IPAT Equation Homework for 2.83/2.813

1. Estimate the change in the amount of fertilizer needed for tomatoes if the number of tomato plants increase by 40% and due to efficiency improvements, the fertilizer used per tomato plant decreases by 30%.

2. Let $\boldsymbol{R} = \boldsymbol{Q} \cdot \frac{\boldsymbol{R}}{\boldsymbol{Q}} = \boldsymbol{Q} \cdot \frac{1}{\boldsymbol{e}}$ where R = Resources used, Q = Quantity of the Goods or Services using resources "R", and e = "eco efficiency", then

   $\boldsymbol{R}_1 = \boldsymbol{Q}_1 \cdot \frac{1}{\boldsymbol{e}_1}$ and $\boldsymbol{R}_2 = \boldsymbol{Q}_2 \cdot \frac{1}{\boldsymbol{e}_2}$

   Write an expression for $\frac{\Delta \boldsymbol{R}}{\boldsymbol{R}_1}$ that is valid even if there are large changes in Q and e. Verify that it gives you the same result as in problem 1.

3. Consider a more general equation for resources used as

   $R = KQ^{\alpha}e^{\beta}$

Here K = constant. This type of equation is not necessarily dimensionally homogeneous, instead it is based upon the fact that R and Q are correlated and R and e are correlated. Furthermore it assumes that Q and e are not correlated. Economists use these kinds of equations to characterize the behavior of complex systems. (A specific example of an equation like this would be the so called Cobbs – Douglass production function). The coefficients α and β can be obtained from regression analysis if one has the data. Furthermore these values are referred to as "elasticities" in the economics literature.

For example the change in resources used R, with respect to a change in the quantity Q is defined as the elasticity $\eta_{\boldsymbol{Q}} = \frac{\partial \boldsymbol{R} / \boldsymbol{R}}{\partial \boldsymbol{Q} / \boldsymbol{Q}} = \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{Q}} \cdot \frac{\boldsymbol{Q}}{\boldsymbol{R}}$

Show that $\eta_{\boldsymbol{Q}} = \alpha$ and $\eta_{\boldsymbol{e}} = \beta$

4. Following on from problem 3, if Q increases to λQ and e increase to λe ($\lambda > 1$) it follows that there should be no increase in R. Mathematically this is

   $R = KQ^{\alpha}e^{\beta} = K(\lambda Q)^{\alpha}(\lambda e)^{\beta}$

   Show that this requires that $\alpha = -\beta$

5. See paper by Waggoner and Ausubel 2002. p. 7862 "Forces Connect with each other".

Verify their proposition that $a + c = b \times a$

6. See Waggoner and Ausubel p. 7863

   Verify the proposition stated in the second paragraph on that page: "If we know the income elasticity b of per capita consumption….

7. See Waggoner and Ausubel p. 7863, fifth paragraph: …Verify their proposition stated as…"Let population in an area be related to income with an elasticity *bp,…..*"