set of principal axes. The stress tensor for these axes contains no off-diagonal (shear) terms – only three principal stresses along the three axes.

Mohr’s circle allows rotation of axes in two dimensions about one principal axis.
2.3 TRESCA CRITERION

This criterion postulates that yielding will occur when the largest shear stress reaches a critical value. Whenever possible, the convention $\sigma_1 > \sigma_2 > \sigma_3$ will be used, but there are cases where this relative comparison is not known a priori. In addition, this convention cannot be maintained rigorously when plots in two- or three-dimensional stress space are considered. This criterion predicts yielding when

$$\sigma_{\text{max}} - \sigma_{\text{min}} = C \quad \text{or} \quad \sigma_1 - \sigma_3 = C \quad \text{if} \quad \sigma_1 > \sigma_2 > \sigma_3 \tag{2-5}$$

To evaluate $C$, a state of uniaxial tension may be used. There, $\sigma_{\text{max}} = \sigma_1$, $\sigma_2 = \sigma_3 = 0$, and yielding occurs when $\sigma_1 = Y$, the yield strength in uniaxial tension. Thus,

$$\sigma_1 - \sigma_3 = Y = C \tag{2-6}$$

In the case of pure shear, $\sigma_{\text{max}} = \sigma_1$, $\sigma_{\text{min}} = \sigma_3 = -\sigma_1$, and $\sigma_2 = 0$. Yielding occurs when the maximum shear stress reaches the yield strength in pure shear, i.e., the shear yield strength $k$. At that time, $\sigma_1 = k$, so

$$\sigma_1 - \sigma_3 = 2\sigma_1 = 2k = C \tag{2-7}$$

and the Tresca criterion becomes

$$\sigma_1 - \sigma_3 = Y = 2k \tag{2-8}$$

2.4 VON MISES CRITERION

This postulates that yielding will occur when some value of the root-mean-shear stress reaches a constant or

$$\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{3}\right]^{1/2} = C_1 \tag{2-9}$$

For example,

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 = C_2 \tag{2-10}$$

Again, uniaxial tension may be used to define $C_2$. Substituting $\sigma_1 = Y$ at yielding, and $\sigma_2 = \sigma_3 = 0$, the constant is $2Y^2$. For pure shear, with $\sigma_1 = k = -\sigma_2$ and $\sigma_3 = 0$, $C = 6k^2$, so the von Mises criterion is expressed as

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 = 2Y^2 = 6k^2 \tag{2-11}$$

In a more general form, this criterion can be written as

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 + \left(6\tau_{xx}^2 + \tau_{xy}^2 + \tau_{xz}^2\right) = 2Y^2 = 6k^2 \tag{2-12}$$
Side Note: For a general state of stress use “effective stress”

2-6 EFFECTIVE STRESS

With either yield criterion, it is useful to define an effective stress denoted as \( \tilde{\sigma} \) which is a function of the applied stresses. If the magnitude of \( \tilde{\sigma} \) reaches a critical value, then the applied stress state will cause yielding; in essence, it has reached an effective level. For the von Mises criterion,

\[
\tilde{\sigma} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}
\]  
\[ (2-16) \]

while for the Tresca criterion,

\[
\tilde{\sigma} = \sigma_1 - \sigma_3 \quad \text{where} \quad \sigma_1 > \sigma_2 > \sigma_3
\]
\[ (2-17) \]

Yielding occurs when \( \sigma_{\text{effective}} = Y \)

Material taken from *Metal Forming*, by Hosford and Caddell
Origin of effective strain

2.7 EFFECTIVE STRAIN

Effective strain is defined such that the incremental work per unit volume is

\[ dw = \bar{\sigma} \, d\bar{\varepsilon} = \sigma_1 \, d\varepsilon_1 + \sigma_2 \, d\varepsilon_2 + \sigma_3 \, d\varepsilon_3 \]  

(2-18)

For the von Mises criterion, the effective strain is given by

\[ d\varepsilon = \frac{\sqrt{2}}{3} \left[ (d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_1 - d\varepsilon_3)^2 \right]^{1/2} \]  

(2-19)

which may be expressed in a simpler form as

\[ d\varepsilon = \left[ \frac{2}{3} (d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2) \right]^{1/2} \]  

(2-20)

If the straining is proportional (with a constant ratio of \( d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3 \)), the total effective strain may be expressed in terms of the total strains as

\[ \bar{\varepsilon} = \left[ \frac{2}{3} (\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) \right]^{1/2} \]  

(2-21)

If the strain path is not constant, \( \bar{\varepsilon} \) must be found from a path integral of \( d\varepsilon \). In this case,

\[ \bar{\sigma} = K \bar{\varepsilon}^n \]

Material taken from *Metal Forming*, by Hosford and Caddell
**Elastic Springback Analysis**

1. Assume plane sections remain plane:
   \[ e_y = -\frac{y}{r} \]  

2. Assume elastic-plastic behavior for material
   
   \[ \sigma = E e \quad \text{for} \quad e < e_{\psi} \]
   
   \[ \sigma = \sigma_y \quad \text{for} \quad e \geq e_{\psi} \]
3. We want to construct the following Bending Moment “M” vs. curvature “1/ρ” curve

\[ M = \frac{M_Y}{(1/\rho)_Y} \left[ \frac{1}{\rho} - \frac{1}{R_1} \right] \]

Springback is measured as

\[ \frac{1}{R_0} - \frac{1}{R_1} \]

Permanent set is

\[ \frac{1}{R_1} \]
4. Stress distribution through the thickness of the beam

Elastic

Elastic-plastic

Fully plastic
5. \[ M = \int_A \sigma y \, dA \]

Elastic region

\[ M = \int \sigma y \, dA = -E \int \frac{y^2}{\rho} \, dA = -\frac{EI}{\rho} \]  
(3)

At the onset of plastic behavior

\[ \sigma = -\frac{y}{\rho} E = -\frac{h}{2\rho} E = -Y \]  
(4)

This occurs at

\[ \frac{1}{\rho} = \frac{2Y}{hE} = \frac{1}{\rho_Y} \]  
(5)

Substitution into eqn (3) gives us the moment at onset of yield, \( M_Y \)

\[ M_Y = -\frac{EI}{\rho_Y} = \frac{EI}{2Y/h} = \frac{2IY/h}{h} \]  
(6)

After this point, the \( M \) vs \( 1/r \) curve starts to “bend over.”

Note from \( M=0 \) to \( M=M_Y \) the curve is linear.
In the elastic – plastic region

\[ M = \int_{0}^{h/2} \sigma y b dy = 2 \int_{0}^{y_Y} Y b y dy + 2 \int_{0}^{y_Y} \frac{y}{Y} Y b dy \]

\[ = 2 Y b \frac{y^2}{2} \bigg|_{y_Y}^{h/2} + 2 \frac{Y}{y_Y} b \frac{y^3}{3} \bigg|_{0}^{y_Y} \]

\[ = Y b \left( \frac{h^2}{4} - y_Y^2 \right) + \frac{2}{3} y_Y^2 Y b \]

\[ M = \frac{bh^2}{4} Y \left[ 1 - \frac{1}{3} \left( \frac{y_Y}{h/2} \right)^2 \right] \]  \hspace{1cm} (7) \]

Note at \( y_Y = h/2 \), you get on-set at yield, \( M = M_Y \)
And at \( y_Y = 0 \), you get fully plastic moment, \( M = 3/2 \, M_Y \)
To write this in terms of $M \text{ vs } 1/\rho$ rather than $M \text{ vs } y_Y$, note that the yield curvature $(1/\rho)_Y$ can be written as (see eqn (1))

$$\frac{1}{\rho_Y} = \frac{\varepsilon_Y}{h/2}$$

(8)

Where $\varepsilon_Y$ is the strain at yield. Also since the strain at $y_Y$ is $-\varepsilon_Y$, we can write

$$\frac{1}{\rho} = \frac{\varepsilon_Y}{y_Y}$$

(9)

Combining (8) and (9) gives

$$\frac{y_Y}{h/2} = \frac{(1/\rho)_Y}{1/\rho}$$

(10)
Substitution into (7) gives the result we seek:

\[
M = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{(1/\rho)_Y}{1/\rho} \right)^2 \right]
\]  \hspace{1cm} (11)

\[
M = \frac{M_Y}{(1/\rho)_Y} \left[ \frac{1}{\rho} - \frac{1}{R_1} \right]
\]  \hspace{1cm} (12)
Now, eqn’s (11) and (12) intersect at $1/\rho = 1/R_0$

Hence,

$$\frac{M_Y}{(1/\rho)_Y} \left[ \frac{1}{R_0} - \frac{1}{R_1} \right] = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{(1/\rho)_Y}{1/R_0} \right)^2 \right]$$

Rewriting and using $(1/\rho)_Y = 2Y / hE$ (from a few slides back), we get

$$\left[ \frac{1}{R_0} - \frac{1}{R_1} \right] = \frac{3}{hE} \frac{Y}{R_0^2} - 4R_0^2 \left( \frac{Y}{hE} \right)^3$$

(13)
60 Ton Discrete Die Press (LMP - Hardt)
The Shape Control Concept
Stretch Forming with Reconfigurable Tool @ Northrop Grumman
Flexible Forming at Ford
Conventional Spinning

Flexible Spinning

Energy & cost: Stamping alum car hoods

- Final part = 5.4kgs
- Total number of parts made = 400
- Die material: cast and machined zinc alloy

Energy. 2.3GJ/pt. Stamping alum. car hoods. 5.4kgO/P. (400pts)

Cost. 136USD/pt

Sheet metal scrapped in factory = 44%