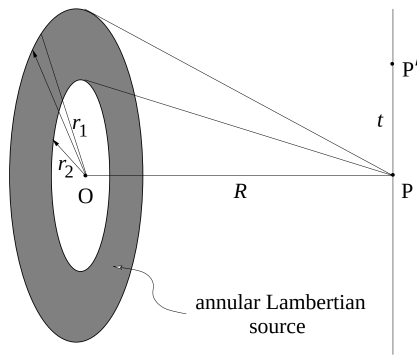
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.997 Optical Engineering **Fall '99**
Assignment #4 Posted Oct. 13, 1999 — Due Oct. 25, 1999

1. Annular Lambertian source. Consider the Lambertian source shown below, which is shaped as a ring with outer radius r_1 and inner radius r_2.

1.a) Calculate the irradiance at point P, located distance R along the normal from the center of the annulus if the radiance of the Lambertian source is $1\text{W}/(\text{m}^2 \text{ sterrad})$, and the dimensions are $r_1 = 10\text{cm}$, $r_2 = 5\text{cm}$, and $R = 1\text{m}$.

1.b) Derive an integral expression for the irradiance at a point P′ not belonging to the normal from the center of the annulus, located distance t from P (measured in the plane parallel to the source). Do not attempt to explicitly evaluate the integral; instead, perform a sanity check to verify that it reduces to the correct form if $t = 0$ and $r_2 = 0$.



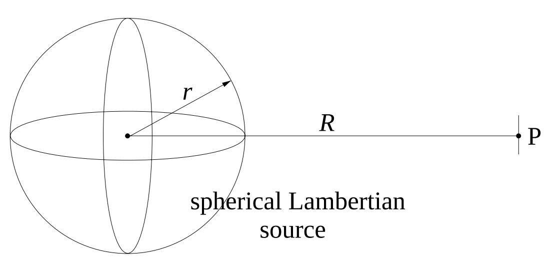
2. Spherical Lambertian source. The spherical-shaped source shown below is Lambertian; *i.e.*, it has uniform radiance N. We seek to calculate the irradiance of this source without performing the tedious integration, but using symmetry arguments instead.

2.a) Show that the total power emitted into space from the surface of the Lambertian sphere is

$$P = 4\pi^2 r^2 N.$$

2.b) We now argue that if we enclose the source with a bigger sphere of radius $R > r$, the entire surface of this exterior sphere must also be uniformly illuminated. Use this argument to calculate the irradiance at distance r.

2.c) Suppose that you are observing this spherical Lambertian source from a distance r. Describe an alternative planar Lambertian source that will appear to you to be equivalent to the actual spherical-shaped source.

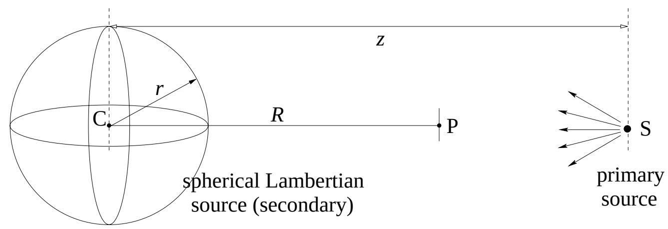


3. Secondary Lambertian scatterer. A uniform spherical Lambertian scatterer of radius r is irradiated by a point source S of intensity I placed at a large distance z ($z \gg r$) from the sphere.

3.a) Find the radiance of the scattered radiation as a function of position and direction.

3.b) Find an integral expression for the irradiance that results from the scattered radiation illuminating a small planar surface placed at P at a distance R from the center C of the scatterer. C, P, and S are not necessarily on a straight line (ignore the effect of the shadow of the small surface on the scatterer). Evaluate this integral in the limit $R \gg r$, and discuss qualitatively how the irradiance changes as the position of S and its orientation are varied.

3.c) Is the surface of the moon a Lambertian scatterer? (Use your own experience to answer this question qualitatively).



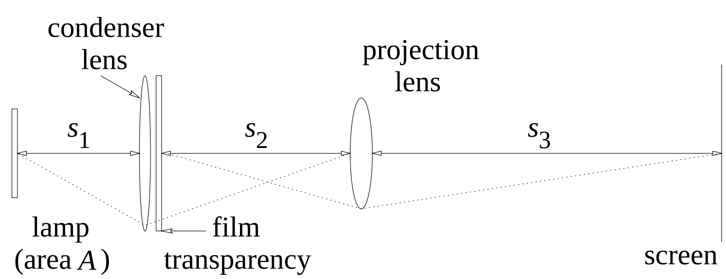
4. The sun as a blackbody. Assuming that the sun is a blackbody of temperature 5800K, and neglecting absorption and refraction in the earth's atmosphere, what is the flux incident on a solar collector of area 1m^2 whose normal points in the direction of the sun?

5. The Köhler projector. The Köhler method of projecting an image from film or transparency onto a screen is shown in the figure below. The illumination is provided by a lamp of luminance L and area A. The condenser lens images the lamp onto the projection lens. The transparency has transmittance T and is imaged onto the screen by the projection lens. The lenses have focal lengths f_c and f_p and speeds $f/f_\text{c}^\#$ and $f/f_\text{p}^\#$, respectively. All lenses are modeled as thin lenses, and we ignore aberrations.

5.a) Derive an expression for the screen illuminance.

5.b) Evaluate this expression numerically for a tungsten source of area $A = 1\text{cm}^2$ that emits like a blackbody of temperature 2800K. The imaging system parameters are $s_1 = 25\text{mm}$, $s_2 = 100\text{mm}$, $s_3 = 5\text{m}$.

5.c) Discuss the effect of the focal lengths and speeds of the lenses on the photometric properties of the projector.



6. Apertures and pupils. Consider the astronomical telescope [WS Fig. 9.1(a), p. 236] with eyepiece, objective focal lengths and radii equal to f_e and a_e, f_o and a_o, respectively. The distance between the two elements is fixed to $D = f_\text{e} + f_\text{o}$, so that both object and image are located at infinity in the paraxial approximation. Assume that $a_\text{e} < a_\text{o}$.

6.a) Derive the locations and lateral sizes (radii) of the aperture, the entrance pupil, and the exit pupil.

6.b) Derive the object and image fields for the cases of no vignetting and complete vignetting.

[See WS exercise 6.3, p. 159 for a numerical example].

7. Interferometry. The device shown below forms the interference pattern between a spherical wave originating at a point source located at distance R from the screen and a plane wave incident at angle θ with respect to the normal to the screen surface. Both waves are at wavelength λ and are mutually coherent. We also assume that the two waves have equal intensities over the entire output screen.

7.a) Derive the shape of the interference pattern $I(x, y)$ and the coordinates (x_0, y_0) where the fringes are stationary (*i.e.*, $\partial I/\partial x = \partial I/\partial y = 0$ at $(x, y) = (x_0, y_0)$).

7.b) Plot the interference pattern on a square of size 50λ centered around the geometrical shadow of the pinhole, sampling every 0.1λ, for the following parameters: *(i)* $R = 1000\lambda$, $\sin\theta = 0.05$; *(ii)* $R = 1000\lambda$, $\sin\theta = 0.1$; *(iii)* $R = 500\lambda$, $\sin\theta = 0.1$.

For the last plots, you may use Matlab code from the following locations:

`http://web.mit.edu/2.997/hw4_7_public.m`

`http://web.mit.edu/2.997/plsph_public.m`

You will have to modify `plsph_public.m` to enter the correct expressions of the plane and spherical waves, respectively.