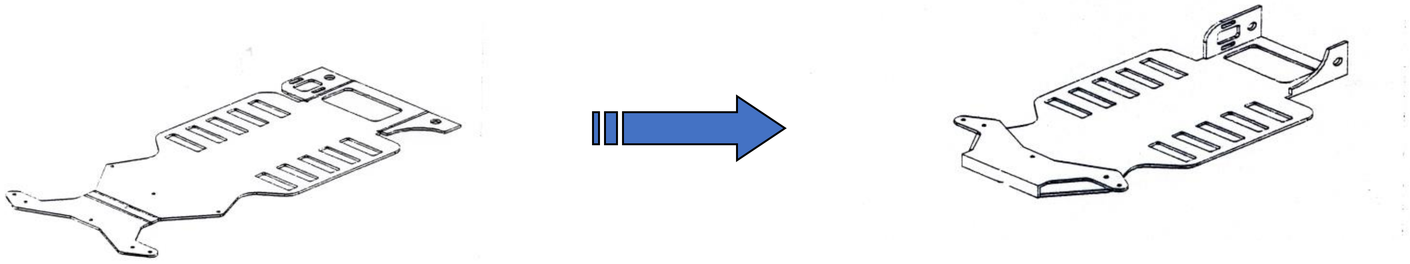


Sheet Metal Forming

2.810

T. Gutowski

- "Sheet Metal Forming" Ch. 16 Kalpakjian
- "Design for Sheetmetal Working",

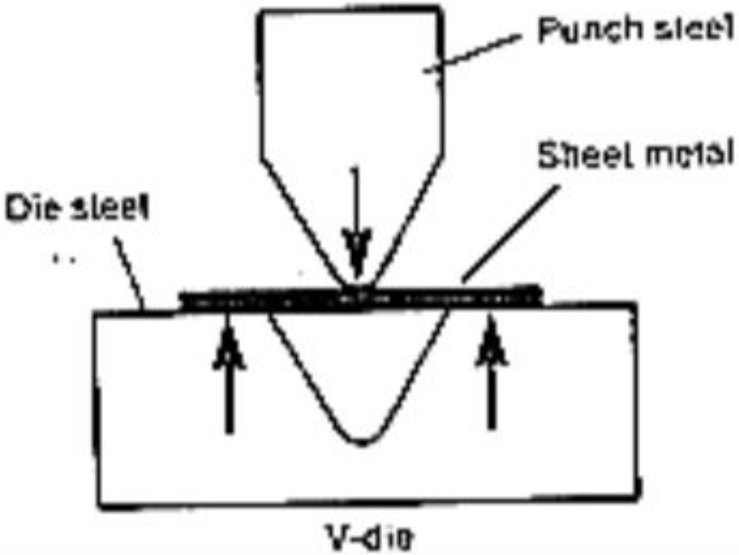
Ch. 9 Boothroyd, Dewhurst and Knight

Outline

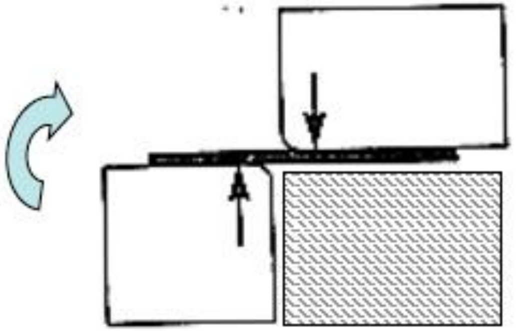
- Examples
  - LMP Shop, Stamping, Stretch forming, hydro-forming, super plastic
- Basic Mechanics
  - Spring back
  - Forming limit diagrams
- Appendix
  - Plastic behavior of metals
  - Spring-back derivation
  - Developing forming technologies

LMP Shop

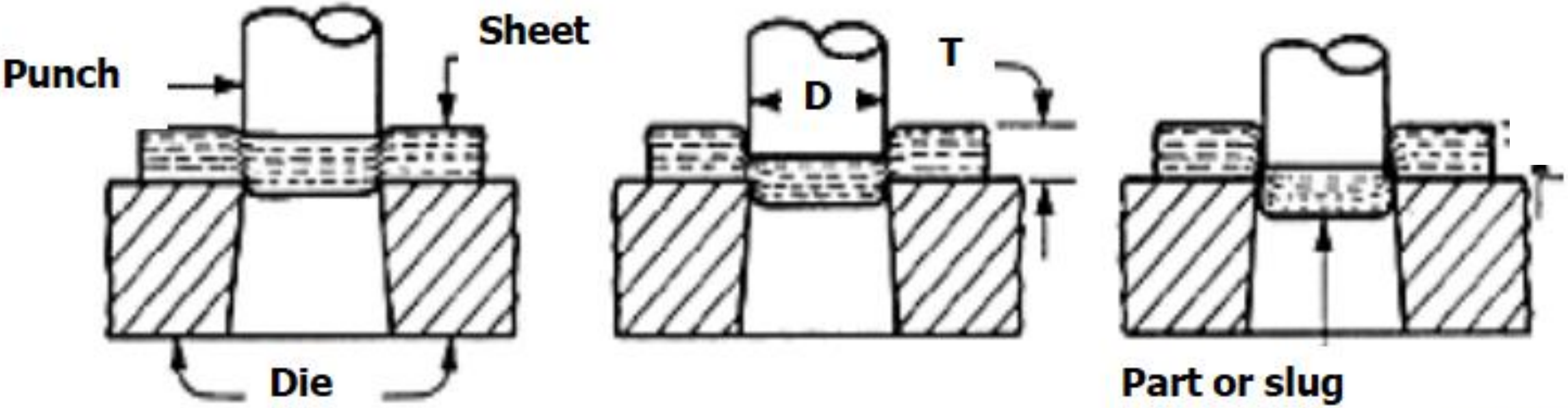
Brake press



Finger brake



Shearing

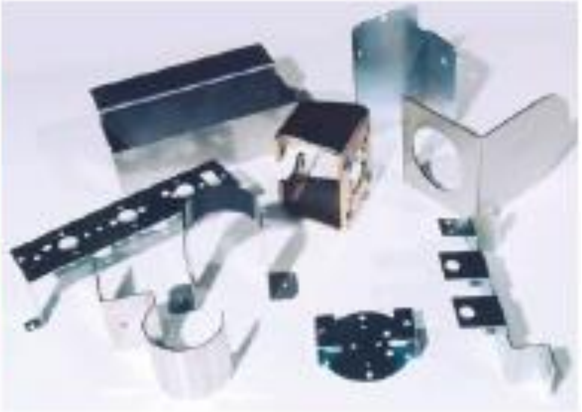


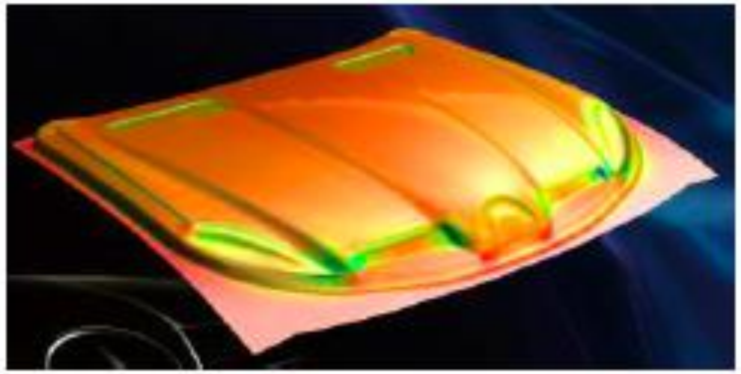
$F = 0.7\ T\ L\ (UTS)$

T = Sheet Thickness
L = Total length Sheared
UTS = Ultimate Tensile Strength of material

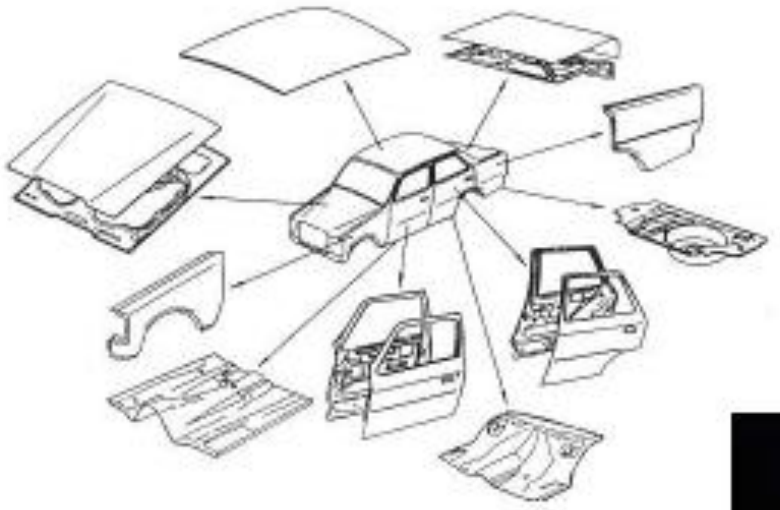
Shear press - LMP Shop

Examples-sheet metal formed



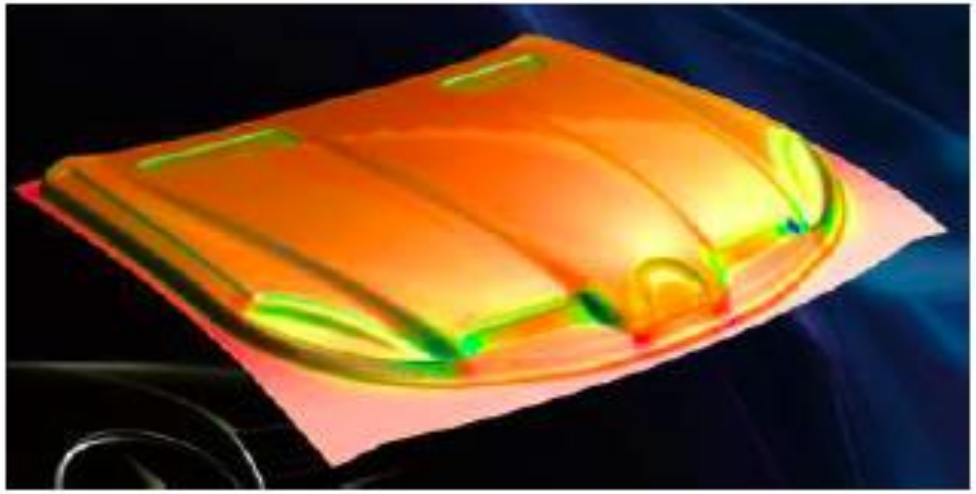


Stamping Auto body panels



- 3 to 5 dies each
- Prototype dies ~ $50,000
- Production dies ~ $0.75-1 M

- Forming dies
- Trimming station
- Flanging station



~ 90 million vehicles produced worldwide every year

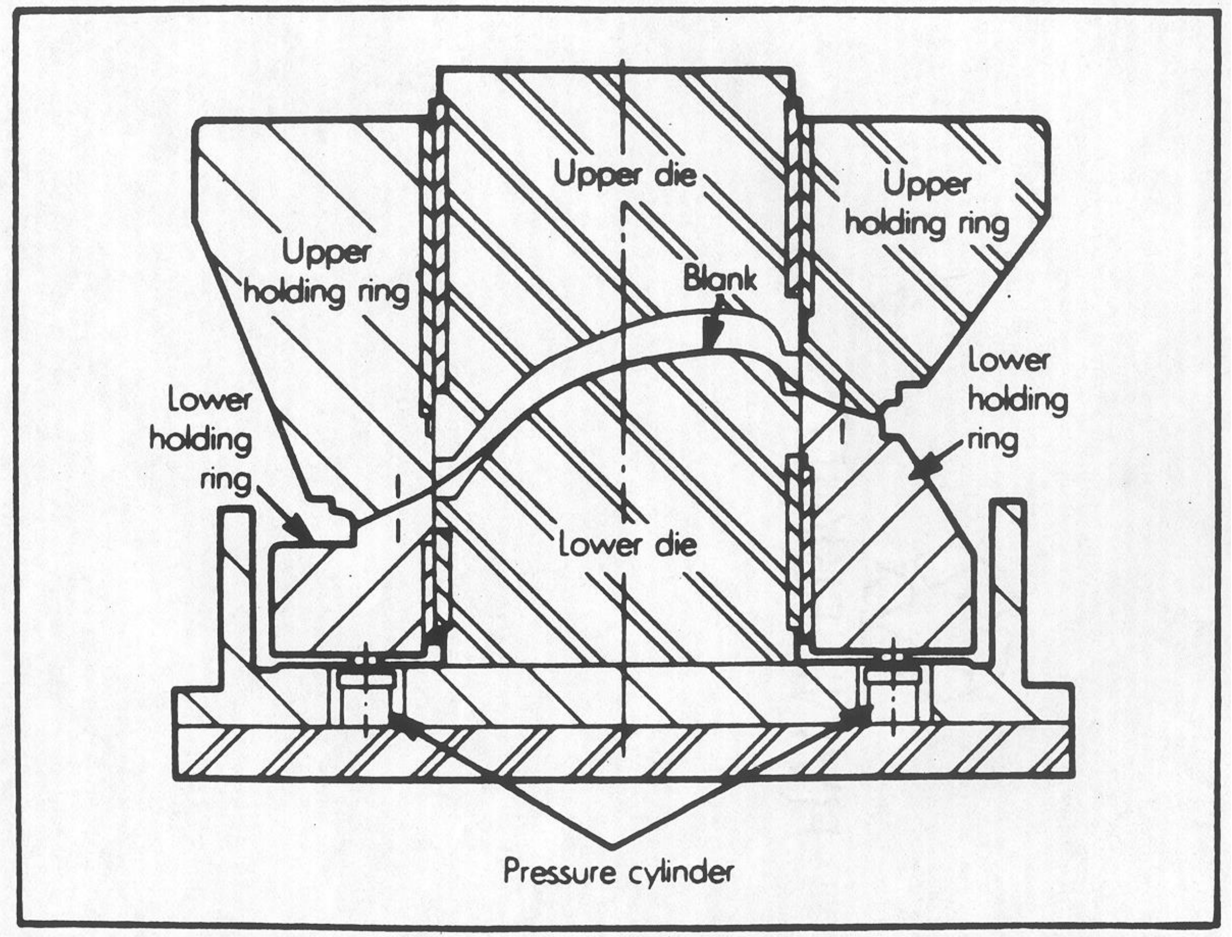
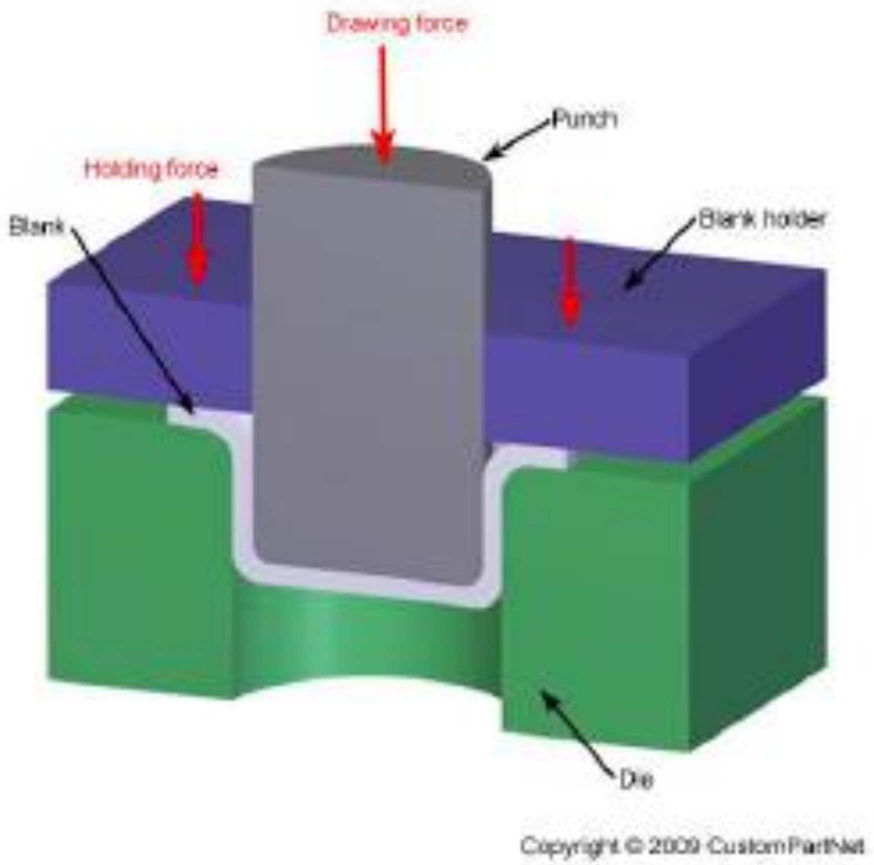


Fig. 7-23 Tooling for stretch-draw forming fenders from steel blanks. *(Oldsmobile Div., General Motors Corp.)*

http://www.thomasnet.com/articles/custom-manufacturing-fabricating/wrinkling-during-deep-drawing

Deep Drawing of beverage cans



Deep Drawing of drinks cans

Hosford and Duncan (can making): http://www.chymist.com/Aluminum%20can.pdf

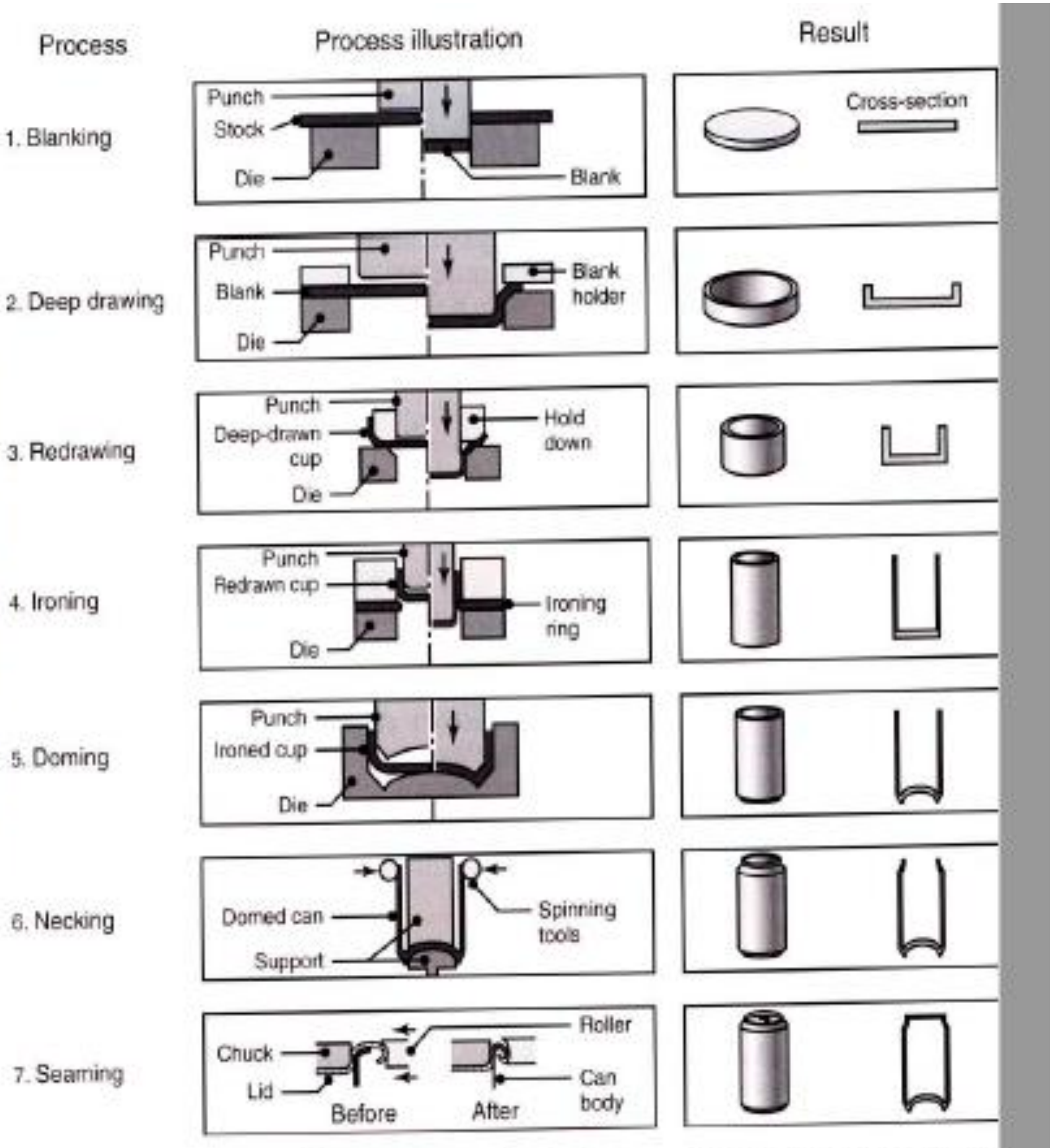
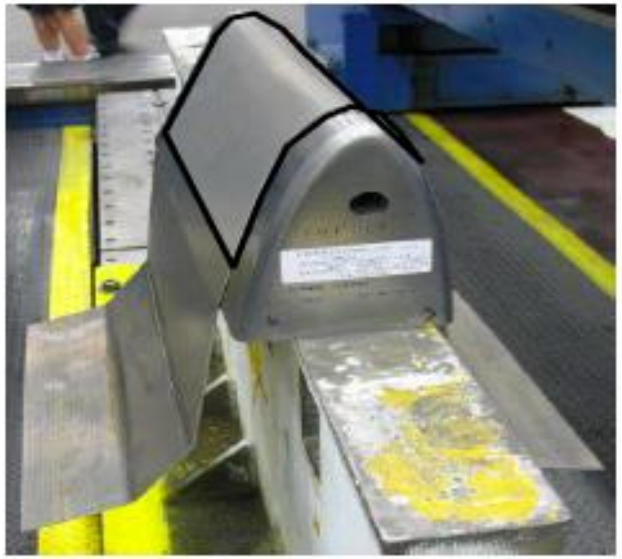


FIGURE 16.31 The metal-forming processes involved in manufacturing a two-piece aluminu beverage can.

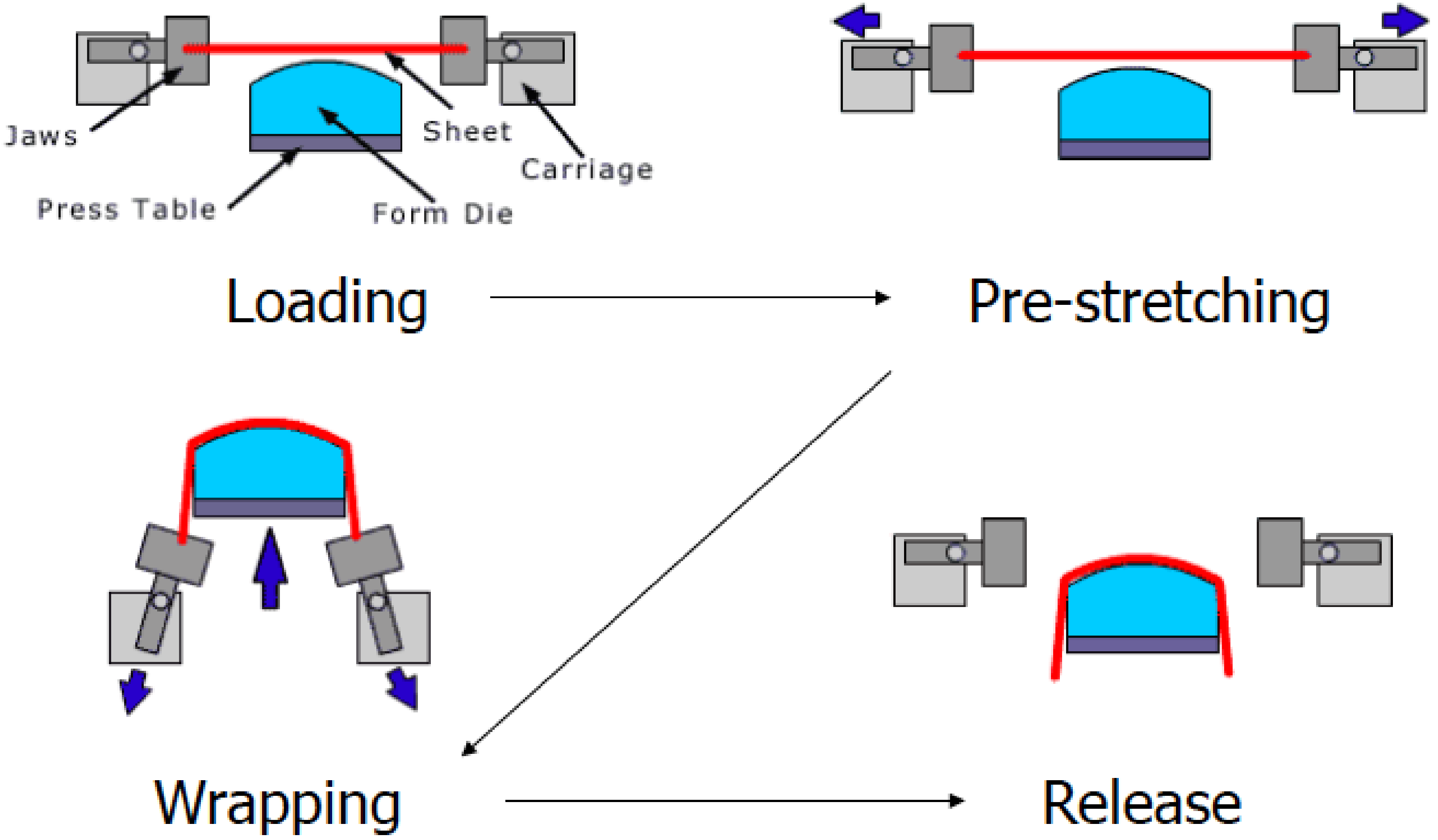
Stretch forming: Forming force



$$F = (Y_S + UTS)/2 * A$$

F = stretch forming force (lbs)
Y_S = material yield strength (psi)
UTS = ultimate tensile strength of the material (psi)
A = Cross-sectional area of the workpiece (in2)

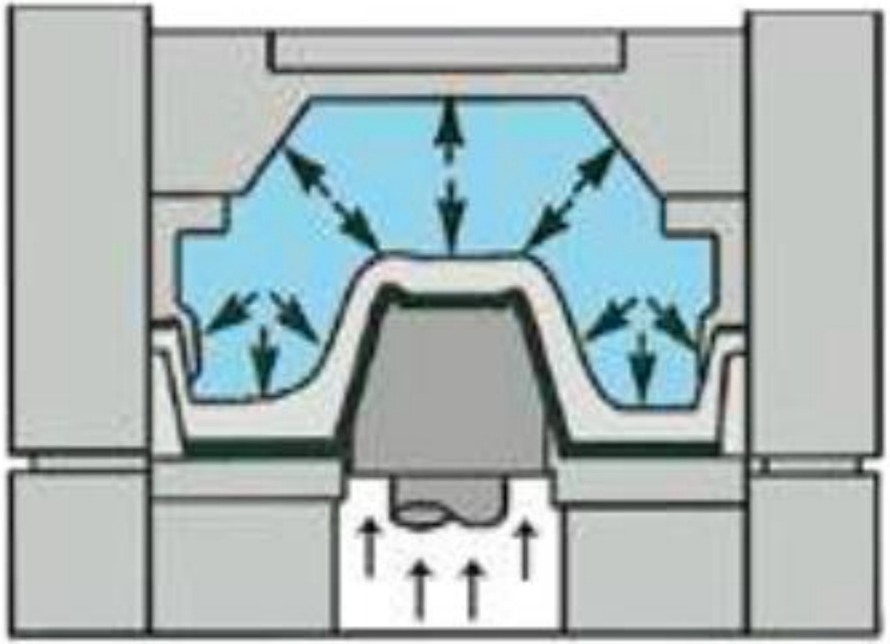
Stretch forming – very cheap tooling, net thinning, slow, low formability, sheet metal up to 15mx9m



* source: http://www.cyrilbath.com/sheet_process.html

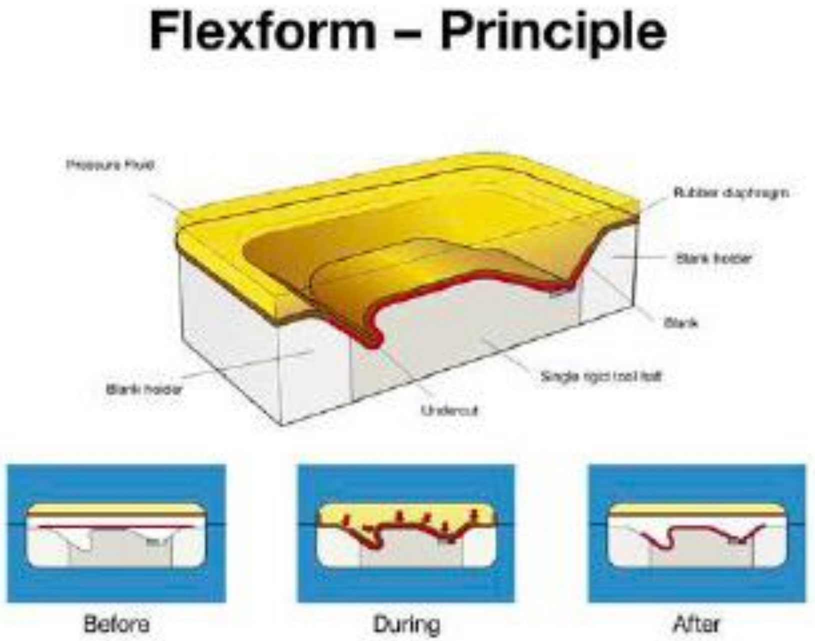
Low volume batches

Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability



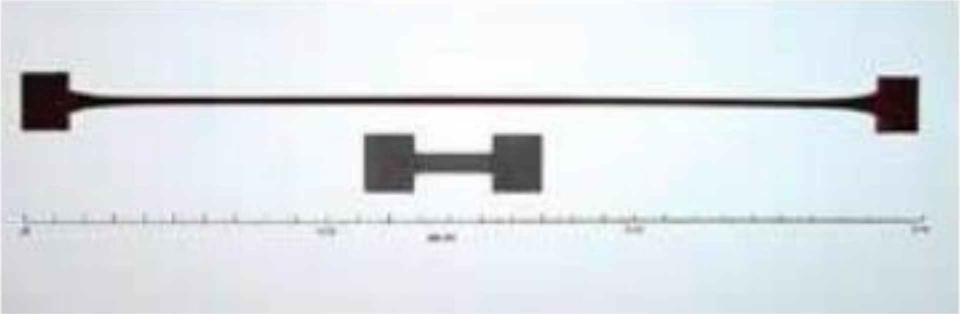
Low volume batches

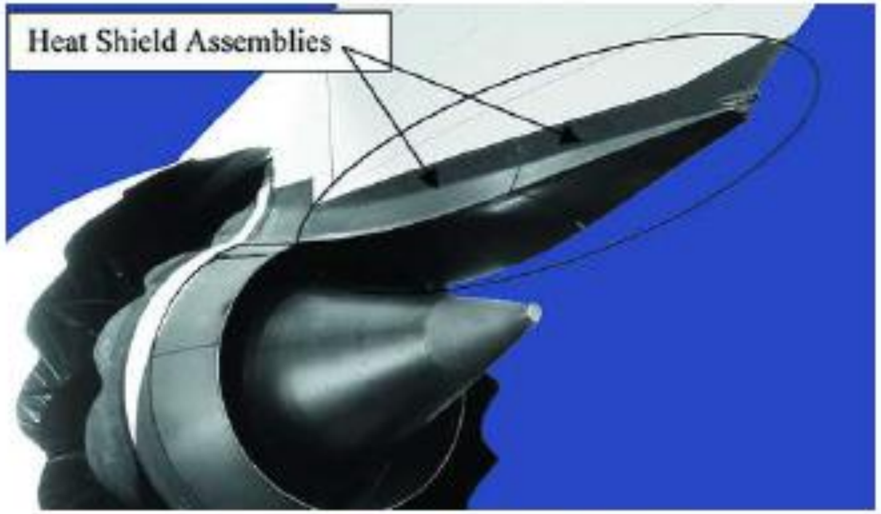
Hydro-forming – cheap tooling, no net thinning, slow(ish), high formability

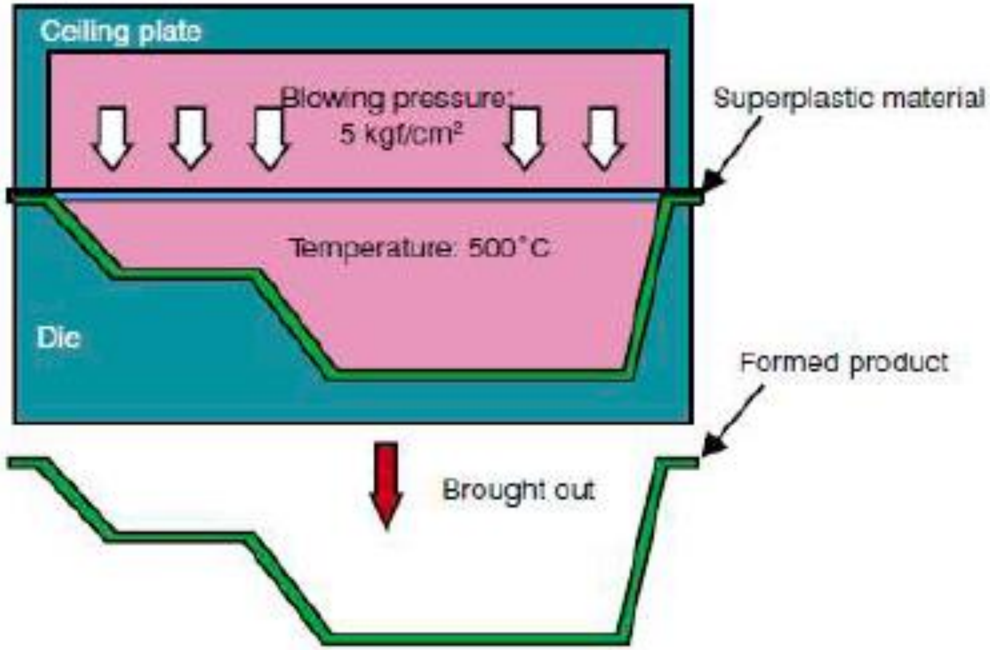


Low volume batches

Super-plastic forming – cheap tooling, net thinning, slow, expensive sheet metal, very high formability

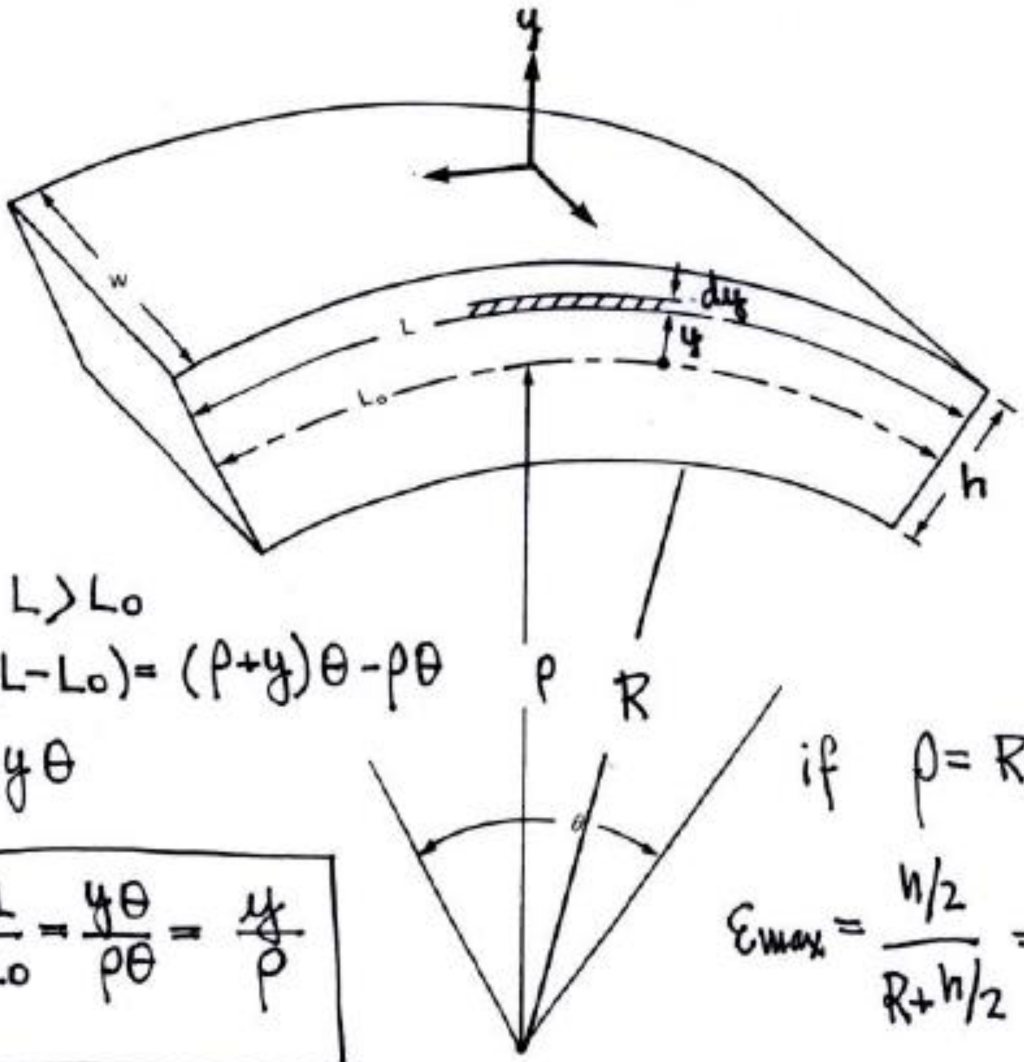






Low volume batches, 0.5-0.75 melting temp

Bending & Spring back



note: $L > L_o$

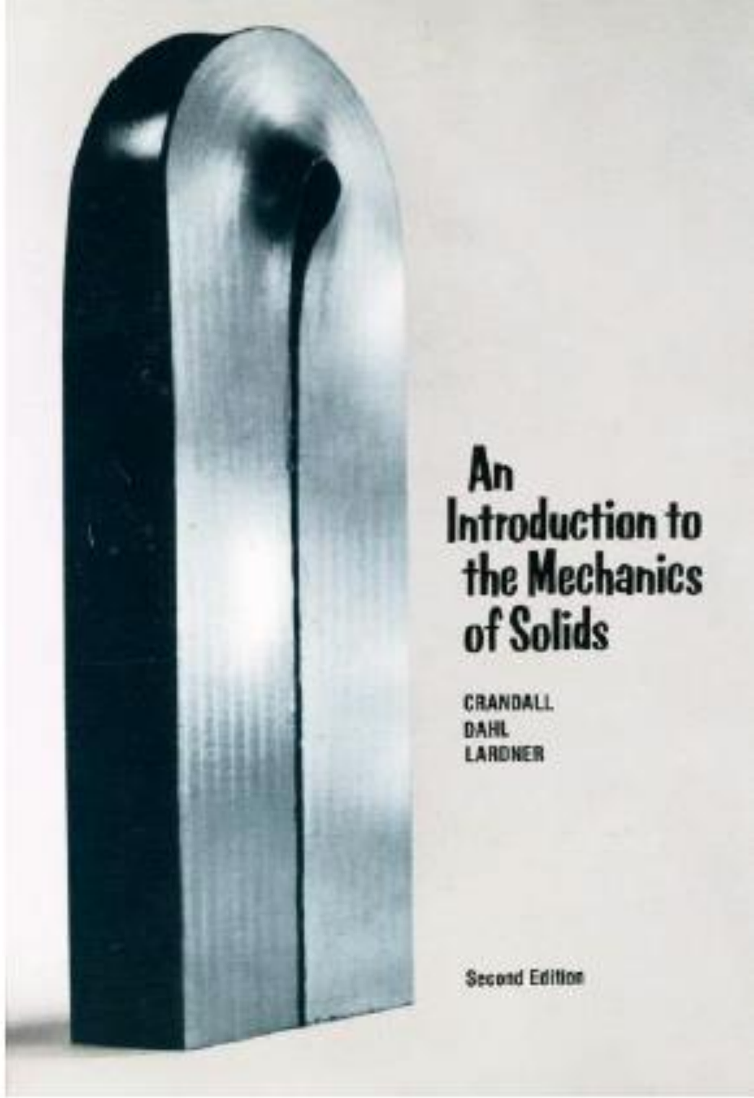
$$\Delta L = (L - L_o) = (\rho + y)\theta - \rho\theta = y\theta$$

$$\varepsilon = \frac{\Delta L}{L_o} = \frac{y\theta}{\rho\theta} = \frac{y}{\rho}$$

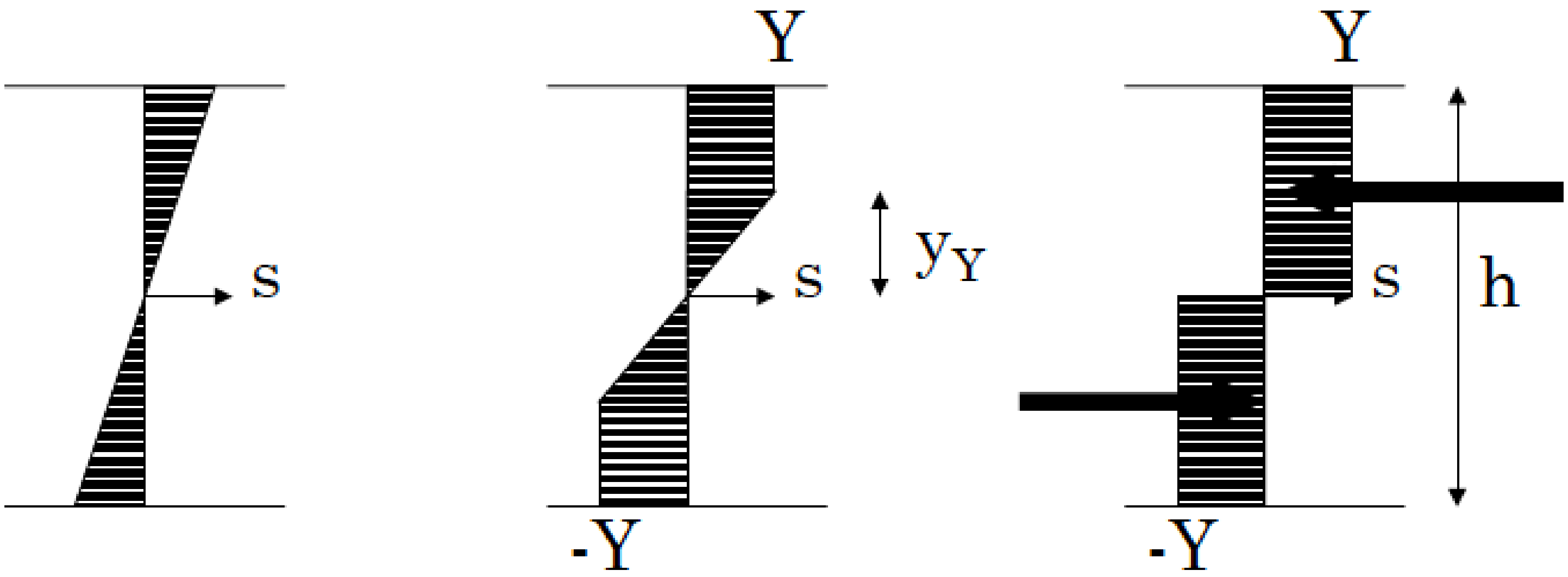
if $\rho = R + \frac{h}{2}$

$$\varepsilon_{max} = \frac{h/2}{R + h/2} = \frac{1}{\frac{2R}{h} + 1}$$

Figure Coordinate system for analysis of bending.



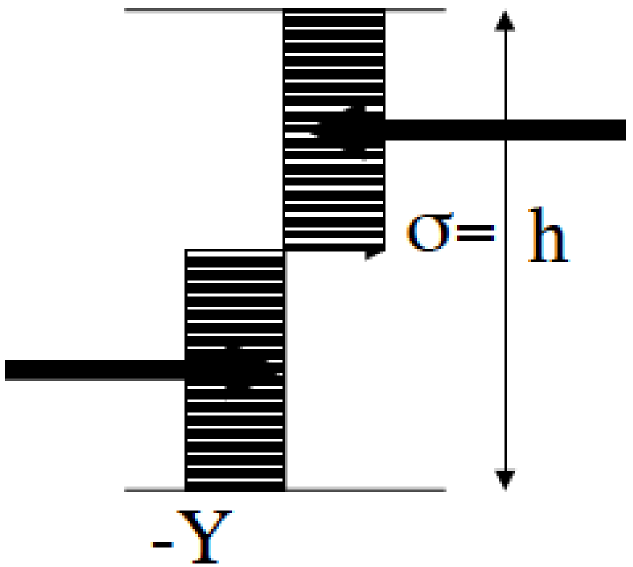
Stress distribution through the thickness of the part



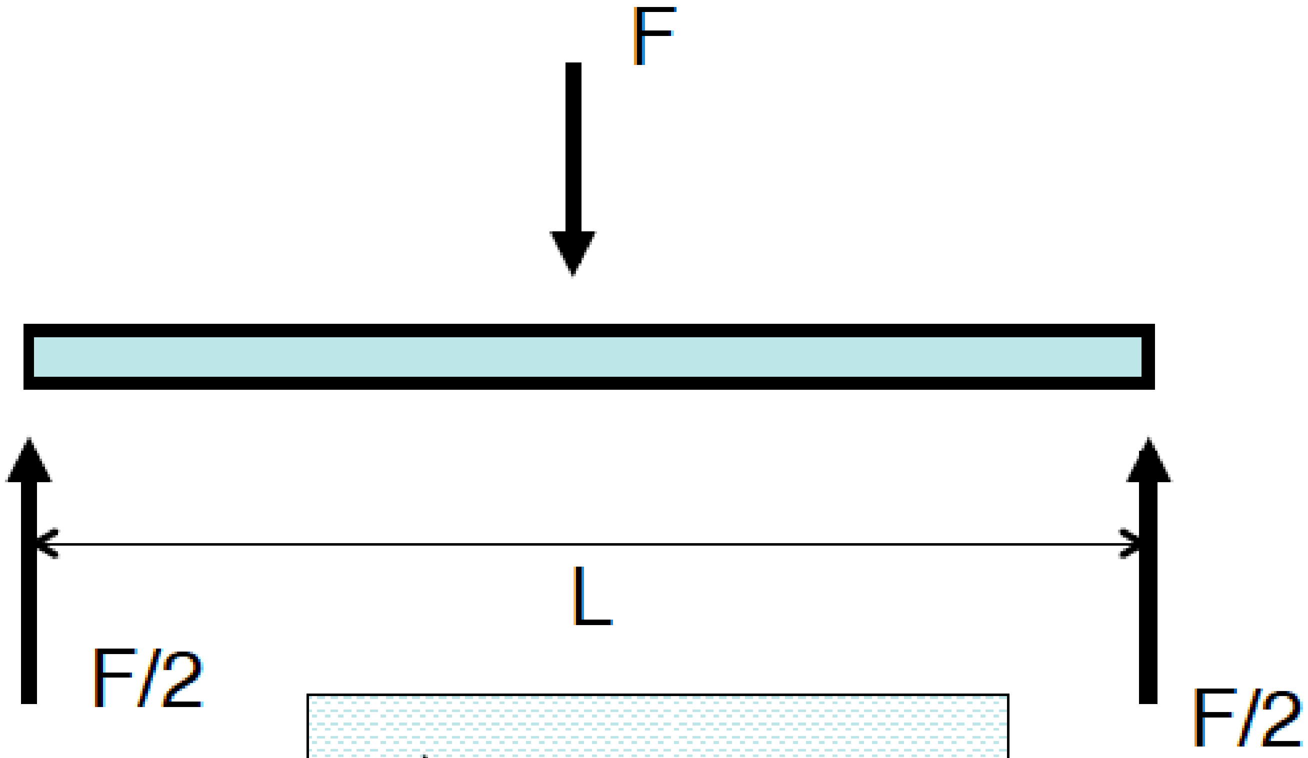
Elastic Elastic-plastic Fully plastic

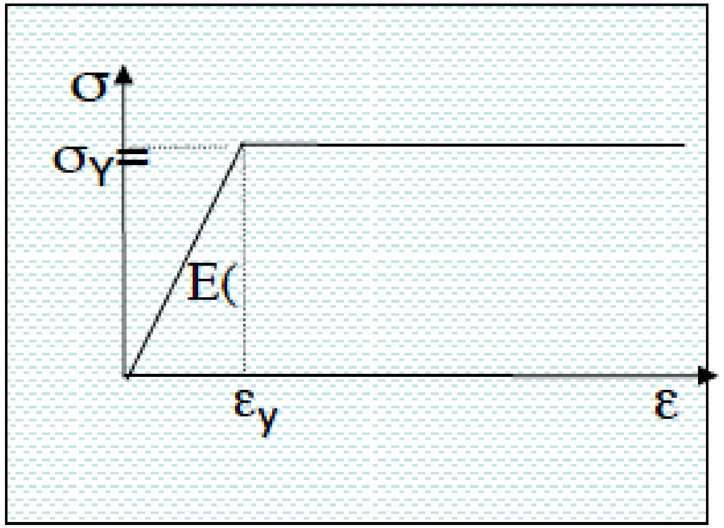
Fully Plastic Moment, $M = Y\,(b\,h/2)\,h/2 = Ybh^2/4$

Balance external and internal moments



Fully plastic

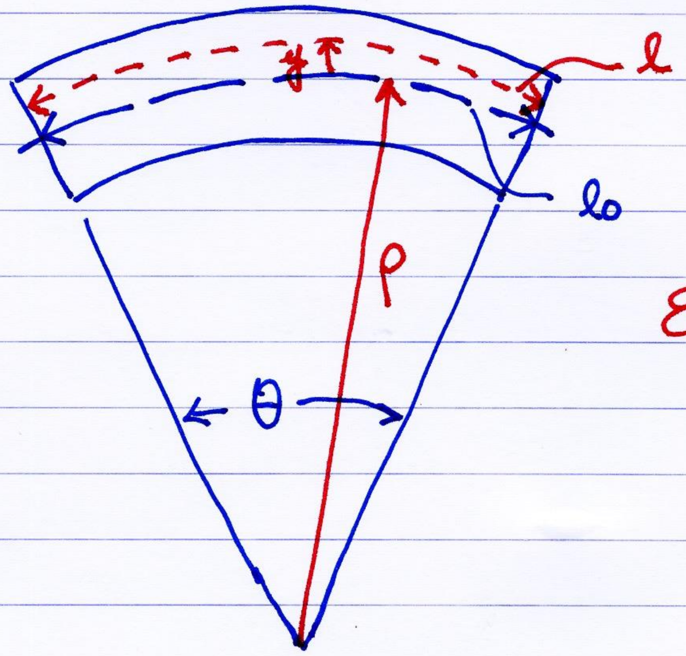




$$Ybh^2/4 = FL/4 = M_{max}$$

$$F = bh^2Y/L$$

Strain:



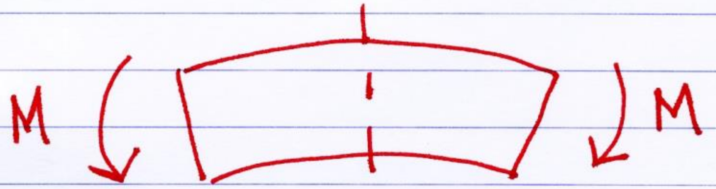
$$\varepsilon = \frac{l - l_0}{l_0}$$

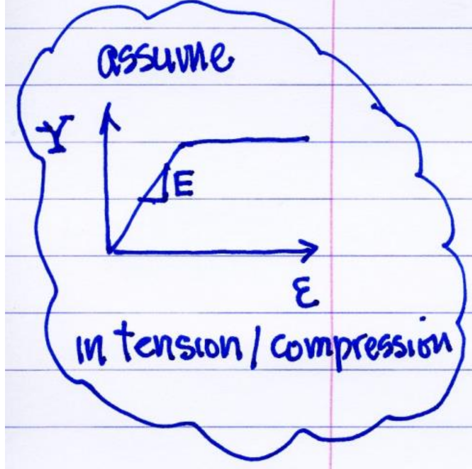
$$l_0 = \rho\theta$$

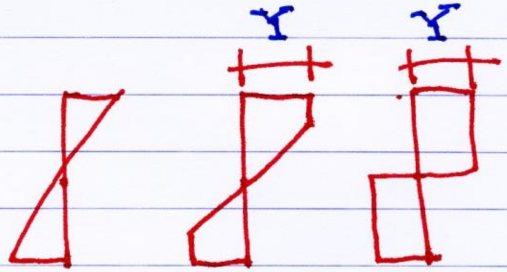
$$l = (\rho + y)\theta$$

$$\therefore \varepsilon = \frac{y}{\rho} \quad \text{—— (1)}$$

Stress

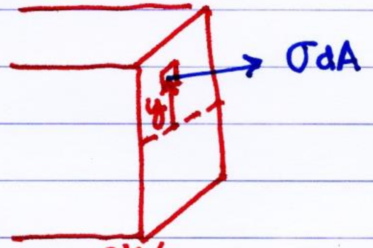






Bending

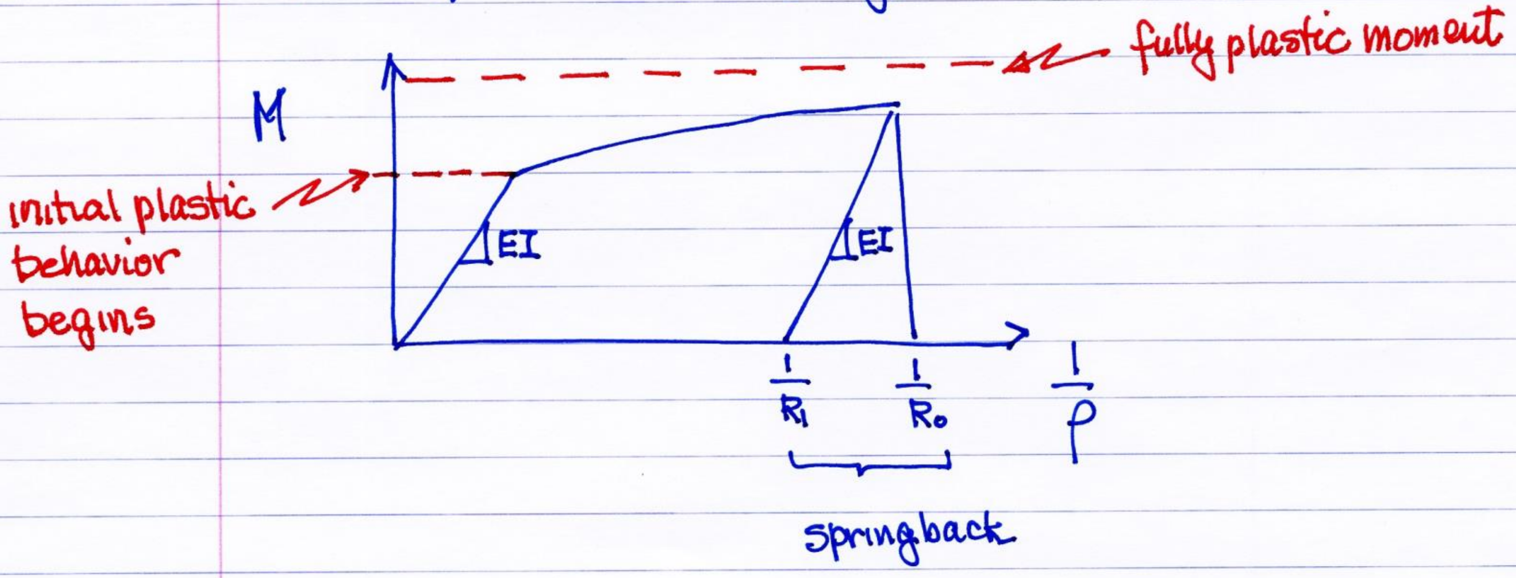
Elastic   Fully Plastic



$$M = \int \sigma \, dA \cdot y \quad \text{—— (2)}$$

$$M = 2\int_0^{h/2} E\frac{y}{\rho} \cdot b\,dy \cdot y = \frac{E}{\rho}\,\frac{bh^3}{12} = \frac{EI}{\rho} \quad \text{—— (3)}$$

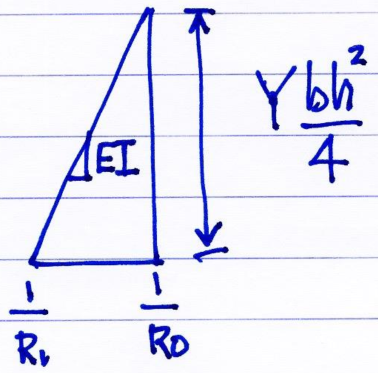
Moment- Curvature Diagram



For a fully plastic moment

$$M_{Max} = 2\int_0^{h/2} Y\, b\, y\, dy = Y\frac{bh^2}{4}$$

A rough estimate of the spring back



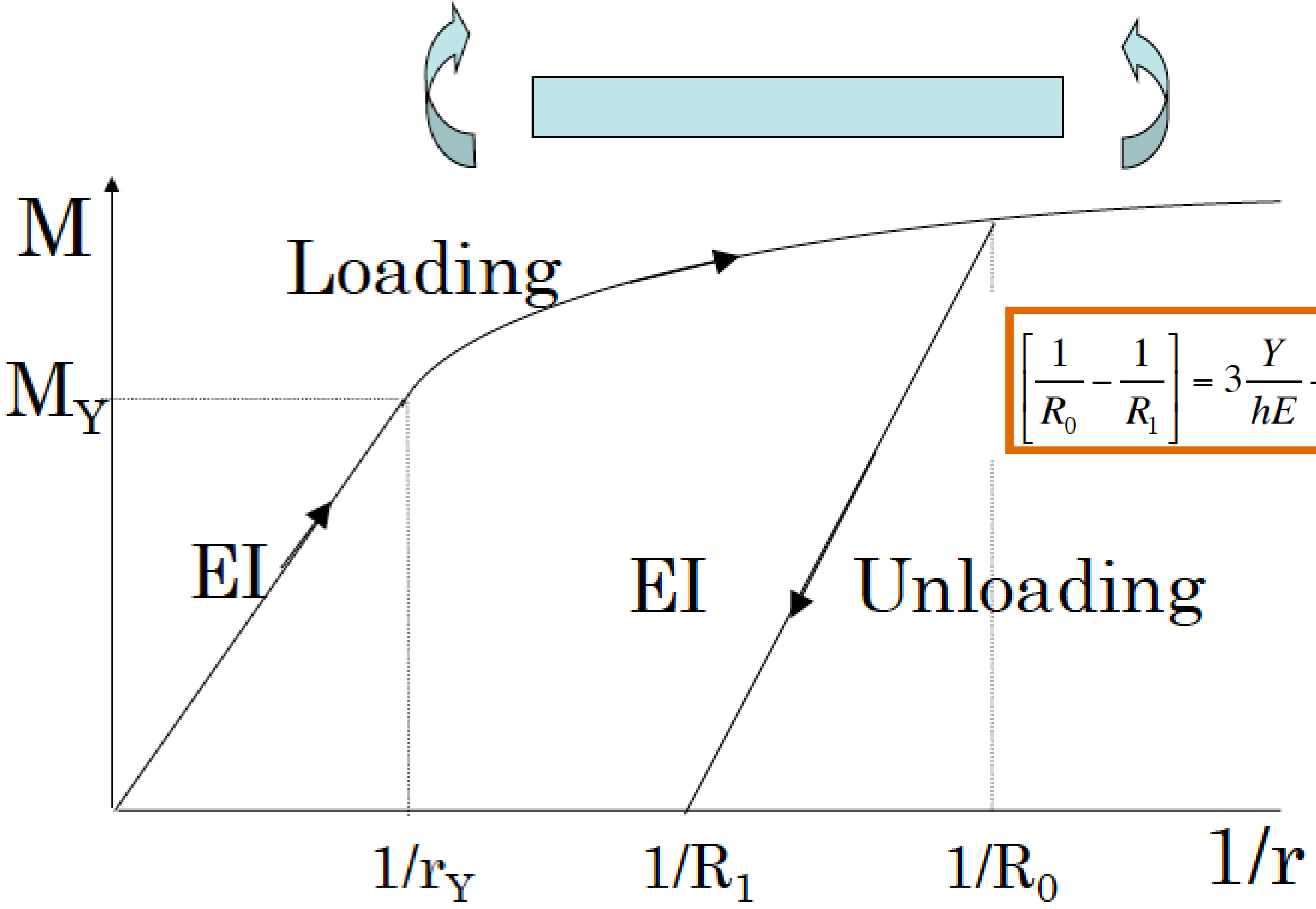
$$\frac{1}{R_0} - \frac{1}{R_1} = \frac{3Y}{Eh}$$

The full solution is:

$$\frac{1}{R_0} - \frac{1}{R_1} = \frac{3Y}{Eh} - 4R_0^2\left(\frac{Y}{Eh}\right)^3$$

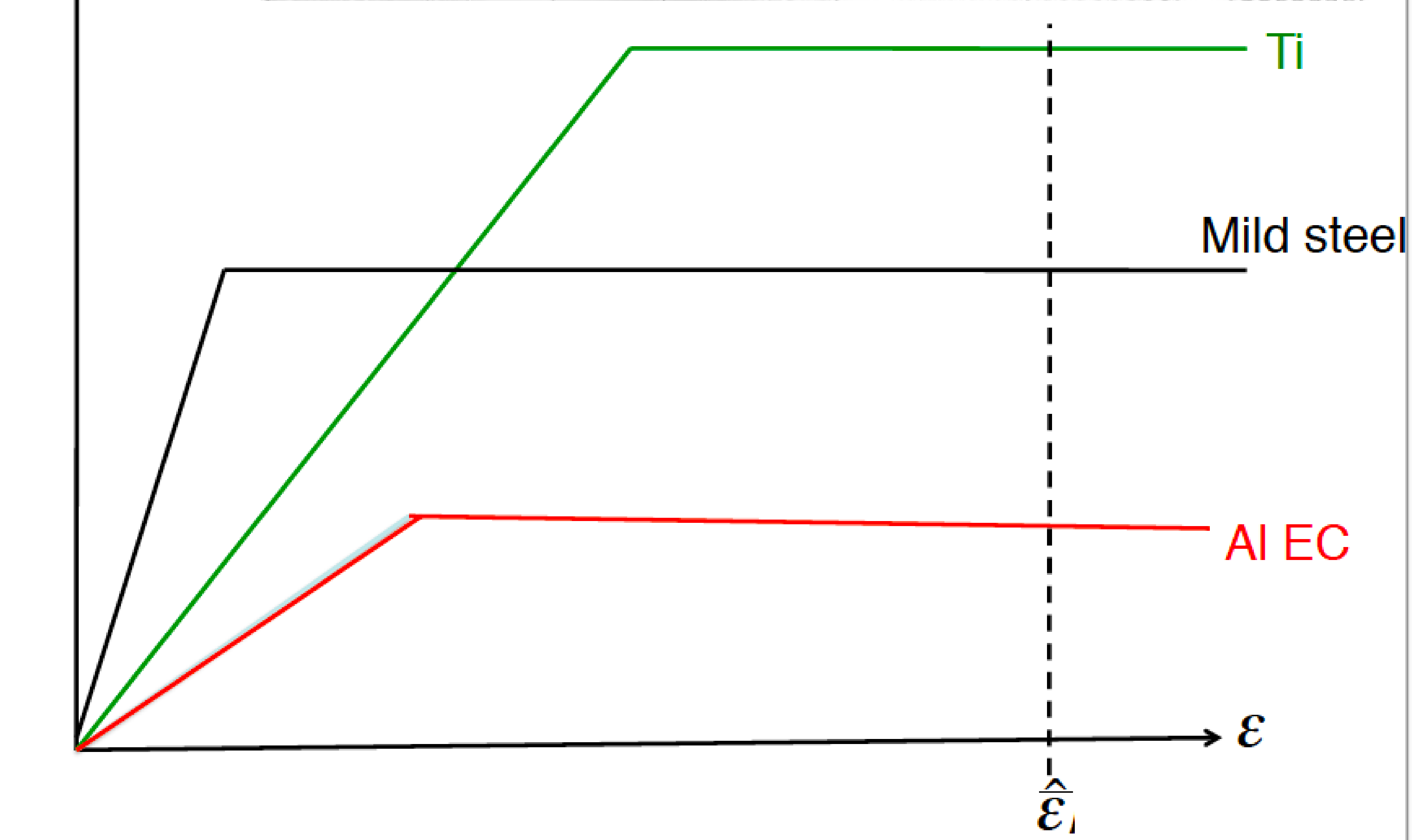
The details are at the end (8 slides)

Bending Moment – Curvature



$$\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=3\frac{Y}{hE}-4R_0^2\left(\frac{Y}{hE}\right)^3$$

		Al 2024 (T3)	Al EC (99.0 %)	Mild Steel	Titanium
Yield Strength, Y (psi)		50000	14000	33000	80000
Modulus, E (psi)		10600000	10000000	30000000	15000000



	Al 2024 (T3)	Al EC (99.0 %)	Mild Steel	Titanium
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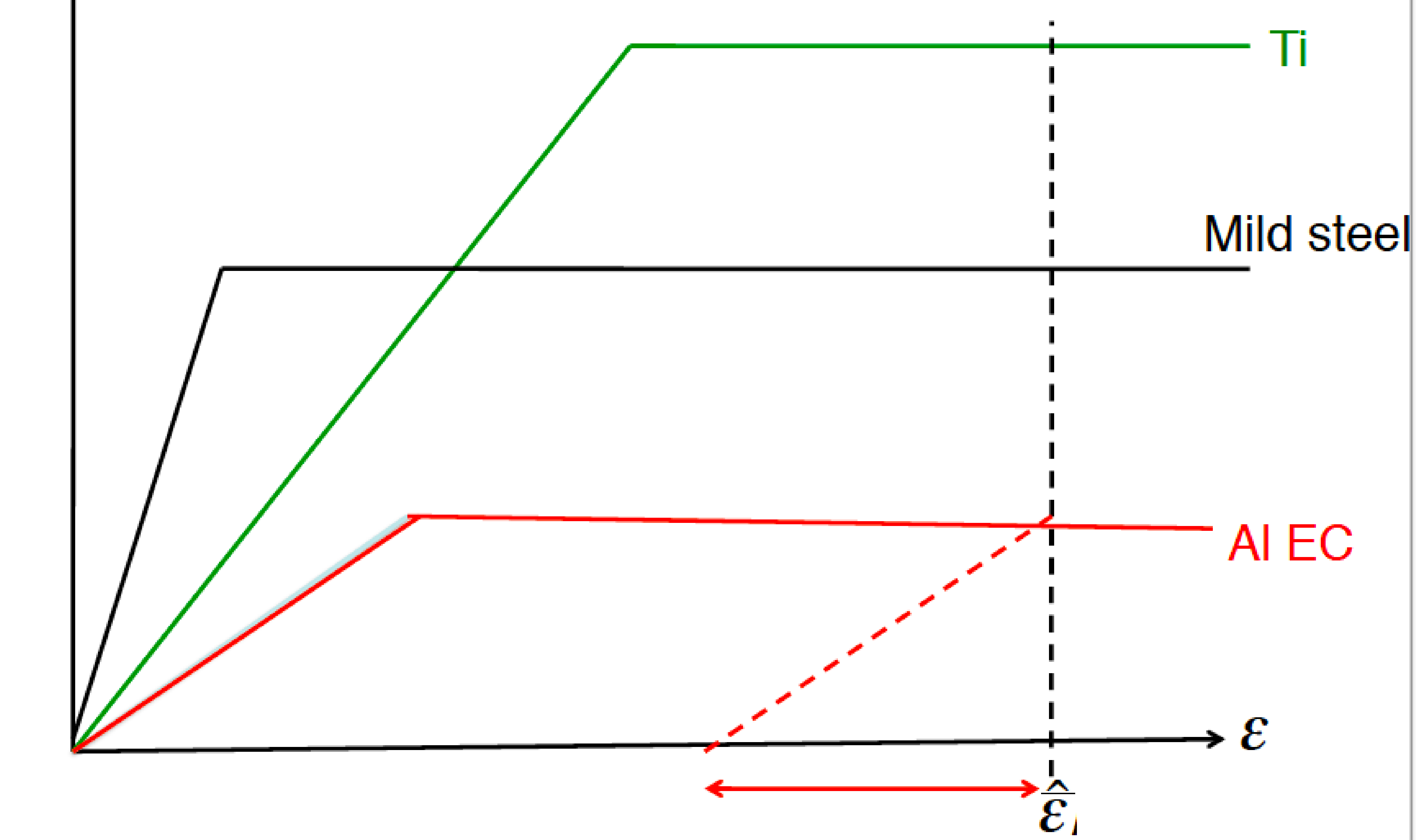

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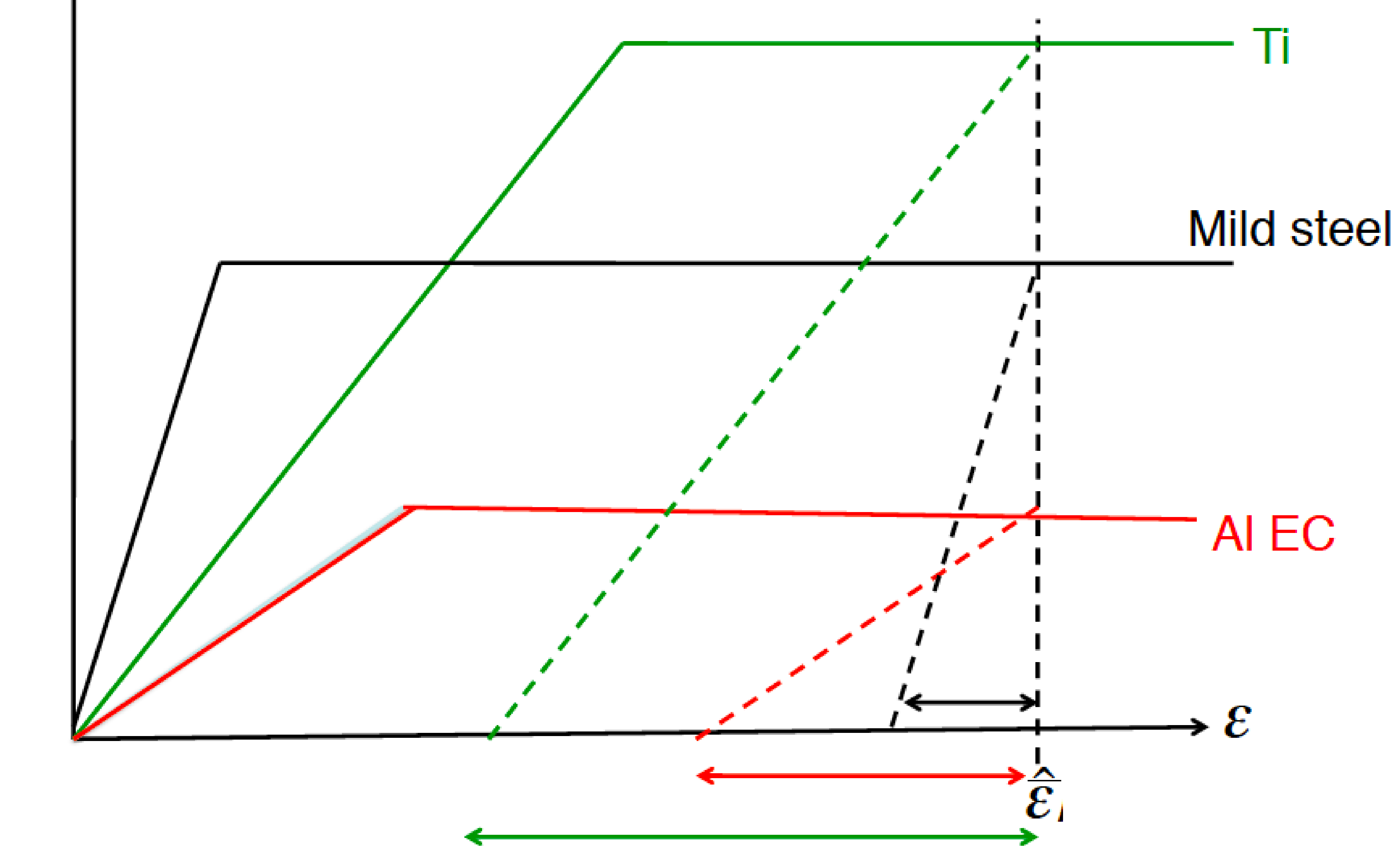
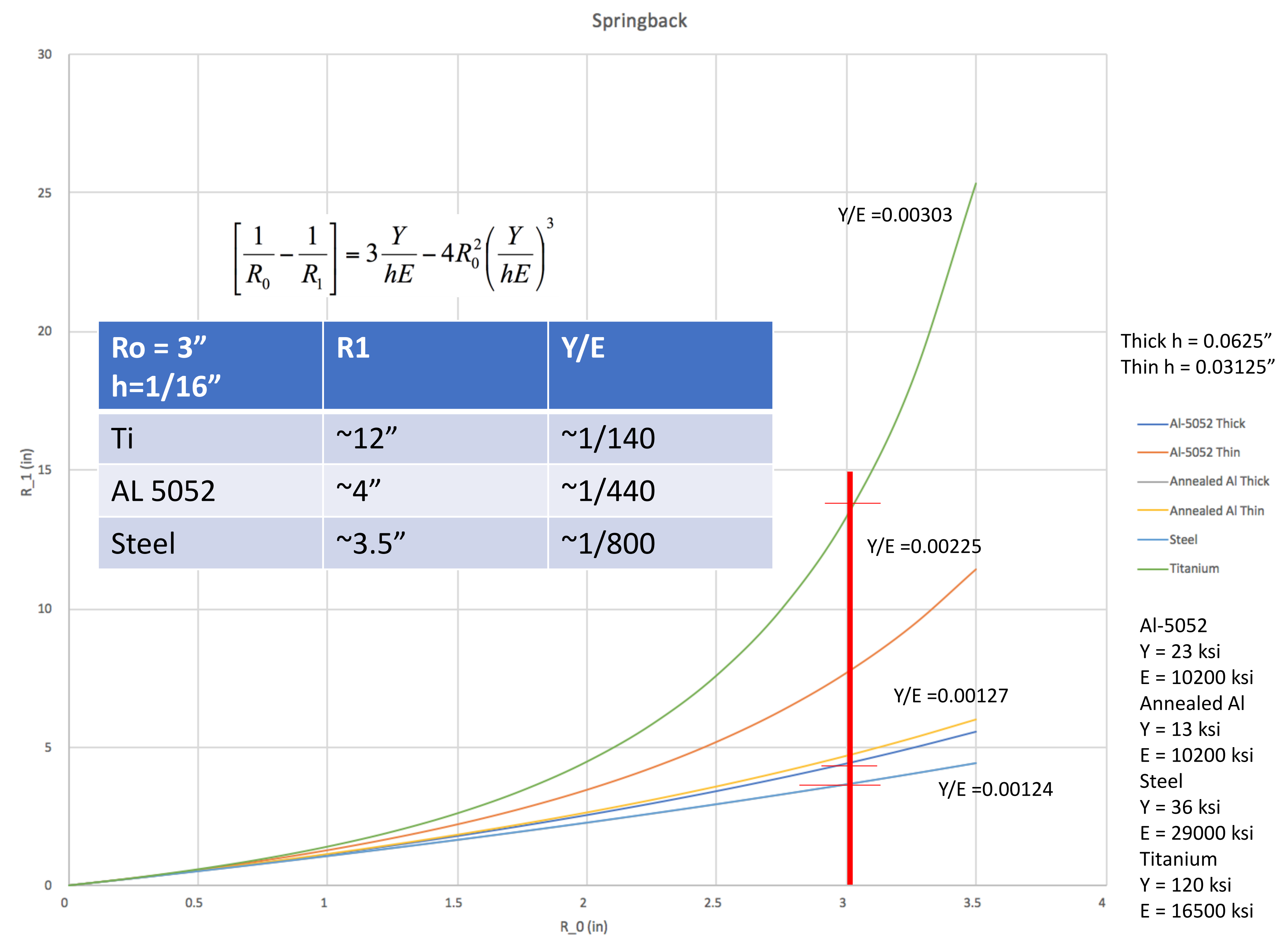




$$\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=3\frac{Y}{hE}-4R_0^2\left(\frac{Y}{hE}\right)^3$$

Ro = 3” h=1/16”	R1	Y/E
Ti	~12”	~1/140
AL 5052	~4”	~1/440
Steel	~3.5”	~1/800

Thick h = 0.0625”
Thin h = 0.03125”

Al-5052
Y = 23 ksi
E = 10200 ksi
Annealed Al
Y = 13 ksi
E = 10200 ksi
Steel
Y = 36 ksi
E = 29000 ksi
Titanium
Y = 120 ksi
E = 16500 ksi

Aluminum Alloys

TABLE 6.3

Properties of Selected Aluminum Alloys at Room Temperature

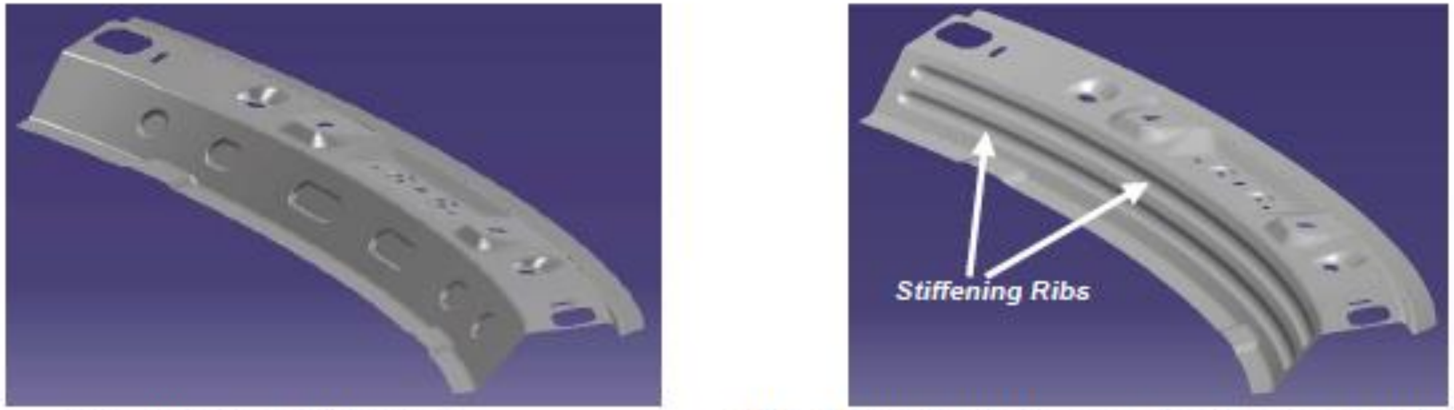
Alloy (UNS)	Temper	Ultimate tensile strength (MPa)	Yield strength (MPa)	Elongation in 50 mm (%)
1100 (A91100)	O	90	35	35–45
	H14	125	120	9–20
2024 (A92024)	O	190	75	20–22
	T4	470	325	19–20
3003 (A93003)	O	110	40	30–40
	H14	150	145	8–16
5052 (A95052)	O	190	90	25–30
	H34	260	215	10–14
6061 (A96061)	O	125	55	25–30
	T6	310	275	12–17
7075 (A97075)	O	230	105	16–17
	T6	570	500	11

Methods to reduce springback

- Smaller Y/E
- Larger thickness
- Over-bending
- Stretch forming
- "coining" or bottoming the punch

Methods to reduce springback

- Smaller Y/E



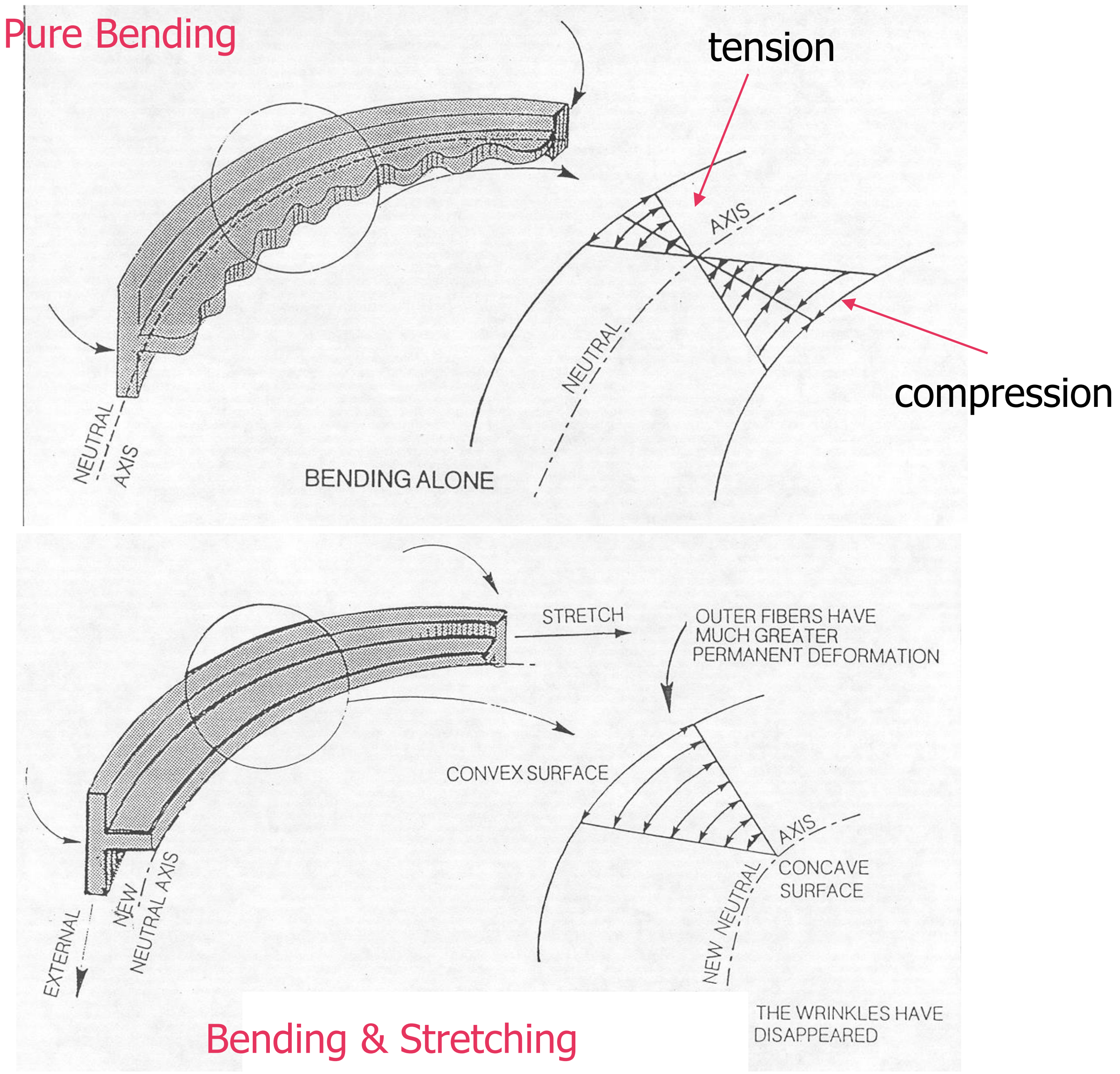
(a). Original Design *(b). Modified Design for Springback Control*

Figure 3. An example of springback control through part design

Apply Tension and Bending-

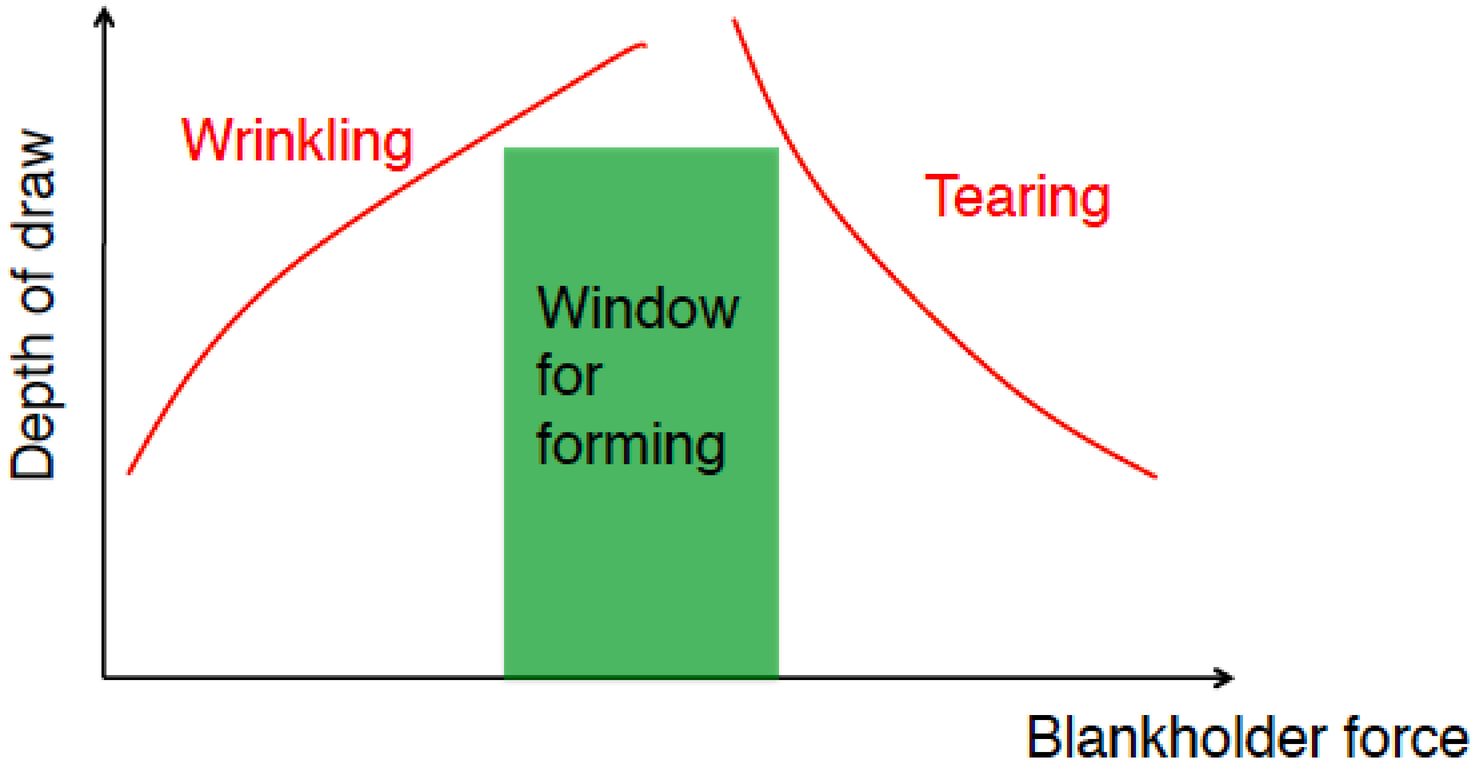
- This limits applicable part geometries

- Advantage of stretch forming

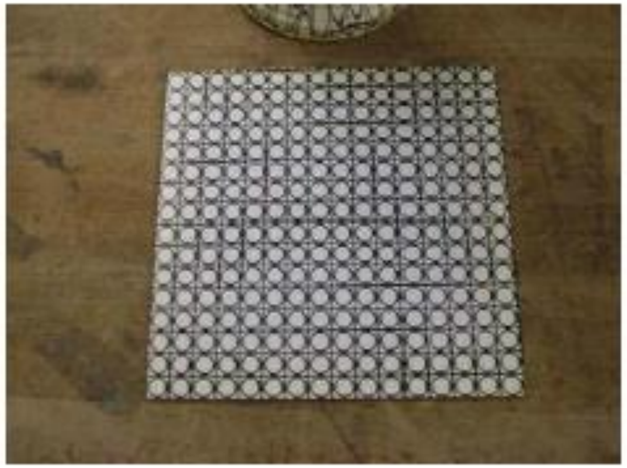


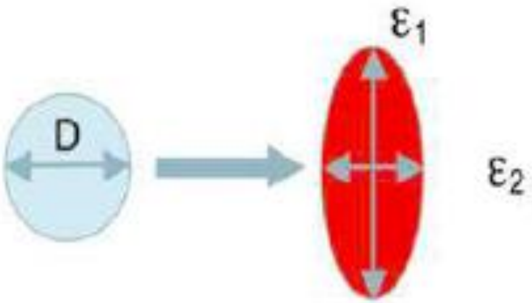
http://www.thomasnet.com/articles/custom-manufacturing-fabricating/wrinkling-during-deep-drawing

Blank holder force: forming window



Forming Limit Diagrams





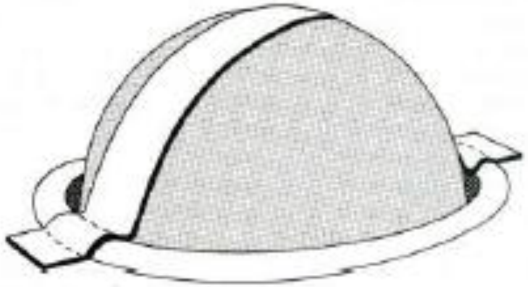
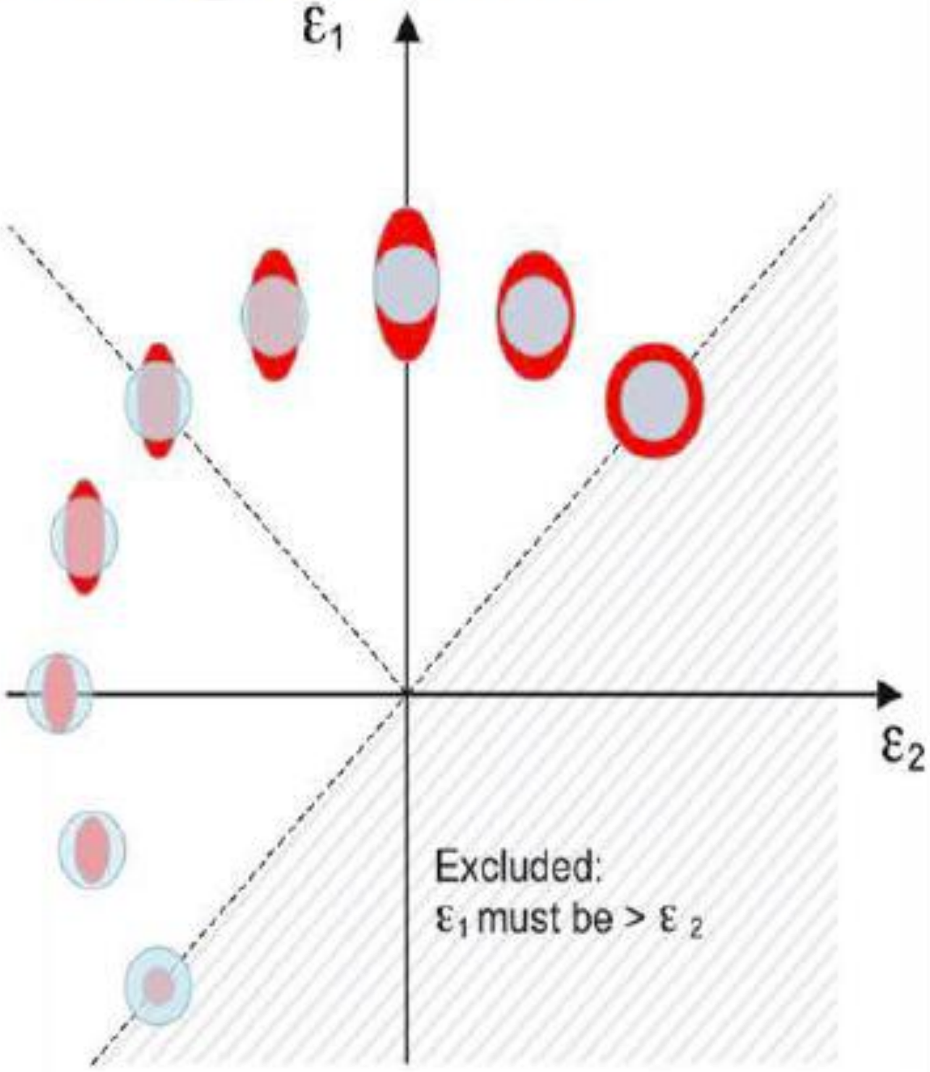


Figure 15-8 Strips of varying width are stretched to obtain different $\varepsilon_2/\varepsilon_1$ ratios.



Forming Limit Diagrams

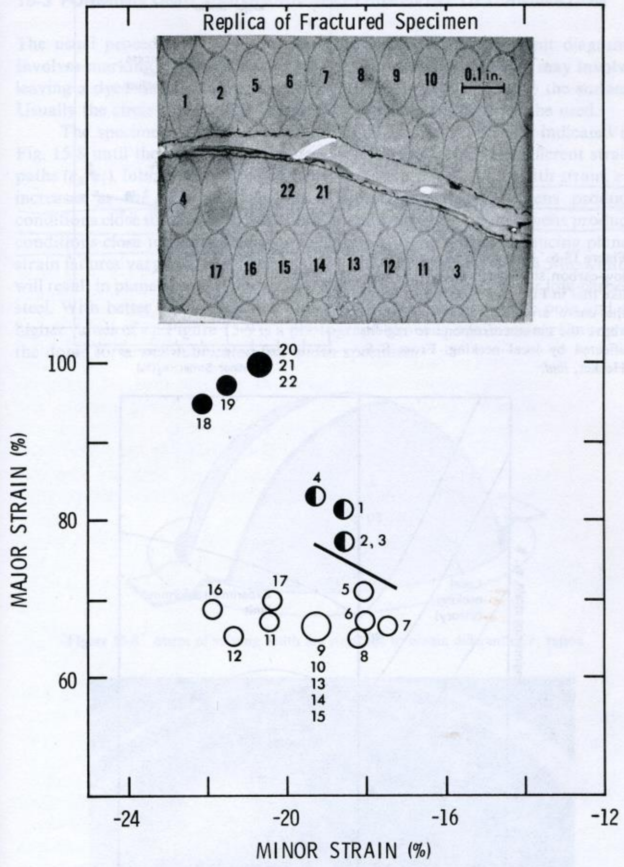


Figure 15-5 Distortion of printed circles near a localized neck and a plot of the strains in the circles. Solid points are for grid circles through which failure occurred, open points are for grid circles removed from failure, and partially filled points are for grid circles very near failure. From S. S. Hecker, *Sheet Metal Ind.*, 52 (1975), pp. 671–75.

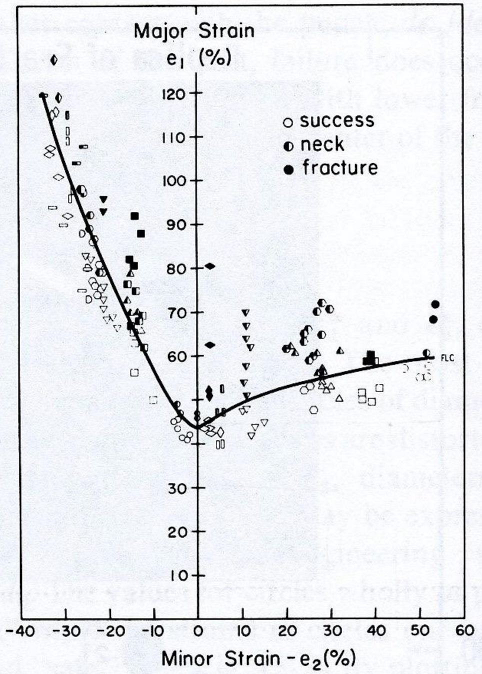
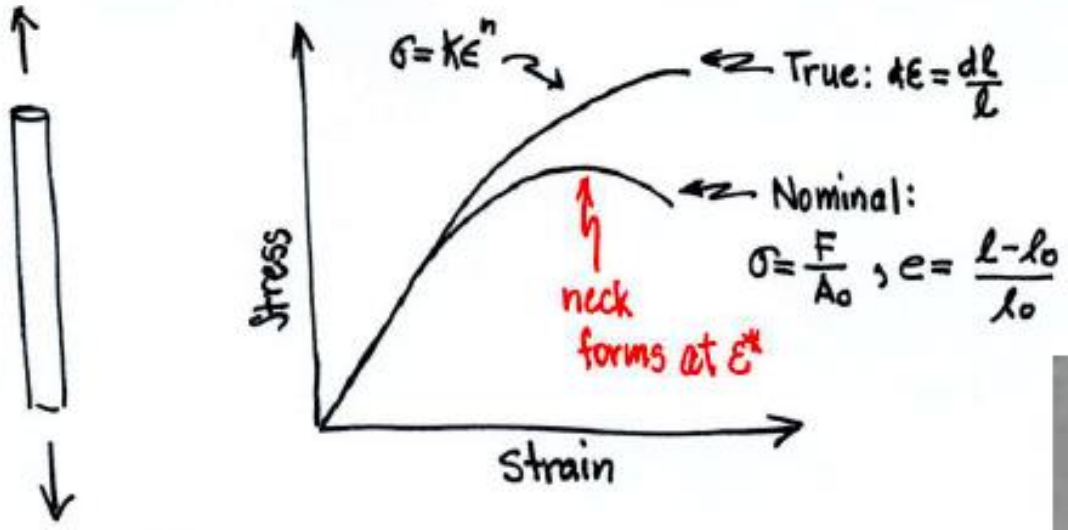
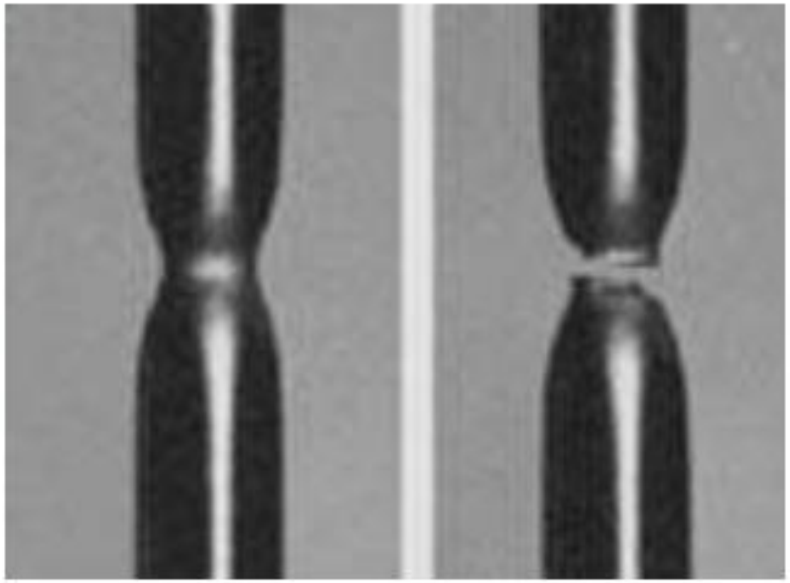


Figure 15-6 Forming limit diagram for low-carbon steel determined from data like that in Fig. 15-5. The strains below the curve are acceptable while those above the curve correspond to regions affected by local necking. From S. S. Hecker, *ibid*.

Figures from Hosford & Caddell

Tensile instability - necking





Tensile instability (1-D)

$F=\sigma A$; so $dF=\sigma dA + A d\sigma = 0$ at max load

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} = d\varepsilon$$

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

With $\sigma = k\varepsilon^n$: $\quad \frac{d\sigma}{d\varepsilon} = nk\varepsilon^{n-1} = \sigma = k\varepsilon^n$

$$\Rightarrow \boxed{\varepsilon^* = n}$$

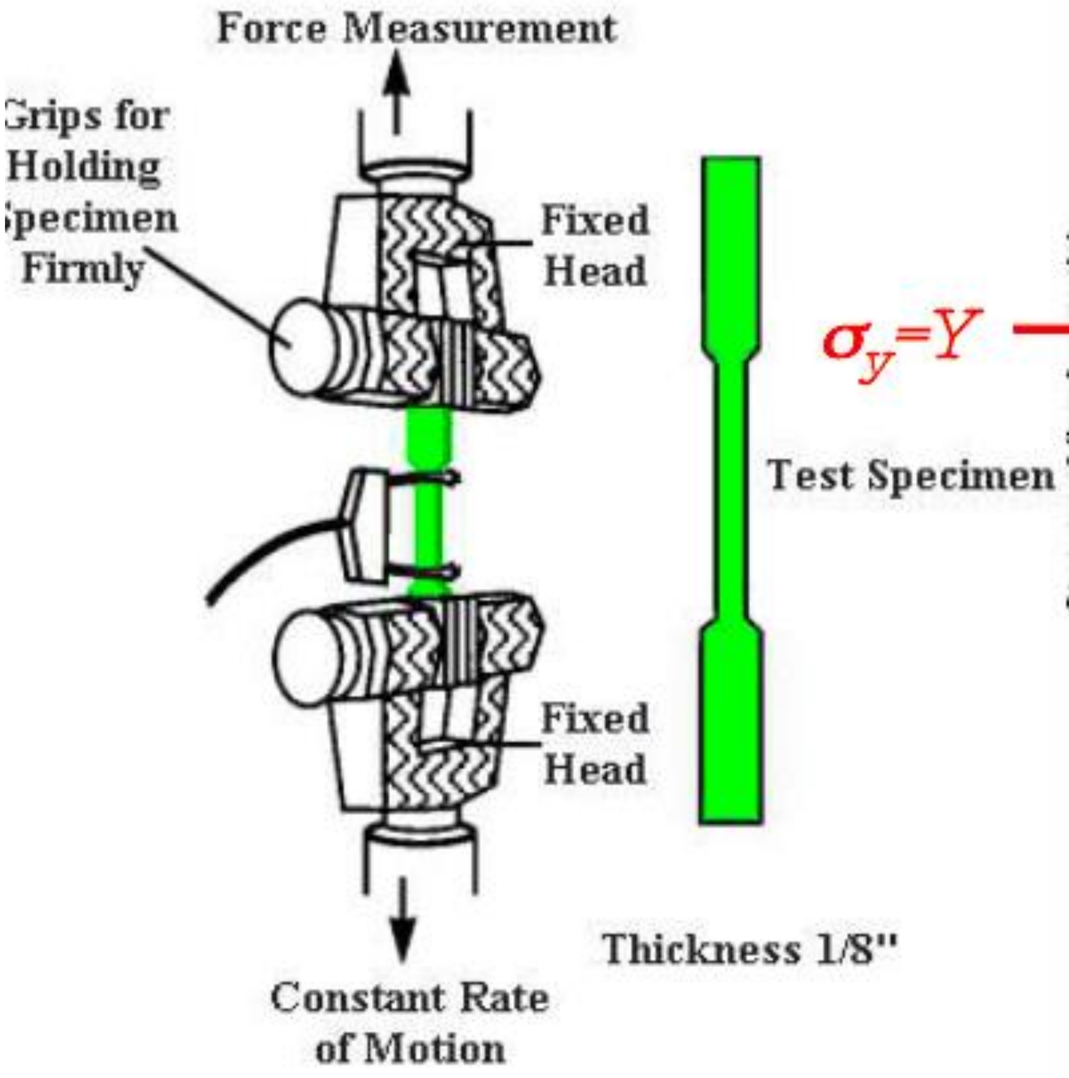
Process Performance

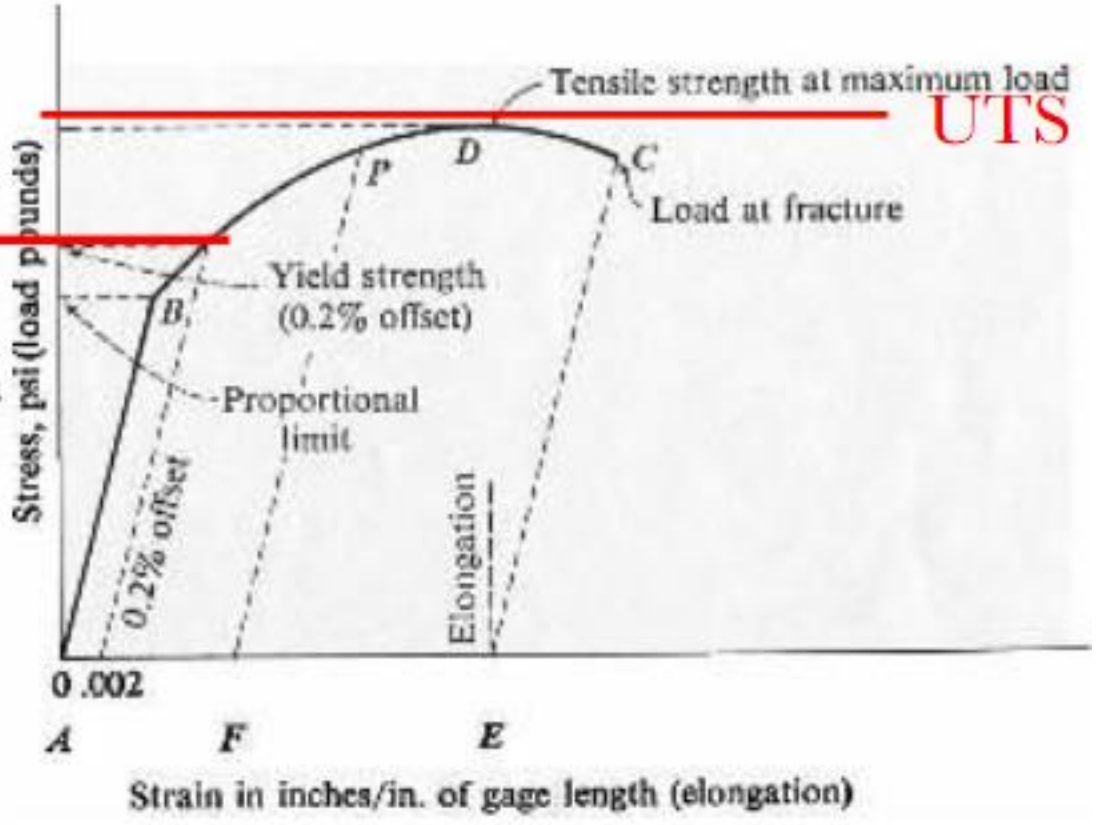
1. Cost – Dies, Material/ Waste
2. Quality – Spring-back, wrinkling, tearing
3. Rate – Lead time for dies
4. Flexibility - Dies

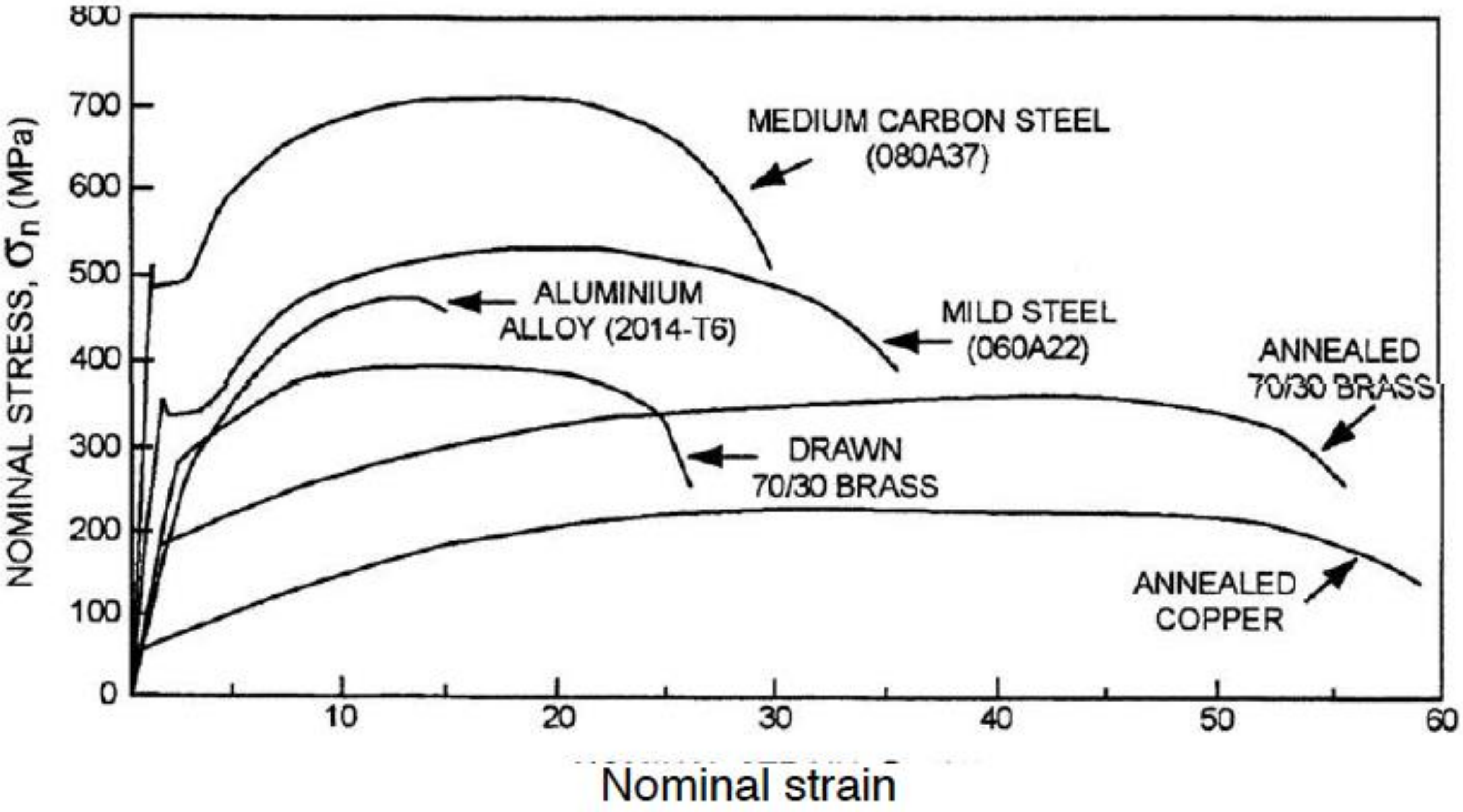
Sheet Metal Appendix

- Plastic behavior of metals
  - Power law behavior
  - 3D and Yield criterion
- Spring back derivation
- Developing technologies
  - Discrete Dies
  - Incremental Sheet Metal Forming
  - Flexible Spinning

Tensile test – the Stress-strain diagram






NOMINAL STRESS, σ_n (MPa)
MEDIUM CARBON STEEL
(080A37)
ALUMINIUM
ALLOY (2014-T6)
MILD STEEL
(060A22)
ANNEALED
70/30 BRASS
DRAWN
70/30 BRASS
ANNEALED
COPPER
0
100
200
300
400
500
600
700
800
10
20
30
40
50
60
Nominal strain

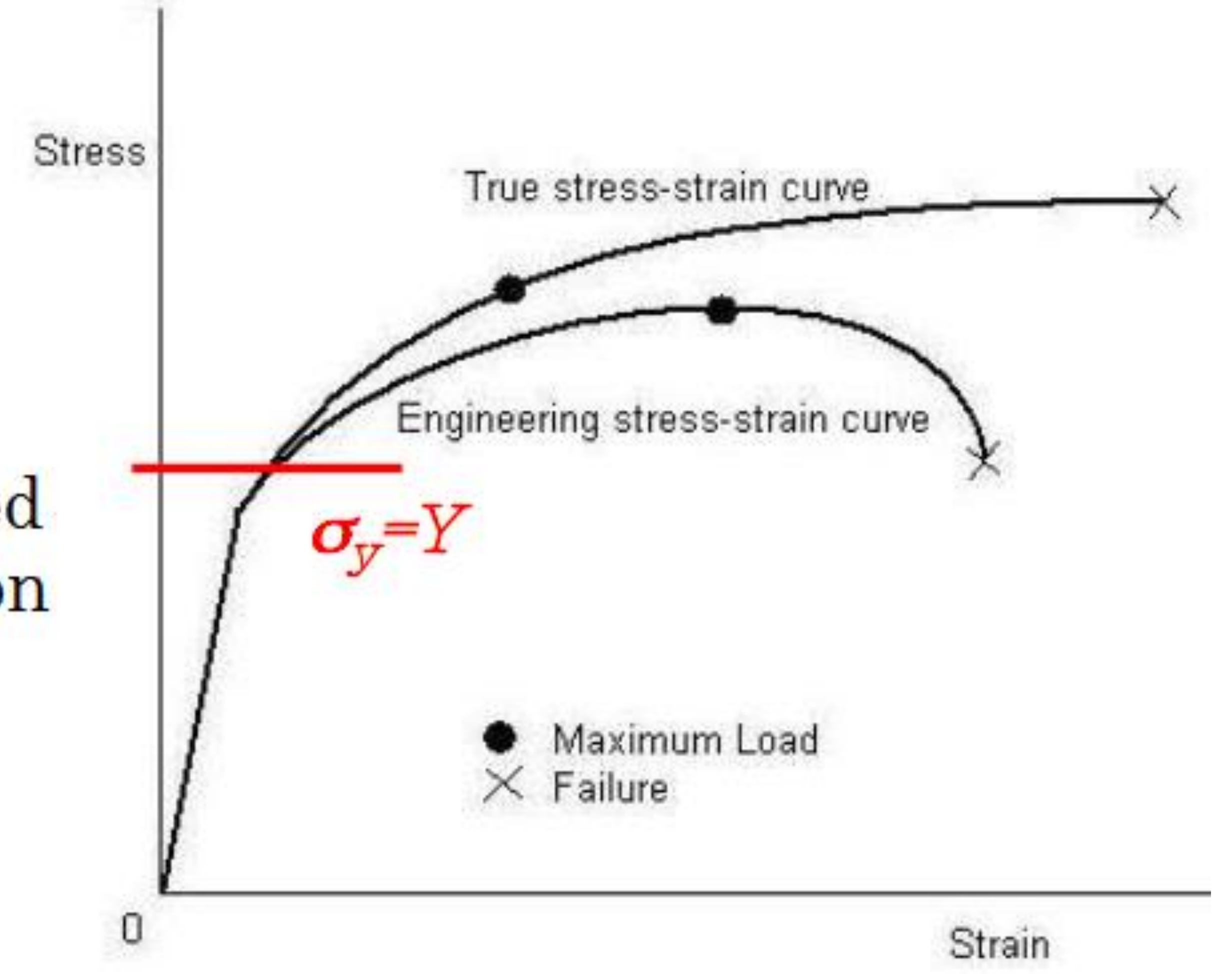
True stress & strain

$$\varepsilon_{tr} = \ln(1+\varepsilon_{en})$$

$$\sigma_{tr} = \sigma_{en}(1+\varepsilon_{en})$$

True stress can be expressed using a power law (Hollomon equation):

$$\sigma_{tr} = K\varepsilon_{tr}^{n}$$



Power-Law Expression (Hollomon equation)

$$\sigma_{tr} = K\varepsilon_{tr}^{n}$$

Can be re-written: $$\log(\sigma_{tr}) = n\log(\varepsilon_{tr}) + \log K$$

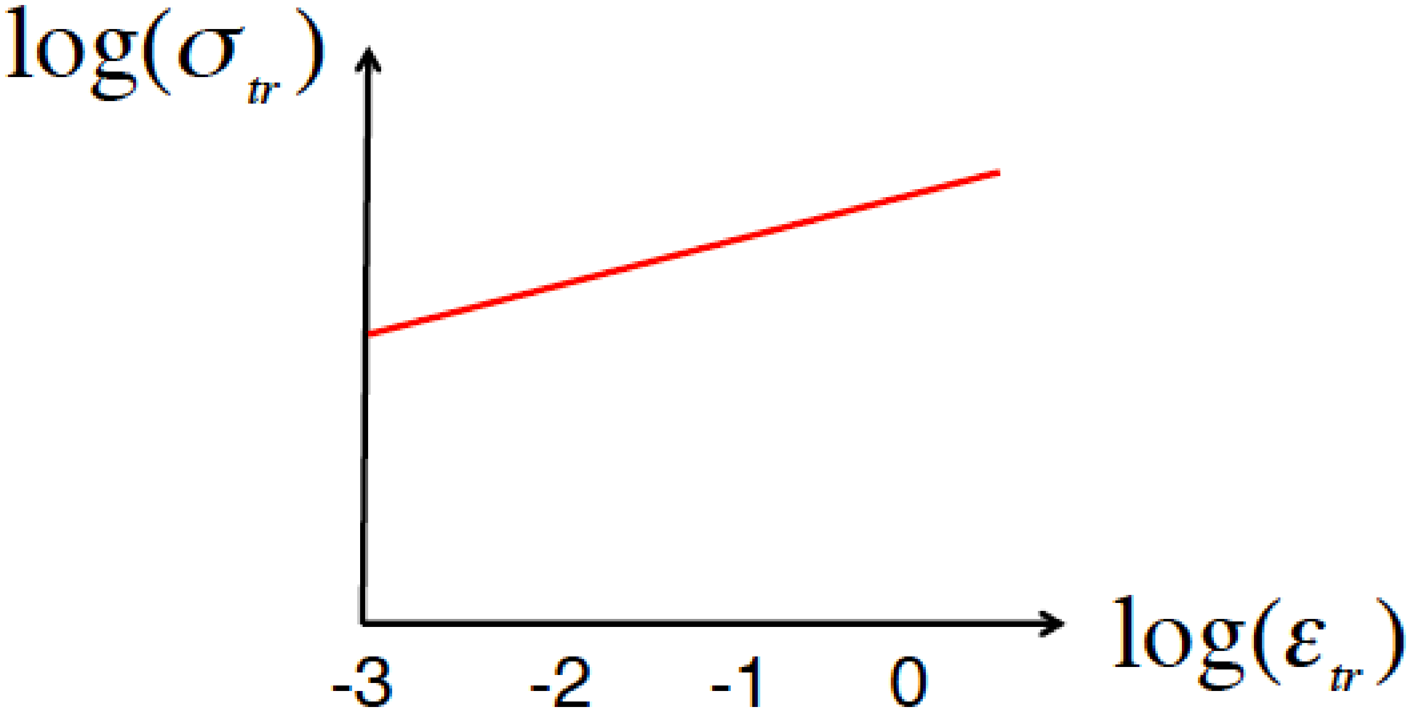
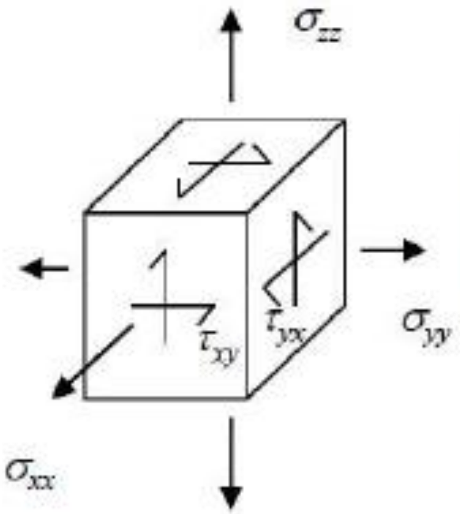


TABLE 2.3

Typical Values for *K* and *n* for Selected Metals

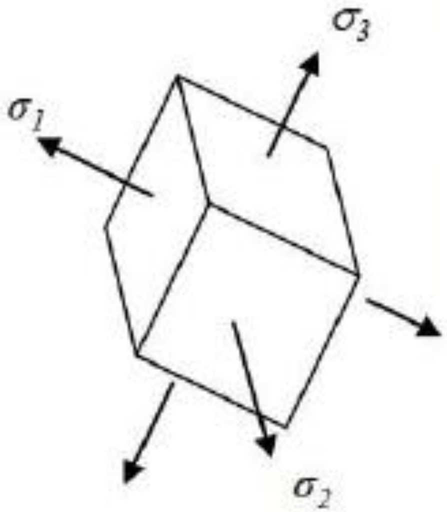
Material	*K* (MPa)	*n*
Aluminum		
1100-O	180	0.20
2024-T4	690	0.16
5052-O	202	0.13
6061-O	205	0.20
6061-T6	410	0.05
7075-O	400	0.17
Brass		
70-30, annealed	900	0.49
85-15, cold rolled	580	0.34
Cobalt-based alloy, heat treated	2070	0.50
Copper, annealed	315	0.54
Steel		
Low-C, annealed	530	0.26
1020, annealed	745	0.20
4135, annealed	1015	0.17
4135, cold rolled	1100	0.14
4340, annealed	640	0.15
304 stainless, annealed	1275	0.45
410 stainless, annealed	960	0.10
Titanium		
Ti-6Al-4V, annealed, 20°C	1400	0.015
Ti-6Al-4V, annealed, 200°C	1040	0.026
Ti-6Al-4V, annealed, 600°C	650	0.064
Ti-6Al-4V, annealed, 800°C	350	0.146

3D Problems



general stress state

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

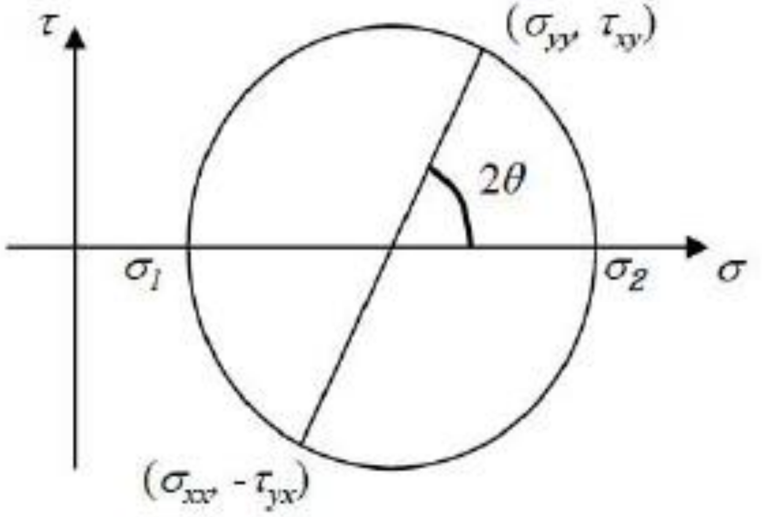


in terms of principal stresses

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

For any general stress state we can find a set of *principal axes*. The stress tensor for these axes contains no off-diagonal (shear) terms – only three principal stresses along the three axes.

Mohr's circle allows rotation of axes in two dimensions about one principal axis



Plastic Yielding Criterion

2-3 TRESCA CRITERION†

This criterion postulates that yielding will occur when the largest shear stress reaches a critical value. Whenever possible, the convention $\sigma_1 > \sigma_2 > \sigma_3$ will be used, but there are cases where this relative comparison is not known a priori. In addition, this convention cannot be maintained rigorously when plots in two- or three-dimensional stress space are considered. This criterion predicts yielding when

$$\sigma_{max} - \sigma_{min} = C \quad \text{or} \quad \sigma_1 - \sigma_3 = C \quad \text{if} \quad \sigma_1 > \sigma_2 > \sigma_3 \tag{2-5}$$

To evaluate C, a state of uniaxial tension may be used. There, $\sigma_{max} = \sigma_1, \sigma_2 = \sigma_3 = 0$, and yielding occurs when $\sigma_1 = Y$, the yield strength in uniaxial tension. Thus,

$$\sigma_1 - \sigma_3 = Y = C \tag{2-6}$$

In the case of pure shear, $\sigma_{max} = \sigma_1, \sigma_{min} = \sigma_3 = -\sigma_1$, and $\sigma_2 = 0$. Yielding occurs when the maximum shear stress reaches the yield strength in pure shear, i.e., the shear yield strength k. At that time, $\sigma_1 = k$, so

$$\sigma_1 - \sigma_3 = 2\sigma_1 = 2k = C \tag{2-7}$$

and the Tresca criterion becomes

$$\sigma_1 - \sigma_3 = Y = 2k \tag{2-8}$$

2-4 VON MISES CRITERION†

This postulates that yielding will occur when some value of the root-mean shear stress reaches a constant or

$$\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{3}\right]^{1/2} = C_1 \tag{2-9}$$

†R. von Mises, *Gött. Nach.*, *math.-phys.*, *Klasse*, (1913), p. 582.

Equivalently,

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = C_2 \tag{2-10}$$

Again, uniaxial tension may be used to define C_2. Substituting $\sigma_1 = Y$ at yielding, and $\sigma_2 = \sigma_3 = 0$, the constant is $2Y^2$. For pure shear, with $\sigma_1 = k = -\sigma_3$ and $\sigma_2 = 0$, $C = 6k^2$, so the von Mises criterion is expressed as

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 = 6k^2 \tag{2-11}$$

In a more general form, this criterion can be written as

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = 2Y^2 = 6k^2 \tag{2-12}$$

Side Note: For a general state of stress use "effective stress"

2-6 EFFECTIVE STRESS

With either yield criterion, it is useful to define an effective stress denoted as $\bar{\sigma}$ which is a function of the applied stresses. If the *magnitude* of $\bar{\sigma}$ reaches a critical value, then the applied stress state will cause yielding; in essence, it has reached an effective level. For the von Mises criterion,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \qquad (2\text{-}16)$$

while for the Tresca criterion,

$$\bar{\sigma} = \sigma_1 - \sigma_3 \quad \text{where} \quad \sigma_1 > \sigma_2 > \sigma_3 \qquad (2\text{-}17)$$

Yielding occurs when $\sigma_{effective} = Y$

Material taken from ***Metal Forming***, by Hosford and Caddell

Origin of effective strain

2-7 EFFECTIVE STRAIN

Effective strain is *defined* such that the incremental work per unit volume is

$$dw = \bar{\sigma}\, d\bar{\epsilon} = \sigma_1\, d\epsilon_1 + \sigma_2\, d\epsilon_2 + \sigma_3\, d\epsilon_3 \qquad (2\text{-}18)$$

For the von Mises criterion, the effective strain is given by

$$d\bar{\epsilon} = \frac{\sqrt{2}}{3}[(d\epsilon_1 - d\epsilon_2)^2 + (d\epsilon_2 - d\epsilon_3)^2 + (d\epsilon_3 - d\epsilon_1)^2]^{1/2} \qquad (2\text{-}19)$$

which may be expressed in a simpler form as

$$d\bar{\epsilon} = \left[\frac{2}{3}(d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2)\right]^{1/2} \qquad (2\text{-}20)$$

If the straining is proportional (with a constant ratio of $d\epsilon_1 : d\epsilon_2 : d\epsilon_3$), the total effective strain may be expressed in terms of the total strains as

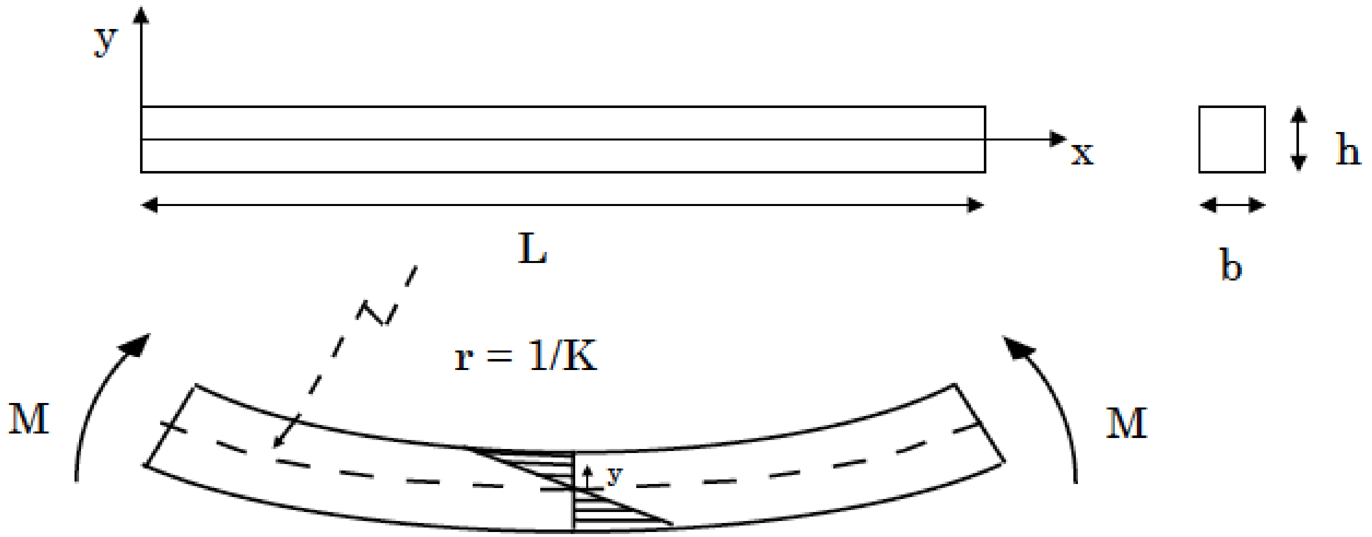
$$\bar{\epsilon} = \left[\frac{2}{3}(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)\right]^{1/2} \qquad (2\text{-}21)$$

If the strain path is not constant, $\bar{\epsilon}$ must be found from a path integral of $d\bar{\epsilon}$. In

$$\bar{\sigma} = K\bar{\varepsilon}^n$$

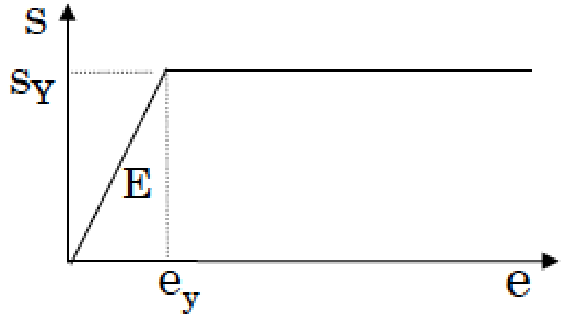
Material taken from ***Metal Forming***, by Hosford and Caddell

Elastic Springback Analysis



1. Assume plane sections remain plane:
   $e_y = - y/r$ (1)

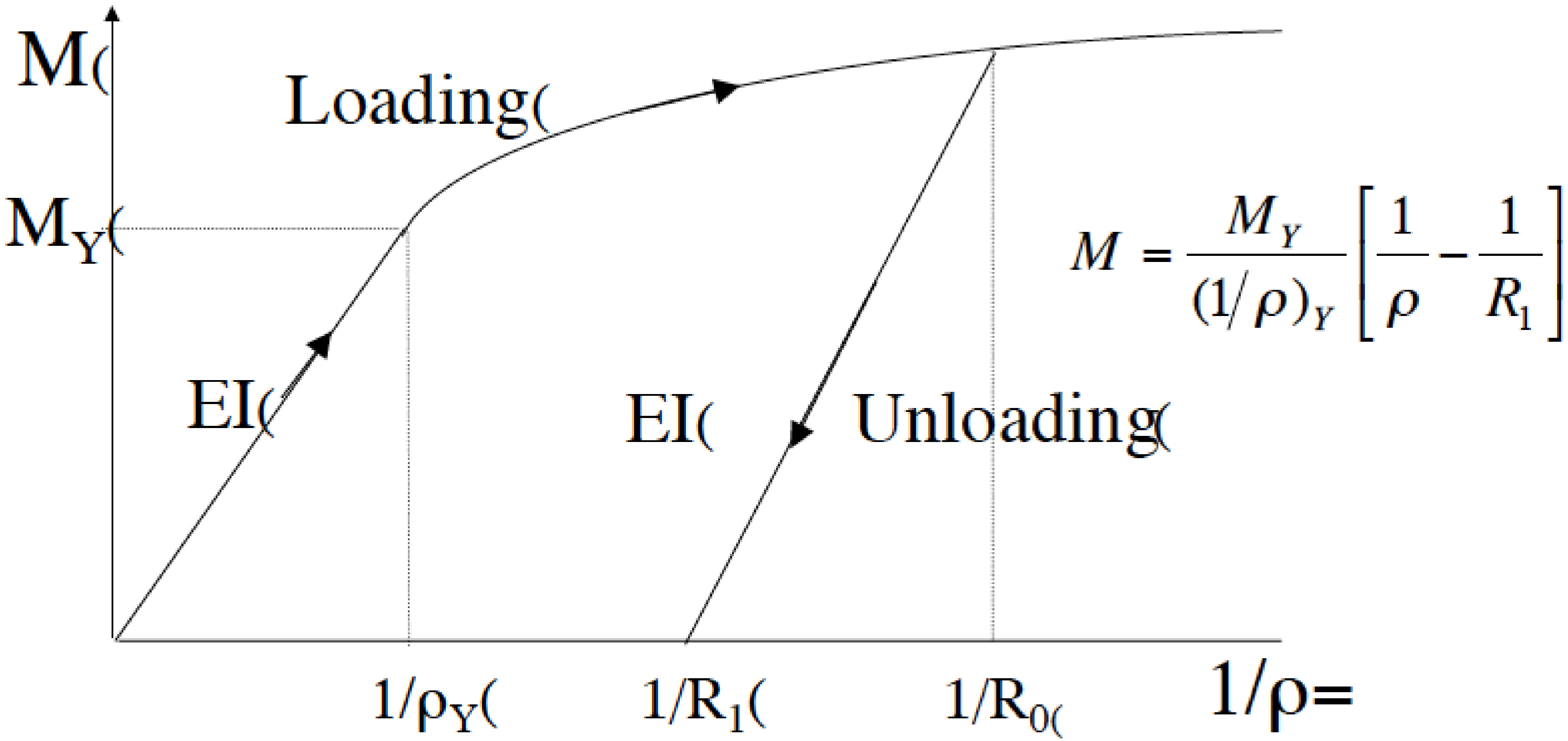
2. Assume elastic-plastic behavior for material



$\sigma = E\, e \qquad e < e_{\psi}$

$\sigma = \sigma_Y \qquad e \underline{\geq} e$

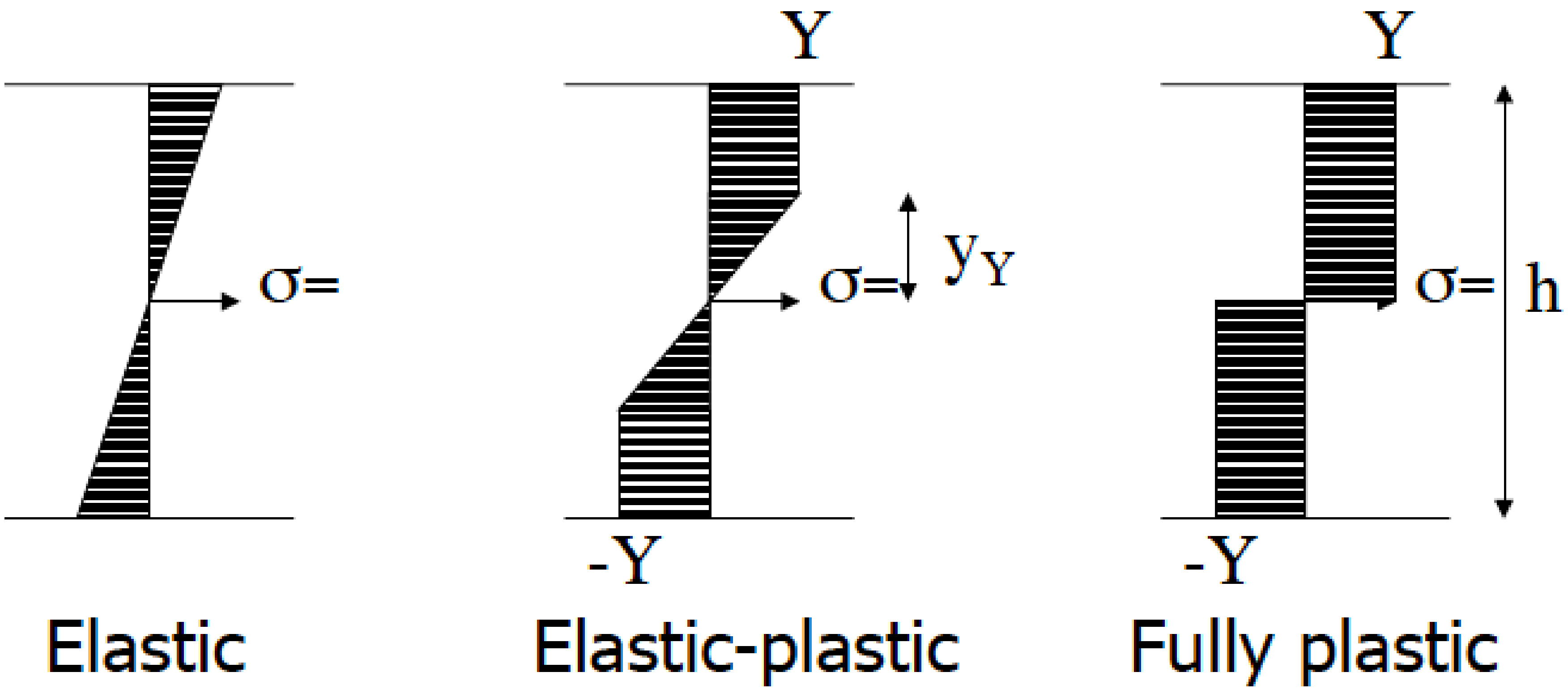
3. We want to construct the following
Bending Moment "M" vs. curvature "1/ρ" curve



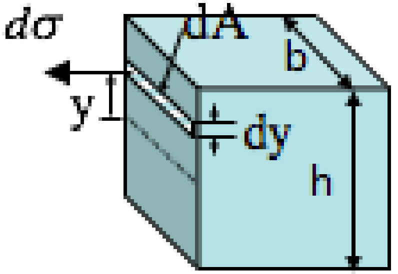
Springback is measured as $1/R_0 - 1/R_1$ (2)

Permanent set is $1/R_1$

4. Stress distribution through the thickness of the beam

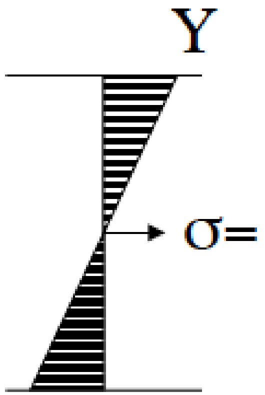


$$5.\ M = \int_A \sigma\, y\, dA$$



Elastic region

$$M = \int \sigma y dA = -E\int \frac{y^2}{\rho} dA = -\frac{EI}{\rho} \qquad (3)$$



At the onset of plastic behavior

$$\sigma = -\ y/\rho\ E = -\ h/2\rho\ E = -Y \qquad (4)$$

This occurs at

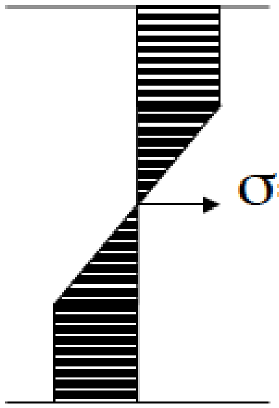
$$1/\rho = 2Y\ /\ hE = 1/\rho_Y \qquad (5)$$

Substitution into eqn (3) gives us the moment at on-set of yield, M_Y

$$M_Y = -\ EI/\rho_Y = EI\ 2Y\ /\ hE = 2IY/h \qquad (6)$$

After this point, the M vs 1/r curve starts to “bend over.” Note from M=0 to M=M_Y the curve is linear.

In the elastic – plastic region



$$M = \int \sigma y b dy = 2\int_{y_Y}^{h/2} Ybydy + 2\int_0^{y_Y} \frac{y}{y_Y} Ybydy$$

$$= 2Yb\frac{y^2}{2}\Bigg|_{y_Y}^{h/2} + 2\frac{Y}{y_Y}b\frac{y^3}{3}\Bigg|_0^{y_Y}$$

$$= Yb(\frac{h^2}{4} - y_Y^2) + \frac{2}{3}y_Y^2 Yb$$

$$M = \frac{bh^2}{4}Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{h/2}\right)^2\right] \qquad (7)$$

Note at $y_Y = h/2$, you get on-set at yield, $M = M_Y$
And at $y_Y = 0$, you get fully plastic moment, $M = 3/2\, M_Y$

To write this in terms of M vs $1/\rho$ rather than M vs y_Y, note that the yield curvature $(1/\rho)_Y$ can be written as (see eqn (1))

$$\frac{1}{\rho_Y} = \frac{\varepsilon_Y}{h/2} \qquad (8)$$

Where ε_Y is the strain at yield. Also since the strain at y_Y is $-\varepsilon_Y$, we can write

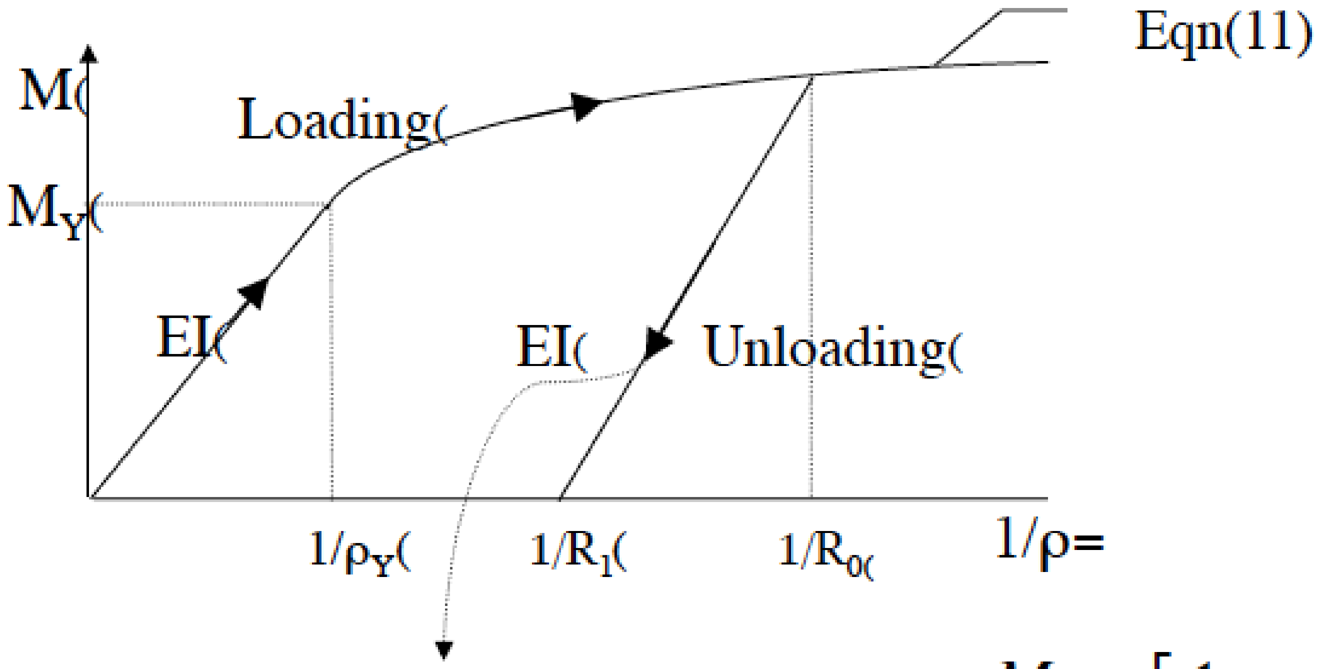
$$\frac{1}{\rho} = \frac{\varepsilon_Y}{y_Y} \qquad (9)$$

Combining (8) and (9) gives

$$\frac{y_Y}{h/2} = \frac{(1/\rho)_Y}{1/\rho} \qquad (10)$$

Substitution into (7) gives the result we seek:

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{(1/\rho)_Y}{1/\rho} \right)^2 \right] \qquad (11)$$



Elastic unloading curve

$$M = \frac{M_Y}{(1/\rho)_Y} \left[\frac{1}{\rho} - \frac{1}{R_1} \right] \qquad (12)$$

Now, eqn's (11) and (12) intersect at $1/\rho = 1/R_0$

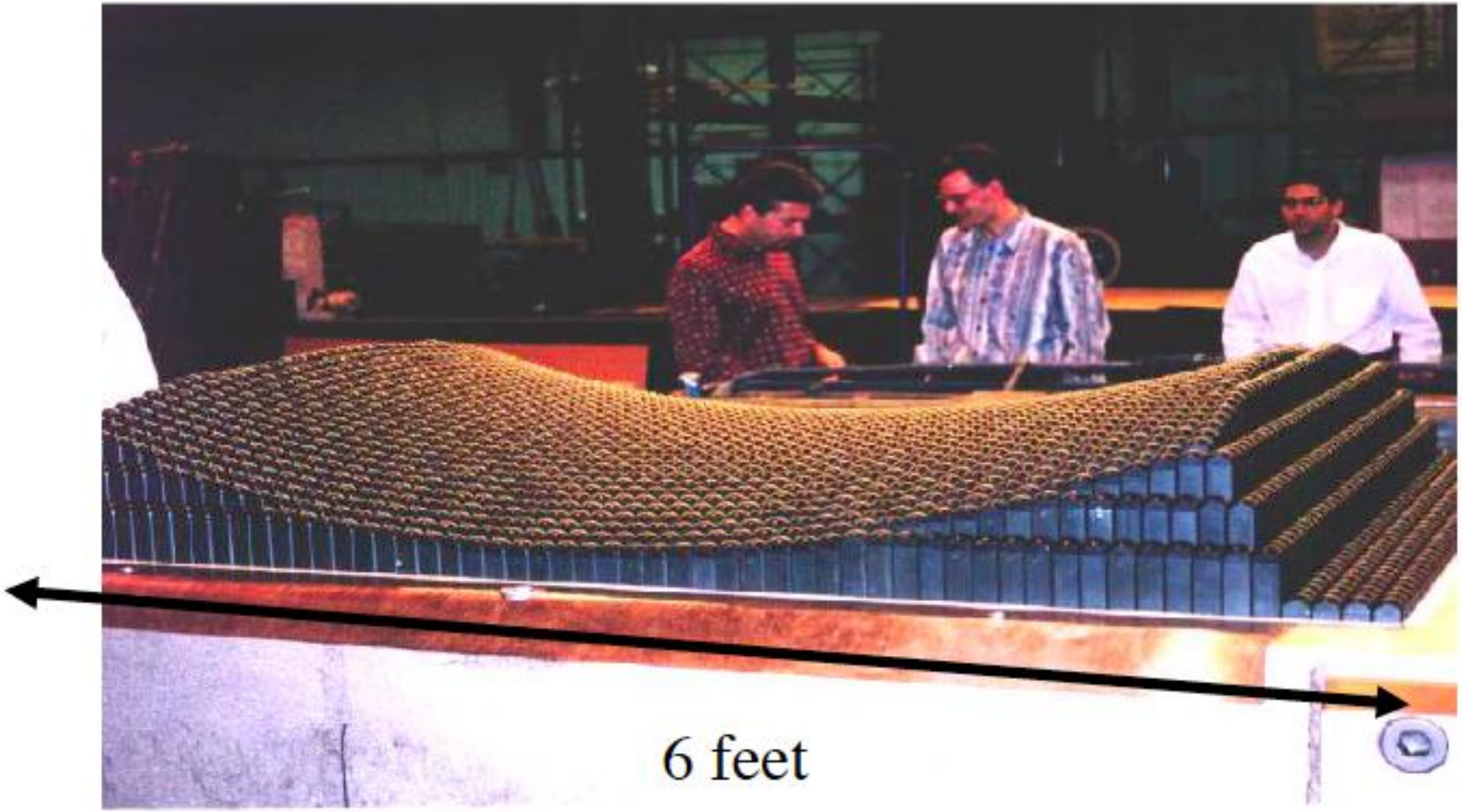
Hence,

$$\frac{M_Y}{(1/\rho)_Y}\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=\frac{3}{2}M_Y\left[1-\frac{1}{3}\left(\frac{(1/\rho)_Y}{1/R_0}\right)^2\right]$$

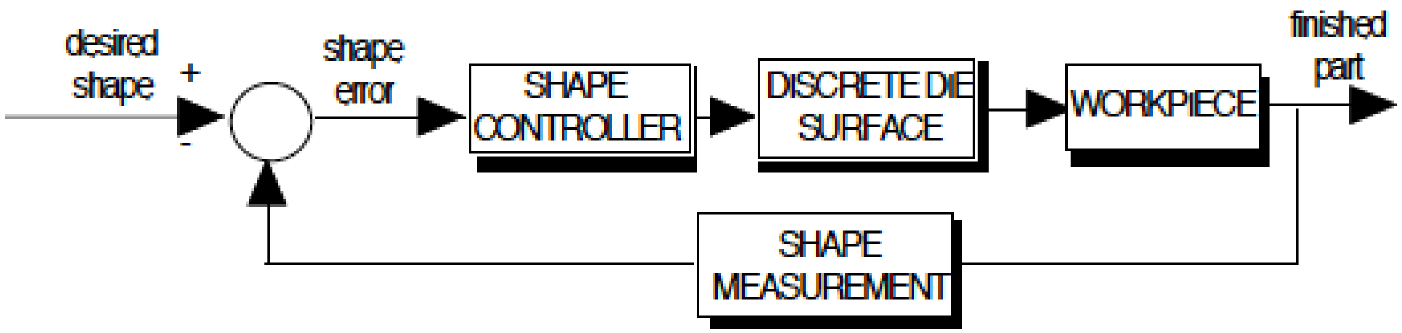
Rewriting and using $(1/\rho)_Y = 2Y / hE$ *(from a few slides back)*, we get

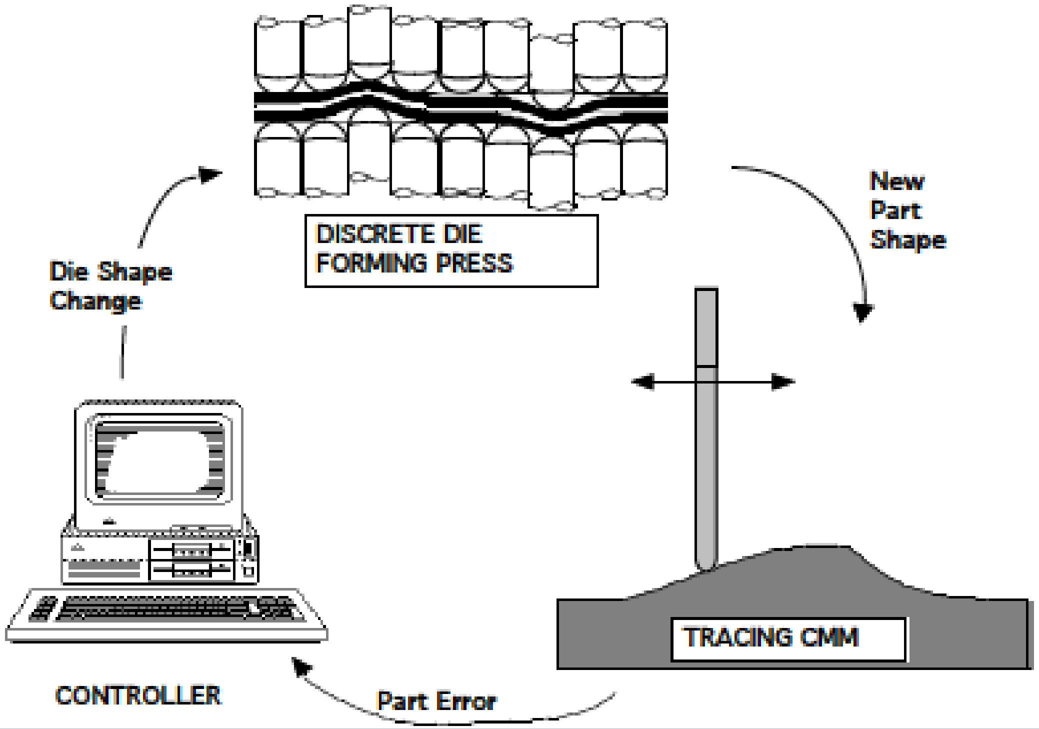
$$\left[\frac{1}{R_0}-\frac{1}{R_1}\right]=3\frac{Y}{hE}-4R_0^2\left(\frac{Y}{hE}\right)^3 \qquad (13)$$

60 Ton Discrete Die Press (LMP - Hardt)



The Shape Control Concept

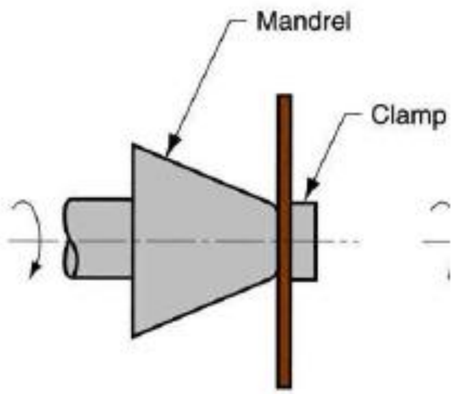


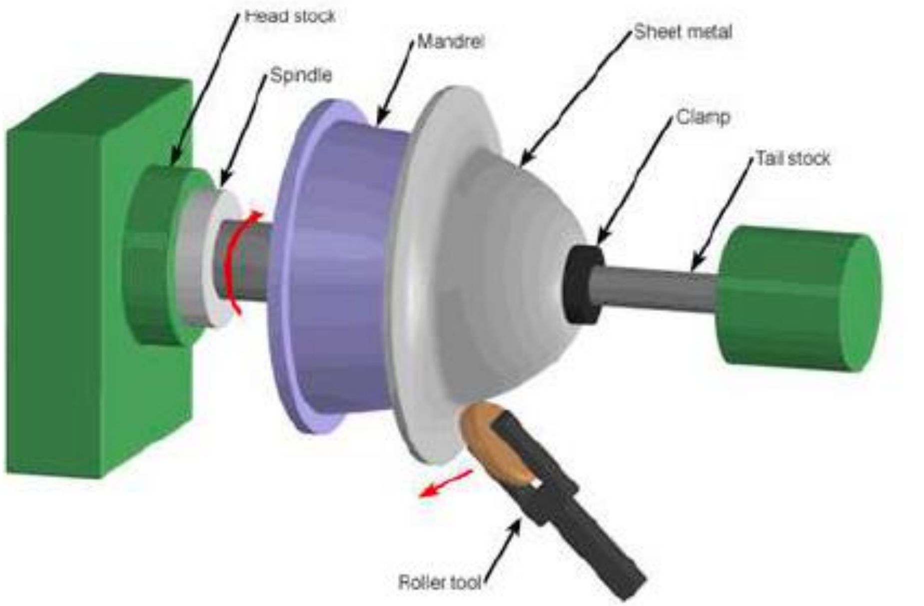


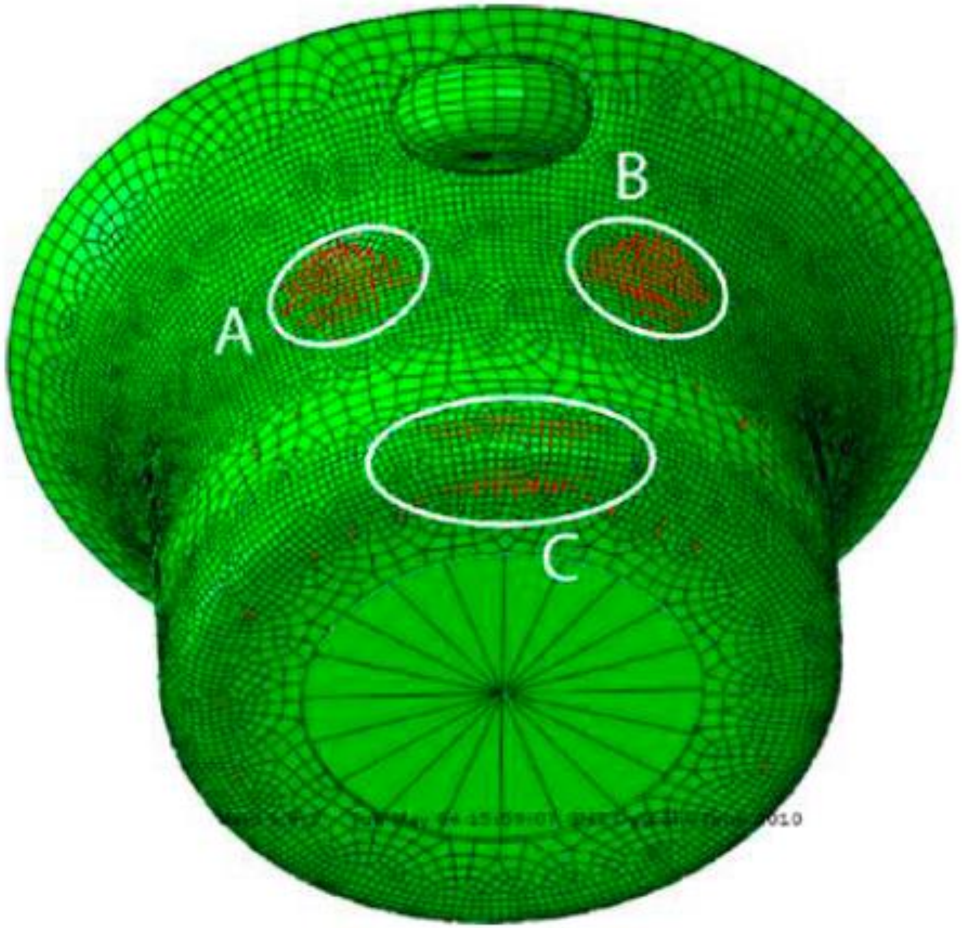
Stretch Forming with Reconfigurable Tool @ Northrop Grumman

Flexible Forming at Ford

Conventional Spinning

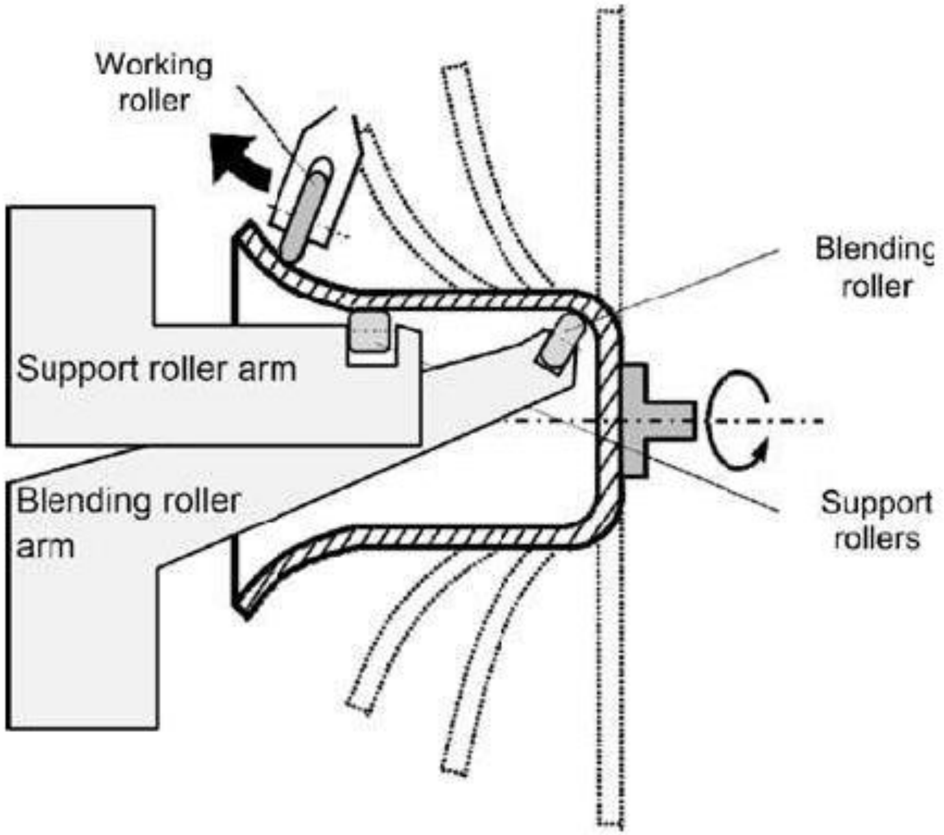




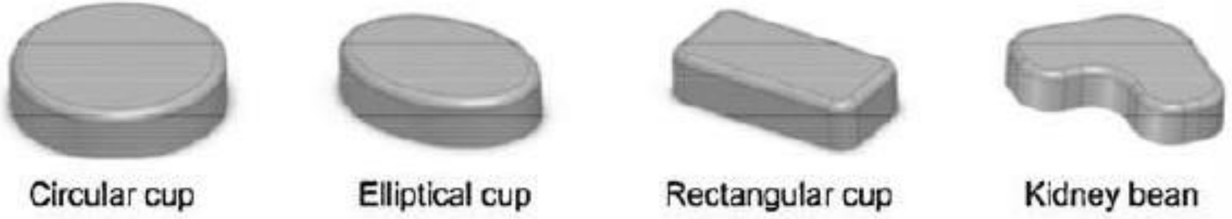


http://www.custompartnet.com/wu/sheet-metal-forming

Flexible Spinning



(b) Machine in operation



Music, O., & Allwood, J. M. (2011). Flexible asymmetric spinning. *CIRP Annals - Manufacturing Technology*, *60*(1), 319–322. doi:10.1016/j.cirp.2011.03.136

Energy & cost: Stamping alum car hoods

- Final part = 5.4kgs
- Total number of parts made = 400
- Die material: cast and machined zinc alloy

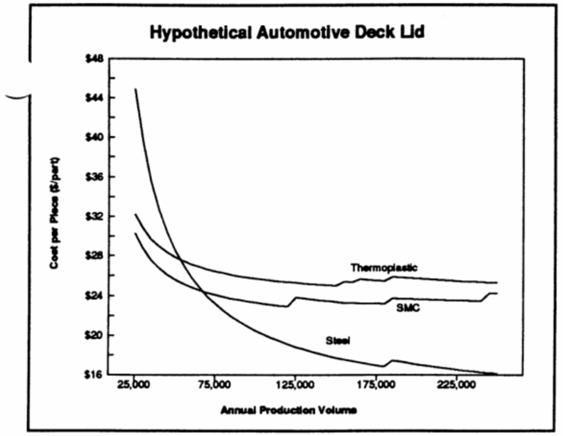
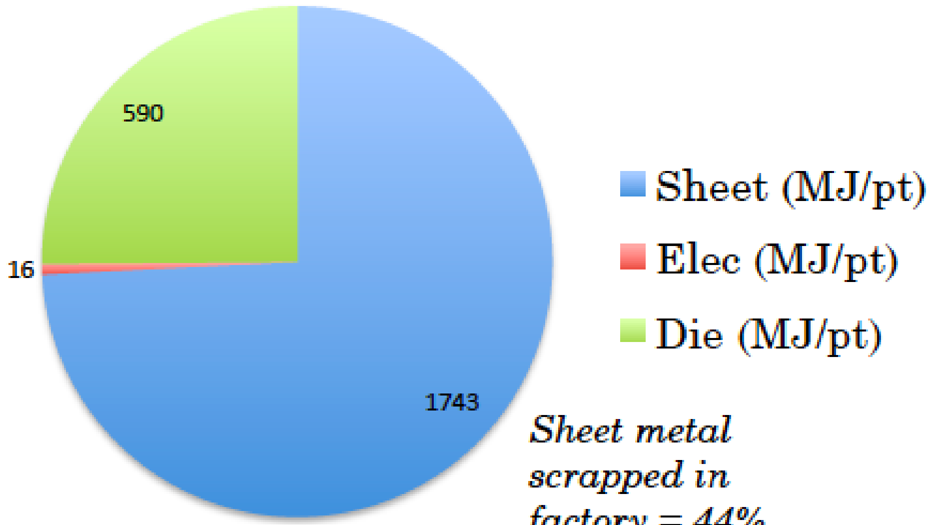


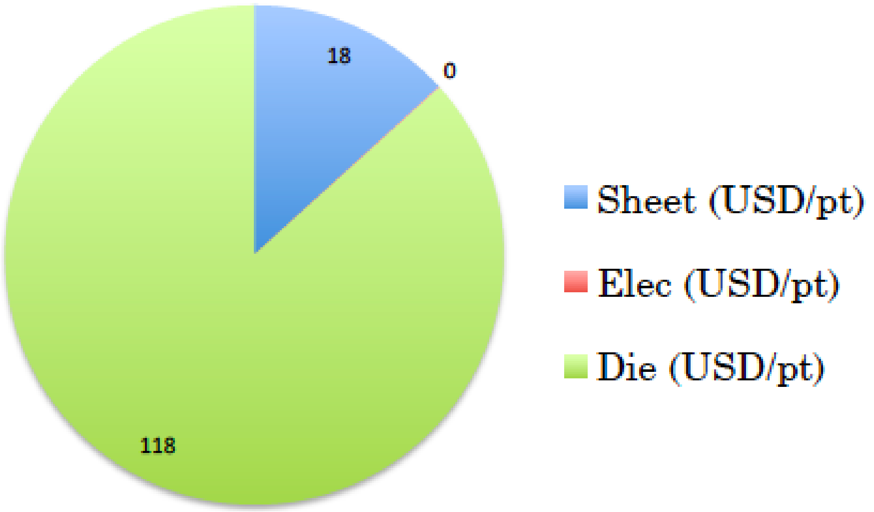
Figure 8: Cost vs. Production Volume

Energy. 2.3GJ/pt. Stamping alum. car hoods. 5.4kgO/P. (400pts)



Sheet metal scrapped in factory = 44%

Cost. 136USD/pt



Source: Unpublished work: Cooper, Rossie, Gutowski (2015)

Excludes equipment depreciation and labor during forming