MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.997 Optical Engineering **Fall '99**

Assignment #3 Posted Oct. 4, 1999 — Due Oct. 13, 1999

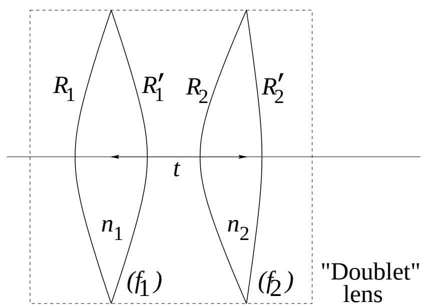
1. WS ch. 3, exercise 1 (p. 85).

2. WS ch. 3, exercise 2 (p. 85).

3. WS ch. 3, exercise 3 (p. 85).

4. WS ch. 3, exercise 4 (p. 85).

5. **Achromatic doublet**. A common method to reduce chromatic aberration is to combine *two* optical elements with different aberration coefficients and optical powers such that the composite optical power equals the desired power of the element and the composite chromatic aberration equals zero. These elements are known as "achromats," or "doublets." Consider the achromat shown below, where the individual elements are thin lenses and there is a distance t between them. We seek to minimize the chromatic aberration of the achromat between two wavelengths of interest λ_a and λ_b.



5.a) Let V_j $(j = 1, 2)$ denote Abbe's V-number [WS eq. 4.10] for the j-the element, and f_j the corresponding focal length. Show that the chromatic dispersion in the optical power of the j-th element is

$$\Delta\left(\frac{1}{f_j}\right) = \frac{1}{V_j\, f_j}$$

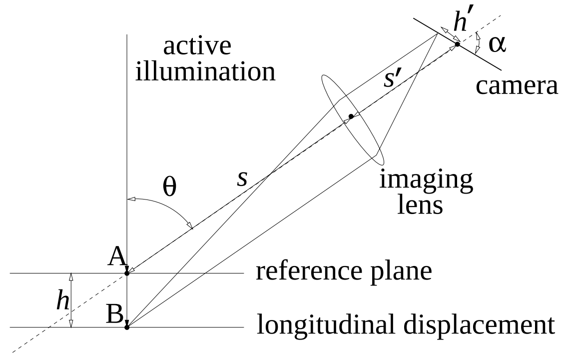
5.b) Derive the thickness t that is required to eliminate chromatic aberration from the composite element in terms of V_j, f_j $(j = 1, 2)$. What is then the composite optical power ϕ of the doublet? What are t and ϕ if the lenses are identical?

5.c) Practical doublets are often cemented (*i.e.*, "glued" together) in order to minimize the possibility of misalignments and other instabilities. Then $t = 0$ and $R_1' = R_2$. Derive the condition that V_j, f_j $(j = 1, 2)$ must satisfy in order to eliminate chromatic aberration in this case. What is then the composite optical power ϕ?

5.d) Design a cemented achromatic doublet for $\phi = 10D$. (You should select the appropriate materials and curvatures for the individual lens components).

5.e) Plot the focal length of the doublet you designed for all wavelengths λ between 0.2μm and 0.7μm and specify the maximum chromatic aberration within this range and the wavelength(s) for which it occurs.

6. **Optical triangulation**. The system shown on the next page is very popular in commercial *surface metrology* systems, *i.e.* systems that measure surface shape very accurately. The surface is illuminated by a thin laser line (shown in cross-section as a dot) and the objective is to infer the height of the surface h with respect to a reference height from the location of the image of the laser line on the camera. Assume that the imaging system is composed of a thin lens with focal length f. The system is calibrated as follows: when the surface is at a reference height and the illumination is incident at A, the image of A appears at the center of a camera (the camera is shown in cross-section as a line). The system is set with distance s between A and the lens, such that the distance s' between the lens and the image of A is given by the thin lens law. The direction of illumination makes angle θ with the optical axis of the imaging system. As shown in the figure, a point B displaced with respect to A by h (but still in focus on the surface) is imaged off-axis onto the camera.



6.a) Explain why the camera must be tilted with respect to the optical axis by an angle α, and derive α in terms of θ and the magnification $m = s'/s$.

6.b) Show that the condition which you derived is equivalent to the *Scheimpflug condition* [WS 2.14, p. 52 and Fig. 2.21, p. 53].

6.c) Due to a positioning error, the camera is tilted with respect to the optical axis by $\alpha + \Delta\alpha$ instead of the correct angle α ($|\Delta\alpha| \ll |\alpha|$). Quantify the effect of the error by plotting the blur of the image of B on the camera as function of h and $\Delta\alpha$. Use an optical system with $f = 1$cm, $f/2$, $m = -0.1$, and $\theta = 30°$.

6.d) Discuss the effects of θ, the $f/\#$ and the camera pixel pitch on depth resolution. If you were to design a lens for this application, which requirements would you impose on the design? Discuss other imperfections that creep in to degrade depth resolution.

This is an example of an *active illumination* imaging system. On the other hand, *passive* systems rely on ambient illumination only and by necessity use two or more cameras to infer depth. Both methods are known as "(2+1/2)D" imaging because they are capable of imaging surface manifolds in 3D space (but they cannot image volumes, in which case they would classify as "3D"). This term was coined by David Marr (1945-1980), MIT Professor of Brain and Cognitive Sciences, who is considered the father of modern Vision science. A good example of passive (2+1/2)D imaging system is human binocular vision.