MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.997 Optical Engineering **Fall '99**

Assignment #1 Posted Sept. 13, 1999 — Due Sept. 22, 1999

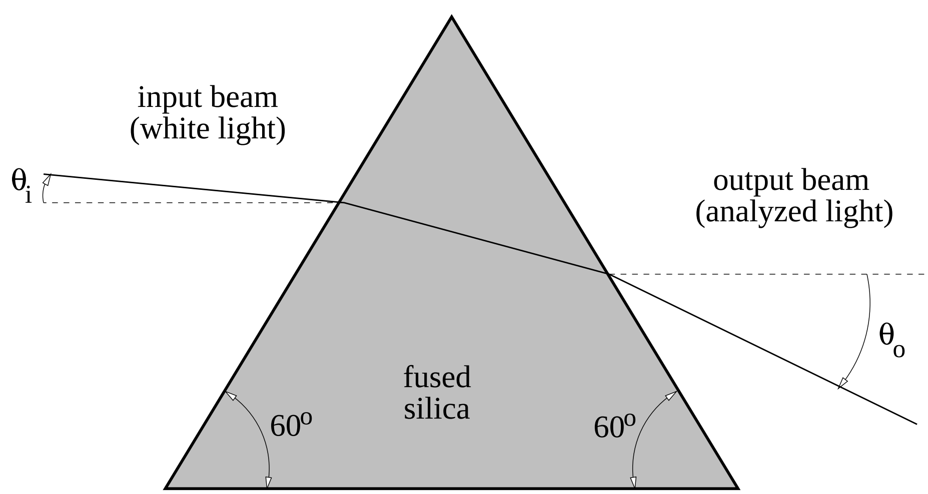
1. **Analyze this.** Newton "analyzed" white light using a prism in a configuration similar to that shown below. A beam of white light is incident on the left-hand face of the equilateral prism at an angle $\theta_{\rm i}$ with respect to the horizontal axis. Because of material *dispersion*, the exit angle $\theta_{\rm o}$ of the light depends on the wavelength; hence, the colors composing the beam are separated beyond the right-hand face of the prism. Assume that the prism is made of fused silica, and use the dispersion data of [WS, Figure 7.6, p.174].

   1.a) Plot $\theta_{\rm o}(\lambda)$ for $0.2\mu\text{m} \le \lambda \le 0.7\mu\text{m}$, separately for $\theta_{\rm i} = 0°, -10°, -50°$.

   1.b) Find the input angle $\theta_{\rm i}$ that disperses the blue Ar^+ line ($\lambda = 488.0\text{nm}$) relative to the red HeNe line ($\lambda = 632.8\text{nm}$) by 1.5°.

   1.c) Plot the minimum deviation angle $\theta_{\rm md}$ for this prism as function of wavelength λ ($0.2\mu\text{m} \le \lambda \le 0.7\mu\text{m}$).

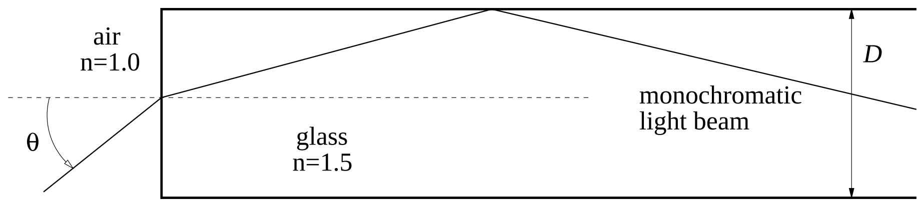
   1.d) Plot as function of wavelength λ ($0.2\mu\text{m} \le \lambda \le 0.7\mu\text{m}$) the critical input angle beyond which light is totally internally reflected at the right-hand face of the prism.



2. **Wanda's world.** Wanda, your goldfish, lives in a spherical ball of water (assume $n_{\rm water} \approx 1.33$). Describe how Wanda might perceive the world.

3. **Inverted Wanda.** Evil wizard Sauron has trapped you inside a bubble of air immersed in water. Describe how objects in the surrounding water appear to you.

4. **Glass waveguides.** Consider the figure shown at the top of next page, where a collimated beam of (monochromatic) light incident at angle θ is coupled into a planar glass slab ($n = 1.5$) of thickness $D = 200\mu\text{m}$. Find the maximum angle $\theta_{\max}$ of the incident illumination for which total internal reflection (TIR) at the slab-air interface keeps the coupled light trapped into the slab. This structure is a *glass waveguide*, and the same principle (with some important modifications) is used in optical fiber communications. Your result $\theta_{\max}$ is referred to as *numerical aperture (NA)* of the waveguide. What happens to the NA as the waveguide index n increases?

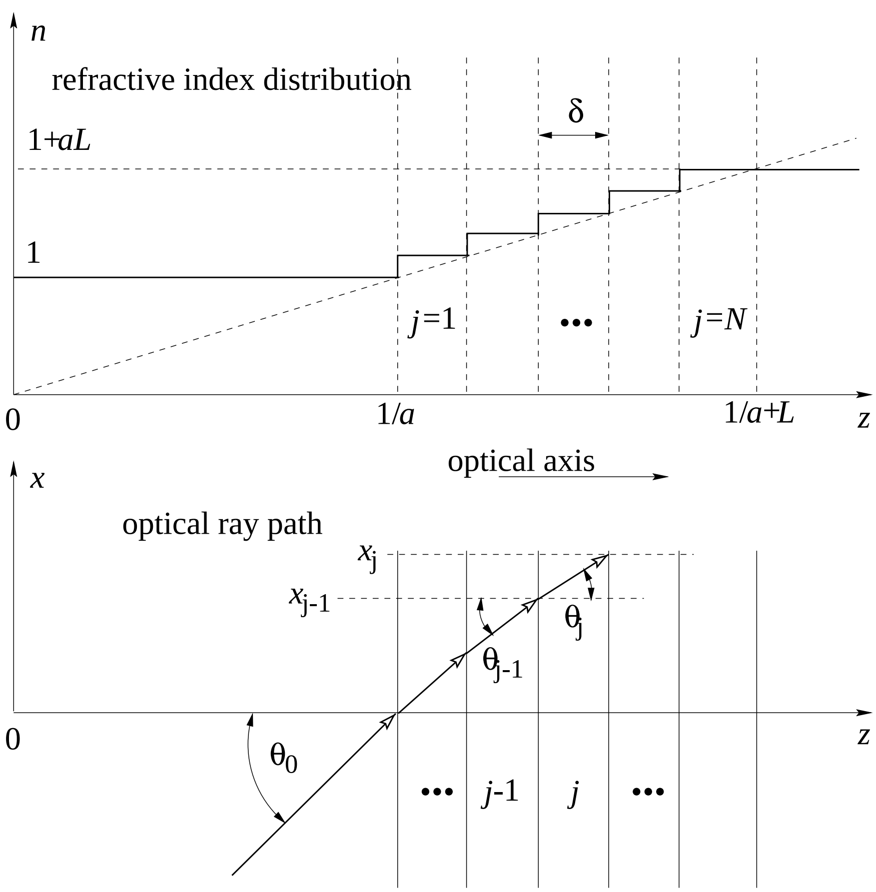


5. Gradient index structures. The figure below depicts a multilayered structure arranged along the optical axis z with different refractive index at each layer. To the left of the structure, the refractive index is $n_0 = 1$. The structure starts at $z = 1/a$ and is composed of N layers, each of thickness δ. The total thickness of the entire structure is $L = N\delta$. We will use j to index the layers ($j = 1, \ldots, N$). Consider a "staircase" distribution of refractive index such that the j-th layer has refractive index

$$n_j = 1 + ja\delta \qquad\qquad j = 1, \ldots, N.$$

The material to the right of the structure is uniform with refractive index $n_{N+1} = 1 + aL$.

5.a) Numerically compute and draw the path of the ray as it traverses the multi-layered structure (ignore reflected rays) for $\theta_0 = 30°$, $a = 0.2\ \text{mm}^{-1}$, $N = 20$, $\delta = 250\ \mu\text{m}$. You will need to define a recursive relationship between the angles θ_{j-1}, θ_j and lateral ray coordinates x_{j-1}, x_j, initialize with θ_0 and $x_0 = 0$, and iterate for $j = 1, \ldots, N$.



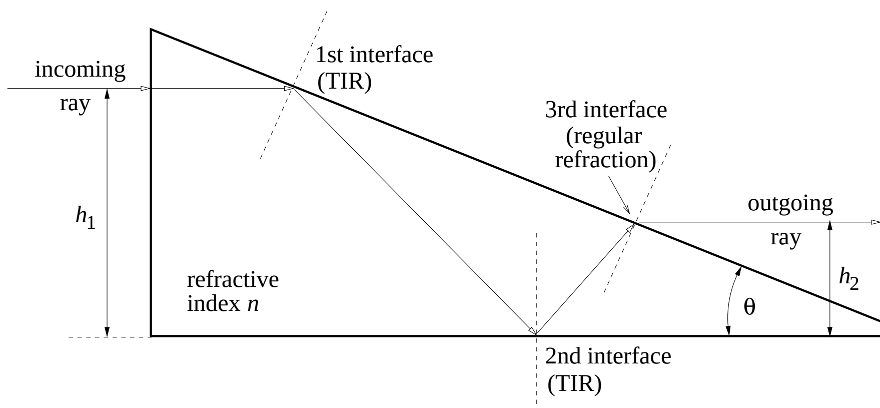
5.b) Consider the continuous version of the above scenario, *i.e.* let $\delta \to 0$, $N \to \infty$ while maintaining $N\delta = L$; also consider a general refractive index distribution $n(z)$ as opposed to the specific staircase-like distribution of the previous question. Derive the ray orientation $\theta(z)$ and lateral coordinate $x(z)$ as function of location along the optical axis for $1/a \le z \le L$. [$x(z)$ may be derived in implicit form only.] This limit is referred to as *GRadient INdex (GRIN)* optics.

5.c) Using the refractive index of question (a) in the GRIN limit (*i.e.*, a ramp function), derive $\theta(z)$ and $x(z)$ for $1/a \le z \le L$. Plot θ and x against z for the parameters specified in question (a).

<u>Hint</u> To answer the questions in this problem, you may need the following integrals:

$$\int \frac{\mathrm{d}u}{\tan u} = \ln\,\sin u \qquad \int \frac{\mathrm{d}u}{\sqrt{u^2 - a^2}} = \ln\left(u + \sqrt{u^2 - a^2}\right).$$

6. An anamorphic demagnifier. Consider the geometry shown below, which uses a single right-angle prism to laterally translate a horizontal incident ray. The ray experiences TIR at the first two glass-air interfaces and exits at the third interface. The prism has index n and tip angle θ.



6.a) Show that a horizontal ray bundle is compressed in the vertical direction only. Such optical systems are called *anamorphic*. Draw a sample input image and how it might look like after passing through the anamorphic prism. Is the image erect or inverted?

6.b) Show that the ray exits with horizontal orientation if either of the following conditions is satisfied:

$$n = \frac{\cos\theta}{\cos 3\,\theta} \qquad \Leftrightarrow \qquad \cos\theta = \frac{1}{2}\sqrt{3 + \frac{1}{n}}.$$

6.c) Show that the above conditions are consistent with the assumption of TIR at the first two interfaces and exit at the third interface.

6.d) Show that the vertical demagnification ratio is

$$f = \frac{h_2}{h_1} = \frac{n}{1 + 2n}.$$

6.e) Plot f and θ as functions of n for $1.33 \le n \le 4$. What are the respective limits of f and θ as $n \to 1$ and $n \to \infty$?

7. Arbitrary rotations. Design a system that may rotate an input image by an arbitrary angle while introducing minimum dispersion for far field objects (*i.e.*, for collinear input ray bundles).