What Gets Recycled: An Information Theory Based Model for Product Recycling

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This work focuses on developing a concise representation of the material recycling potential for products at end-of-life. To do this we propose a model similar to the "Sherwood Plot", but for products rather than for dilute mixtures. The difference is reflected in the material composition and the processing systems used for the two different applications. Cost estimates for product recycling systems are developed using Shannon's information theory. The resulting model is able to resolve the material recycling potential for a wide range of end-of-life products with vastly different material compositions and recycling rates in the U.S. Preliminary data on historical trends in product design suggest a significant shift toward less recyclable products.

1. Introduction

In 1994, Allen and Behmanesh proposed that the "Sherwood Plot" could be used to indicate the recycling potential of material waste streams (*1*). More recently, Johnson et al. consider applying this same approach to product recycling (*2*). The "Sherwood Plot" is named after Thomas K. Sherwood, a chemical engineer who published a figure in 1959 that indicated a close relationship between the price of a material and its dilution (1/concentration) in the feed stream (*3*). Since that time, this relationship has been confirmed for a wide variety of materials in dilute solutions including metals, biological materials, and pollutants (*4−8*). Because of the large range in prices and concentrations, the plot is always presented in log−log form, but the relationship is essentially linear between material value "k_v" ($/kg) and dilution "$1/c_v$", where c_v is the mass fraction concentration of the target material (valuable material) in the feed stream (entire mixture).

The Sherwood plot can be derived from a simple economic model for revenues and costs. Profitability requires that revenues from the sale of the target material exceed the costs of extraction and processing. Assuming these costs scale with the amount of material processed, one gets the requirement for profitable extraction that,

$$k_v m_p c_v > k_c m_p \qquad (1)$$

where k_v is the market value of the target material ($ per kg of target material), m_p is the total mass of mixture processed (kg of mixture), c_v is the concentration of the target material in the mixture (kg of target material per kg of mixture), and k_c is the cost of processing the mixture ($ per kg of mixture).

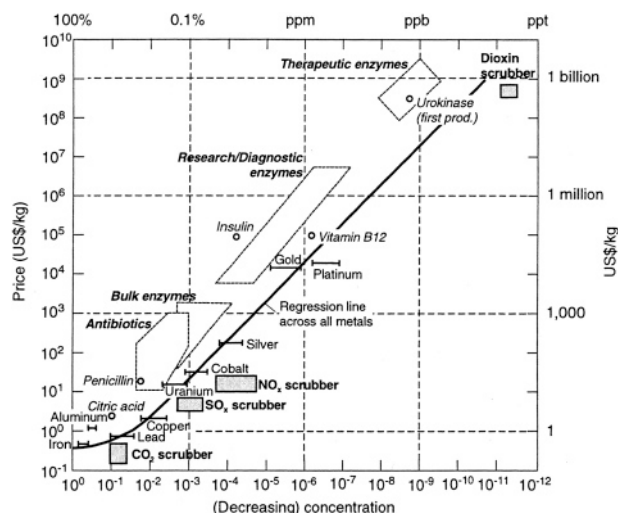* Corresponding author phone: 617 253-2034; fax: 617 253-1556; e-mail: gutowski@mit.edu.

FIGURE 1. Sherwood plot showing the relationship between the concentration of a target material in a feed stream and the market value of the target material. Figure from Grübler (*8*).

Simplification yields

$$k_v > \frac{1}{c_v} k_c \qquad (2)$$

which can be written as an equality if a multiplicative constant is included to account for profit.

Further work by Holland and Petersen shows that the cost of metals prepared from dilute ores is dominated by mining and milling costs, whereas the cost of metals prepared from concentrated ores is dominated by smelting and refining costs. This work shows that the cross over occurs between 1 and 10% (*7*). Hence, in the dilute region ($c_v < 0.05$), the Sherwood Plot captures the essential cost scaling. In other words, $1/c_v$ represents the transport and processing of large quantities of materials that dominates over all other cost factors for the extraction of a target material from a very dilute mixture. Figure 1 shows an extensive Sherwood Plot prepared by Grübler (*8*). The central line is the curve fit for metals provided by Holland and Petersen (*7*). Note that the curve flattens in the concentrated ore regime where the $1/c_v$ cost scaling is no longer valid. In the dilute region, the plot reveals three different groups. Parallel to the central line, one could draw a line through the biological materials with a slope of about $1/kg of mixture. Similarly a line through the scrubber data would have a slope of about 0.1¢/kg of mixture. The central line through the metals data has a slope of about 1¢/kg of ore in the dilute region. These trends can be used to assess the potential profitability of separation operations in a gross sense. Ores and dilute mixtures located below and to the right of the appropriate line would be unprofitable; those above and to the left, profitable.

2. Data Collection And Model Development

Products vary widely in their material recycling potential. A survey of the products used by the typical household will reveal that most products are not recycled (*9*). Those that are economically recycled must satisfy the requirement that their potential revenues exceed the costs of collection, processing, and transport. To understand these costs, we studied 10 different recycling systems for a variety of products and waste streams, from automobile recycling to curbside collection to computer recycling (*10, 11*). What we found were complex
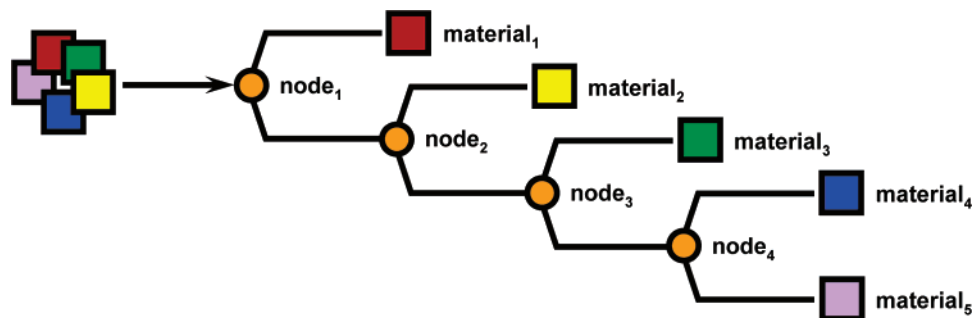
FIGURE 2. Example of a tree diagram showing a material separation system for five materials using four binary separation steps labeled as nodes 1 through 4.

systems that attempted to separate many different materials, primarily by binary separation steps, then shipped these materials to various locations for further processing. Conceptually, all of these systems could be modeled as tree diagrams, with the trunk as the input stream and the branch ends as the material outputs, as shown in Figure 2. We observed that these "tree" systems produced a variety of valuable output materials, the input source was highly variable, the waste stream required collection, monitoring, and disposal, and the concentration of valuable materials in the input stream was high, in every case over 50% ($c_v > 0.5$). This led us to the conclusion that recycling system costs for products would, by and large, scale in a different way from the Sherwood Plot. Simply put, ideal "Sherwood material systems" are made up of a valuable target material in a dilute solution. On the other hand, products are made up of concentrated mixtures with many target materials, all of which must be separated and transported to make the system viable. We will come back to the distinction between these two material systems at the end of the paper and show that they are essentially mutually exclusive.

2.1. Proposal For a Product Recycling Heuristic. These observations led us to a proposal for an alternative cost scaling heuristic for the mixed material systems which are characteristic of products. The core features of this proposal are that (1) the processing costs scale like some measure of the complexity of the separation system, (2) all materials are targeted (this includes the mixed waste stream), (3) material counting corresponds to the categories established by recycled materials markets, (4) all separation processes are equal in cost, and finally, (5) this model only considers material recycling (component recycling is mentioned at the end of this paper). Additional comments on these assumptions can be found in the Supporting Information. Although this representation of recycling systems may seem overly simplistic, we will show that it appears to be able to clearly differentiate between those products which are currently recycled, and those products which are not.

To scale the complexity of the separation system, several alternatives are possible. Perhaps the simplest is to propose that these costs scale with the number of target materials "M", or the minimum number of binary separation steps to separate these materials, "$M - 1$". Alternatively a thermodynamic approach could also prove profitable. Ayres notes that log price in the Sherwood plot scales like the thermodynamic work of extraction (negative the work of diffusion) for a single material of interest (*12*). In our case, it seems reasonable to assume that separation costs for mixed material systems would scale as the thermodynamic work of separation (negative the work of mixing for an ideal solution). We already know that both of these approaches would provide useful results. Nevertheless, we propose an alternative approach to this problem, based upon information theory (*13*, *14*). It will become clear that the information theory approach contains both the material counting approach and

the thermodynamic approach. Furthermore, even though our model is at a rather abstract level, it provides more physical insight into why it works. Finally, we have observed that the information theory approach outperforms the material counting approach in its ability to differentiate between products which are recycled and those which are not (*20*).

2.2. Information Theory and Recycling Systems. During our study of product recycling systems, we realized that there were a number of features which are very similar to how Shannon conceptualized communication systems (*15*, *16*). Perhaps the most important similarity is that both material separation systems and message coding systems can be represented as tree diagrams. Furthermore, both systems often employ binary steps. For example, in recycling systems, a material is either magnetic or it is not, while in coding systems, a character is either a "1" or a "0". We emphasize that binary steps are not required for either recycling or coding, but they occur frequently in both, thus making the analogy that much easier to interpret. A tree diagram representing the separation of a mixture of five materials was shown in Figure 2. The system has four nodes or separation points and five branch ends or material outputs. Other configurations for separating five materials are possible, but all can be represented as tree diagrams. The analogy in information systems is that tree diagrams can be used to represent code words. In both cases the size and the complexity of the tree diagram represents the level of effort required to perform the intended task (e.g., decode a message, or separate the materials in a product). A useful measure of this effort can be obtained by tracing the path to each end point and counting the number of nodes traversed, n_i. This is then multiplied by the probability of occurrence (mass concentration in our case), and then summed over all paths. The result, "$\bar{n}$" is the average word length in information theory, or a measure of the operating cost for a recycling system, and is given by

$$\bar{n} = \sum_{i=1}^{M} c_i n_i \tag{3}$$

where M is the number of materials in the product, c_i is the mass concentration of material i, and n_i is the number of separation steps necessary to isolate material *i*.

This measure could serve as our metric for the cost of separation, but it requires some knowledge about how recycling systems are structured and this could vary widely between different systems. There is an alternative approach, which follows from information theory. This is to define a measure of mixing, based only on the materials and concentrations in the product, and then show that this measure is a lower bound for the cost metric $\bar{n}$. This procedure follows directly from Shannon's development of a measure of uncertainty, H, and then uses his Noiseless Coding
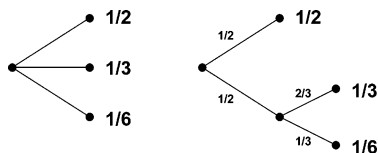
FIGURE 3. Illustration of the weighted sum property for mixtures of mixtures.

Theorem, which shows that H is a lower bound for the average word length for any coding scheme (*13, 17*).

The first step in our development then, is to propose a measure of material mixing, H, which has the following properties, (and here we paraphrase Shannon):

(1) H should be continuous in the c_i.

(2) If all the c_i are equal, $c_i = 1/M$, then H should be a monotonic increasing function of M. (With equal concentrations, there is more mixing when there are more materials.)

(3) H should be additive. Thus, if a mixture can be broken down as a mixture of mixtures, then the final H should be the weighted sum of the individual component values of H. The meaning of this is illustrated in the equality below and figure 3. On the right side of Figure 3 we see a mixture of mixtures. On the left we see the final three-component mixture. For this special case, making the two representations of this problem equal requires that

$$H\left(\frac{1}{2},\frac{1}{3},\frac{1}{6}\right) = H\left(\frac{1}{2},\frac{1}{2}\right) + \frac{1}{2}H\left(\frac{2}{3},\frac{1}{3}\right)$$

Shannon showed that the only H satisfying these three assumptions is of the form

$$H = -K \sum_{i=1}^{M} c_i \log c_i \qquad (4)$$

where K is a constant, M is the number of materials, and c_i is the concentration of material i.

By convention, we set $K = 1$, and take logarithms to the base two, yielding H in bits.

For our purposes, we will use H as a measure of material mixing. H can be interpreted as the average number of binary separation steps needed to obtain any material from the mixture. Of course, this function is also quite similar to the thermodynamic work of separation for an ideal solution, in which case we would use mole fractions instead of mass fractions and the constant would be different. The final step is to show that H is a lower bound for $\bar{n}$. As already mentioned, Shannon shows this in his Noiseless Coding Theorem (*15–17*). The assumptions in information theory can be applied directly to recycling (*13*). One important assumption is that each branch end result is a unique material. In fact, the sequence of separation steps defines the material. The result is that

$$\sum_{i=1}^{M} c_i n_i \geq \sum_{i=1}^{M} c_i \log_2 c_i \qquad (5)$$

or

$$\bar{n} \geq H \qquad (6)$$

We will use H, our measure of material mixing, as our estimate of the cost of separation. From a practical point of view, this result greatly simplifies the cost calculation, because H only requires knowledge about aspects of the material counting

TABLE 1. Product Data Used in Figure 4[a]

product	$\sum m_i k_i$ ($)	H (bits)	recycling rate
automobile battery	$10.95	1.30	96%
automobile	$358.61	2.22	95%
catalytic converter	$107.54	.699	95%
refrigerator	$34.69	1.67	90%
newspaper	$0.028	.095	70%
automobile tire	$1.85	.575	66%
steel can	$0.004	.060	63%
aluminum can	$0.019	.001	45%
HDPE bottle (no. 2)	$0.012	.163	27%
PET bottle (no. 1)	$0.008	.476	23%
glass bottle	$0.002	.003	20%
desktop computer	$17.69	2.36	11%
television	$7.05	2.09	11%
laptop computer	$2.79	2.89	11%
aseptic container	$0.005	1.10	6%
cell phone	$0.908	2.91	1%
work chair	$12.19	2.27	0%
fax machine	$6.43	2.09	0%
coffee maker	$0.535	1.93	0%
cordless screwdriver	$0.130	1.80	0%

[a] Recycled Material Values from refs *18* and *19*. References for the product bills of materials and recycling rates can be found in the Supporting Information.

scheme for the product, and no detailed knowledge about the nature of the recycling system.

3. Results

Using the results from information theory, our profitability requirement for product recycling is,

$$\sum_{i=1}^{M} m_i k_i > H k_b \qquad (7)$$

where m_i is the mass of material i (kg), k_i is the value of material i ($ per kg), k_b is the processing cost per bit ($ per bit), H is the measure of material mixing (bits).

To test (*7*), we analyzed 20 products with widely different material compositions and recycling rates in the United States. Material counting was based upon identifying all valuable recycled materials which could be separated from the product. The material values were obtained from market data on recycled materials, as reported by the RecycleNet Corporation (*18*), and estimated from market data on virgin materials, as priced on the New York spot market (*19*). The amounts of the materials and their concentrations were obtained primarily from published bills of materials for each of the products. Note that the recycling rates for these products ranged from 0% for coffee makers, cordless screwdrivers, fax machines, and work chairs, to 96% for automobile batteries. Values for H varied from 0.001 bits (aluminum can) to 2.91 bits (cell phone). A summary of the data for each product is given in Table 1. The materials that were counted are listed in Table 2. The references for product materials, composition and recycling rates are all given in the Supporting Information.

The results of these calculations are plotted in Figure 4. Here we see material value in $ along the y axis, and material mixing, H, in bits, along the x axis. The recycling rates for each product are conveyed by the area of the circle around each data point. Products with no circle have recycling rates of essentially zero. It is readily apparent that the data are segregated, with the products with high recycling rates in the upper left corner, and the products with the very low recycling rates in the lower right. This trend is particularly clear for products with $H > 0.5$, where the recycling rates range from 66 to 96% in the upper left, and from 0 to 11%

TABLE 2. Material Counting Scheme (25 Materials) Used for the Results Given in Figure 4

nonprecious metals	precious metals	plastics	nonmetal, nonplastic
aluminum	gold	acrylonitrile/butadiene/styrene (ABS)	glass
copper	palladium	polyamide (PA)	paper
iron	platinum	polycarbonate (PC)	rubber
lead	rhodium	polyethylene (PE)	other
nickel	silver	polyethylene terephthalate (PETE)	
steel		polypropylene (PP)	
tin		polystyrene (PS)	
zinc		polyvinyl chloride (PVC)	

in the lower right, with an abrupt transition zone between them. To emphasize this point, we have labeled a line as the "apparent recycling boundary". Although our model does not give the exact location of this boundary, it can be seen that in the region where the products are complex mixtures, with approximately $H > 0.5$, there is a rapid change in recycling rates confined to a diagonal region in the vicinity of this line.

In the lower left corner of Figure 4 things are a little less clear. This is due in part because we have not included many additional low value items which have very low, or zero recycling rates such as Styrofoam cups, plastic bags, staples, straws, gum wrappers, etc. These would all lie below 10^{-3} dollars and have H values below 0.5. The inclusion of these items would have made the straight extension of the "apparent recycling boundary" below $H = 0.5$ look quite

secure. Nevertheless, the group of products in this region, made up of bottles, cans, and newspaper, does have a mix of recycling rates (20−70%) which calls into question exactly where the boundary should fall. For these reasons we hesitate to extend the boundary below about $H = 0.5$ at this time. We return to this issue in the next section.

The results shown in Figure 4 are sensitive to the material counting scheme employed. In general, materials should be counted as they are valued and separated, and one should take care not to double count. While the accounting for material value is relatively straightforward, the effect of material counting on the material mixing parameter H is less clear. In particular, we were concerned that the relative positions of products might change for different counting schemes and thus alter the results shown in Figure 4. To test this we investigated counting both fewer and more materials
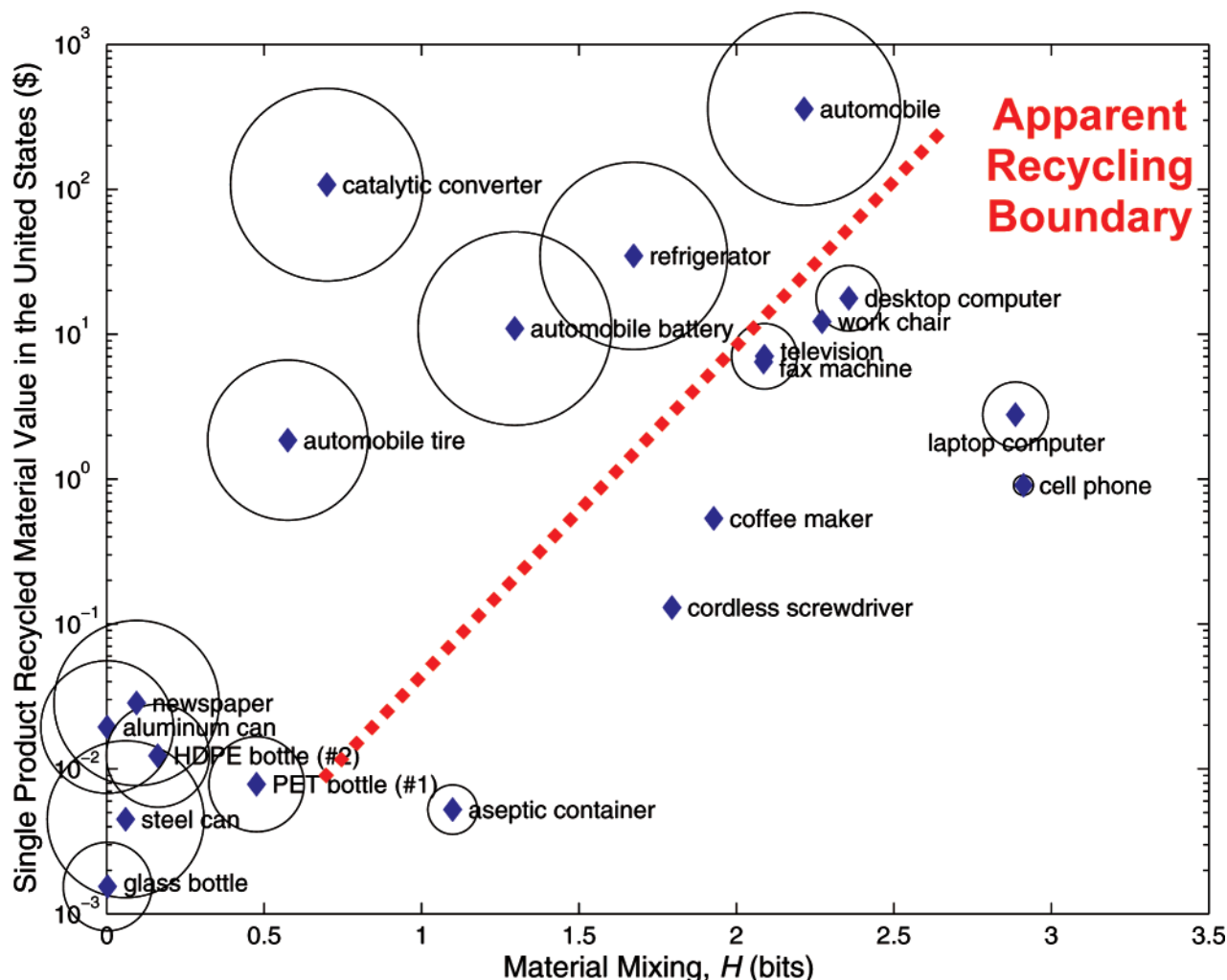

FIGURE 4. A plot of single product recycled material values ($\sum m_i k_i$), material mixing (H), and recycling rates (indicated by the area of the circles) for 20 products in the U.S. The "apparent recycling boundary" is shown.

TABLE 3. Four Different Material Counting Schemes Corresponding to 4, 10, 25, and 40 Materials, as Used to Generate Figure 5

low-level	mid-level	high-level	ultra high-level
ferrous metals	aluminum	aluminum	aluminum
non-ferrous metals	copper	copper	antimony
plastics	iron	gold	beryllium
other	lead	iron	brass
	nickel	lead	cadmium
	steel	nickel	chromium
	tin	palladium	cobalt
	zinc	platinum	copper
	plastics	rhodium	gold
	other	silver	iron
		steel	lead
		tin	magnesium
		zinc	mercury
		acrylonitrile/butadiene/styrene (ABS)	nickel
		polyamide (PA)	palladium
		polycarbonate (PC)	platinum
		polyethylene (PE)	rhodium
		polyethylene terephthalate (PETE)	silver
		polypropylene (PP)	stainless steel
		polystyrene (PS)	steel
		polyvinyl chloride (PVC)	tin
		paper	zinc
		glass	acetal
		rubber	acrylic resin
		other	acrylonitrile/butadiene/styrene (ABS)
			epoxy resin
			phenolic resin
			polyamide (PA)
			polycarbonate (PC)
			polyethylene (PE)
			polyethylene terephthalate (PETE)
			polymethyl methacrylate (PMMA)
			polyoxymethylene (POM)
			polypropylene (PP)
			polystyrene (PS)
			polyvinyl chloride (PVC)
			paper
			glass
			rubber
			other

than used in Figure 4. Four different material counting schemes are listed in Table 3. Figure 5 shows the results of using these different schemes for four different products. It can be seen that, while the H values of the products shifted upwards as the number of materials counted was increased, the relative order of the products did not change for schemes that counted between 10 and 40 materials. Furthermore, the
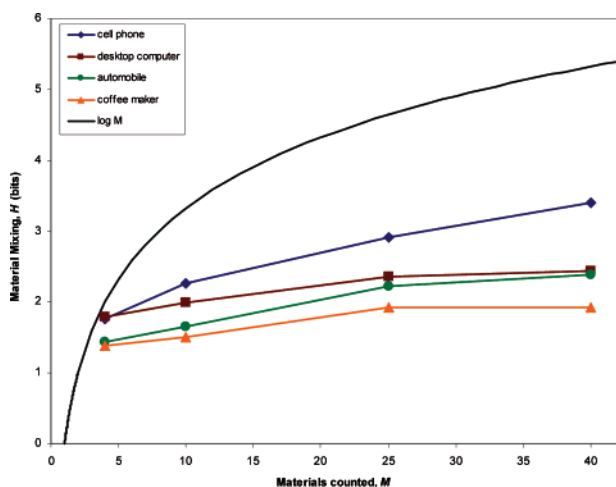

FIGURE 5. Material mixing, H, versus number of materials counted for the four different material counting schemes given in Table 3. The upper line, log M, represents the upper limit on material mixing values.

changes in H indicated in Figure 5 would not change the basic conclusion drawn from Figure 4. (Here we are assuming that material values are properly averaged and hence do not change with different counting schemes.) This indicates that the materials counted must be specified, but that the basic result indicated in Figure 4, that there is a clear transition from products with very low recycling rates to products with very high recycling rates in the region $H > 0.5$, appears to be robust to changes in the counting schemes for H. Note that the material value of a product can be influenced by a small amount of high valued material, and should be included. Furthermore, in the special case where a small concentration of valuable material is separated from a product, and the rest is treated as waste, the product could be considered a dilute solution, and treated as in the Sherwood Plot. We return to this issue in the next section. For more details on materials counting, see the Supporting Information and ref *20*.

4. Discussion

4.1. Modeling Issues. This paper presents a cost scaling scheme for product recycling based upon a Shannon Information type mixing metric. A feature of this work is that we differentiate between material mixtures that are dilute, and therefore can be treated on the Sherwood Plot, and the concentrated mixed material systems typical of products. We would like to explore the differences between these two situations. This can be done by writing an expression for the largest value of H obtainable for a very general mixture made up of M materials. Of the M materials in this mixture,
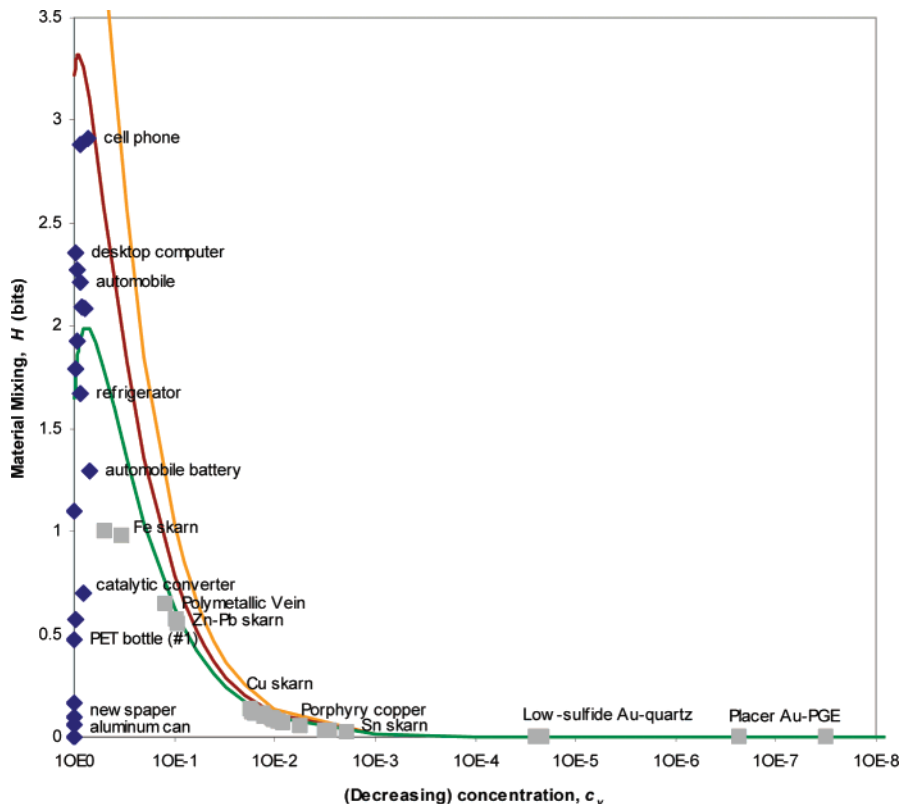
FIGURE 6. Plot showing various products and metal ore deposits in terms of H and $1/c_v$ along with eq 8 for $M = 4$ (green), 10 (red), and 50 (yellow). Select data points are labeled with the names of products, as in Figure 4, or names of mineral deposit models per Cox and Singer (**21**).

$M - 1$ materials are considered of value, and together have a mass concentration c_v, and the one remaining material is waste, with a mass concentration $1 - c_v$. The largest value of H obtainable for any given mixture of this type, would be the one with the $M - 1$ valuable materials evenly distributed within their mass fraction c_v. Using the additive property of H, this can be written as,

$$H \leq (1 - c_v)\log_2 \frac{1}{1 - c_v} + c_v\log_2 \frac{(M - 1)}{c_v} \qquad (8)$$

This equation says that as a solution or ore gets increasingly dilute (the kind that the Sherwood Plot can treat), H becomes smaller and smaller. In fact in the limit as $c_v \rightarrow 0$, $H \rightarrow 0$, for any value of M. As a practical example consider a relatively nondilute mixture near the lower bound of what can be treated as a "Sherwood material" with $c_v = 10^{-3}$ but with 9 co-mined valuable materials, i.e., $M = 10$. Equation 8 above gives the upper bound on H, as 0.015 bits. This is a very small value of H, about 2 orders of magnitude smaller than the typical complex products we analyze in our paper and well below the range where we draw our "apparent recycling boundary" i.e., $H > 0.5$. Nevertheless, this value does overlap with some of the simple products we analyze.

We can gain further insight into this problem if we create a plot for the two different material types. This is shown in Figure 6 above. Here, we have plotted a large number of mixed-material products (blue diamonds) and co-mined ores (gray squares). The x axis is as plotted in Figure 1 and corresponds to the dilution parameter $1/c_v$. This number implies a value statement concerning what part of a mixture is of value and worth capturing, and what part is not. The y-axis is the measure of material mixing H, measured in bits. The material counting scheme for calculating H includes all of those materials that are separated including the waste stream.

The results show that the two material systems discussed in this paper, essentially lie along different axes. That is, in general, dilute mixtures are confined to the x axis with only very small values of H, whereas the products are confined to the y axis and are quite concentrated. Near the origin, however, there is a third region of concentrated, relatively simple material systems. This region includes concentrated ores such as iron and aluminum, and simple products such as bottles and cans. We can also plot eq 8 with different values of M to show the upper bounds on H. This is done for $M = 4$, 10, and 50. As can be seen, the three lines converge in the dilute region, eliminating the possibility of a dilute solution with a large value of H.

It is in the third region however, where we will eventually run into the limits of both models. As already mentioned, the lower bound for the Sherwood plot for metals is in the range of concentrations between 1 and 10% (**7**), whereas for the products, it is probably somewhere below $H = 0.5$. It is in this region ($H < 0.5$) where we are unable to resolve the details of the recycling rates for the simple products in the lower left-hand corner of Figure 4. This is likely to be related to the low values of H, and the growing importance of other factors. Nevertheless, the fact that there are many very low monetary value, low H value products with essentially zero recycling rates below this group, e.g., Styrofoam cups, paper cups, plastic bags, staples, etc., and there are several relatively high monetary value, low H value products with significant recycling rates within this group, e.g., aluminum cans 45%, steel cans 63%, and newspapers 70%, means that there still is a transition zone in the region, but it is less definite than the one for $H > 0.5$. Tentatively then, we expect the transition zone for $H < 0.5$ to run near the bottom of the group of products shown, but to be rather broad, encompassing many of those products. This zone has mixed recycling rates, which depend on factors other than H. Future work will be needed
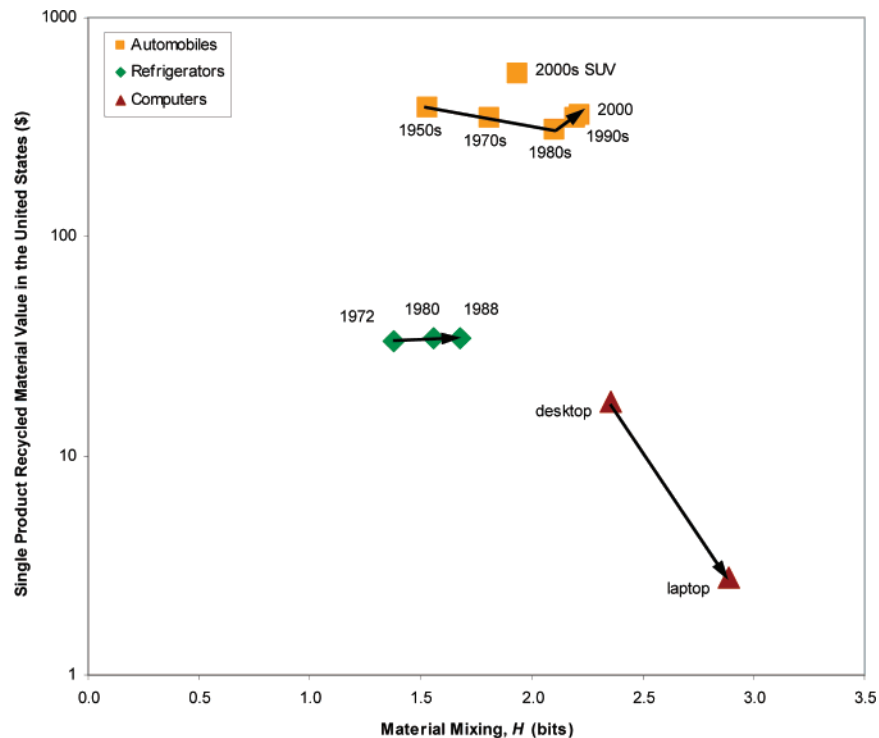
FIGURE 7. Design trends in refrigerators, automobiles, and computers.

to more fully resolve the issues in this area. For $H > 0.5$ however, the data in Figure 4 indicates a rather clear transition from low recycling rates to high.

4.2. Design Trends. The points shown in Figure 4 are meant to be representative of recently retired products. Simply stated the results imply that products with high material values are recycled, provided their mixture is not too complicated. Of course the material composition of products can vary significantly by manufacturer, make, model, and year. As additional data are gathered, they will provide further insight into the model proposed here. Currently, we have collected historical bills of material data for three products, automobiles, refrigerators, and computers, and plotted them in Figure 7. The results illustrate a rather significant design trend. In general, all products have become materially more complex, which is shown as a large displacement along the H axis. The ironic exception is the sports utility vehicle (SUV) which has both increased in material value and decreased in material mixing, both for the same reason: the addition of many kilograms of aluminum and steel. In addition to changes in material mixing, we also see changes in material value. These are due mostly to changes in product size, and, to a lesser extent, material composition. In general, refrigerators and SUVs have gotten bigger, while computers and 1950s to 1980s era cars have gotten smaller. Overall, the trends show an apparent remarkable reduction in the recyclability of products due primarily to greater material mixing. Given the rather significant resources devoted to developing complex material mixtures for products, compared to the rather modest resources focused on how to recapture these materials, it appears that there is reason for concern. As a consequence, recent policy actions such as take-back laws and "extended producer responsibility" appear to be clearly warranted to reclaim the materials in these products. An important part of these policy actions is that there be a feedback loop back to design, so that the consequenses of complex designs are understood. After further development, we hope the methods presented here could help by providing a simple way to evaluate a design early in the design stage.

4.3. Other Recycling Issues. The model proposed here appears to leave out two potentially important aspects of recycling: (1) the resale of components and (2) policy. Here we address them briefly. Concerning the first issue, it is well-known that for some products the resale of functioning components can add significantly to the total recycling revenues. This is certainly the case for automobiles. The structure of the automobile recycling industry however, is segmented, and involves two stages: disassembly and shredding. The first sells components whereas the second sells materials. The two are interdependent; one cannot exist without the other (*22*). In other words, selling components will not work without a cost-effective means of handling the rest of the product. Often this means materials recycling. In our treatment of the data, the automobile plotted in Figures 4 and 7 has already been stripped of its catalytic converter, battery and tires. Other products can be treated in a similar way.

With regards to policy, there have been many policies which have influenced recycling. To take one example, landfill bans have had a significant effect on promoting the recycling of some products. This has been the case for automobiles and many of their components, and is likely to be the case for computers. The model presented here cannot take this directly into account, but it should be able to identify likely and unlikely candidates for economical recycling. For example, in spite of various policy help, the products shown above the line in Figure 4 can usually be recycled today in a profitable manner. Those below the line require a recycling fee to be profitable or simply are not recycled. The model does allow one to combine various waste streams and/or strip various material components from a product to see if the situation improves. The model may also account for improved recycling technology by shifting the recycling boundary down and to the right.

Acknowledgments

Supporting Information Available

(1) sample tree diagrams for recycling systems, (2) comments on model assumptions, including materials counting, (3) expanded product data table, including a 3-d plot of Figure 4, and (4) product references. This material is available free of charge via the Internet at http://pubs.acs.org.

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