Fundamental questions in nuclear physics

Physics of nuclei:

- How do nucleons interact?
- How are nuclei formed? How can their properties be so different for different A?
- What’s the nature of closed shell numbers, and what’s their evolution for neutron rich nuclei?
- What is the equation of state of dense matter?
- Can we describe simultaneously 2, 3, and many-body nuclei?
What is nuclear matter? Easy, an infinite system of nucleons!

- Infinite systems
- Symmetric nuclear matter: equal protons and neutrons
- Pure neutron matter: only neutrons
- W/o Coulomb: homogeneous
- Nuclear matter saturates (heavy nuclei, “bulk”)
- Neutron matter positive pressure
- Properties of infinite matter important to constrain energy density functionals
Low-density neutron matter and Cold atoms

“Low-density” means $\rho \ll \rho_0$
Ultracold Fermi atoms

- Dilute regime $\rightarrow$ mainly s-wave interaction
- $T$ fraction of $T_F \rightarrow T \sim 0$
- Experimentally tunable interaction
- Crossover from weakly interacting Fermions (paired) to weakly repulsive Bosons (molecules)
Dilute regime → mainly $s$-wave interaction

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Scattering length

Two-body system with attractive interaction:

\[ a < 0 \]

No bound states

\[ a = \infty \]

Bound state with \( E_b = 0 \)

\[ E_b \sim \frac{\hbar^2}{2m a^2} \]

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Very Low Density Neutron Matter: cold atoms

Low density neutron matter $\rightarrow$ unitary limit:

$$ r_{\text{eff}} \ll r_0 \ll |a|, \quad r_{\text{eff}} = 0, \quad |a| = \infty $$

Only one scale: $\rightarrow E = \xi E_{FG}$

- NN scattering length is large and negative, $a = -18.5$ fm
- NN effective range is small, $r_{\text{eff}} = 2.7$ fm
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Nuclear and neutron matter 7 / 29
Exact calculation of $\xi$ using AFMC:

\[ \rho^{1/3} = \alpha N^{1/3}/L \]

$\xi = 0.372(5)$ Carlson, Gandolfi, Schmidt, Zhang, PRA 84, 061602 (2011)

$\xi = 0.376(5)$ Ku, Sommer, Cheuk, Zwierlein, Science 335, 563 (2012)

Validation of Quantum Monte Carlo calculations
Fermi gas and neutron matter

Carlson, Gandolfi, Gezerlis, PTEP 01A209 (2012).

Ultracold atoms very useful for nuclear physics!
BCS (Bardeen-Cooper-Schrieffer) pairs: an arbitrarily small attraction between Fermions can cause a paired state of particles and the system has a lower energy than the normal gas.
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The (condensate) pairs may then form a superfluid:

The pairing gap is basically the energy needed to "break" a pair, and then excite the system to its normal energy.
Pairing gap and neutron stars

The pairing gap is fundamental for the cooling of neutron stars.

Neutron star crust made of nuclei arranged on a lattice surrounded by a gas of neutrons.

Specific heat suppressed by superfluidity (similarly to the superconducting mechanism).

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Cooling dependent to the pairing gap!
Dilute neutron matter

The pairing gap is the energy cost to excite one particle from a BCS (collective) state.

Pairing gap of low–density neutron matter vs cold atoms:

Gezerlis, Carlson (2008)

Cold atoms results confirmed by experiments!
Dense neutron matter

“Dense” means $\rho \sim (0.5–\text{few times})\rho_0$
Non interacting two-components Fermi gas (non-relativistic):

\[ \frac{E}{N}(k_F) = \frac{3}{5} \frac{\hbar^2}{2m} k_F^2 = E_{FG}(k_F), \]

where \( k_F = (3\pi^2 \rho)^{1/3}. \)
Fermi gas (1/2)

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For a system made of neutrons and protons, define:

\[ \rho = \rho_n + \rho_p, \quad \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}, \]

Useful relations:

\[ \rho_p = \frac{1 - \alpha}{2} \rho, \quad \rho_n = \frac{1 + \alpha}{2} \rho. \]

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The nuclear matter energy is given by:

\[
\frac{E}{A}(\rho, x) = \frac{N}{A} \frac{E}{N}(k_F^{(n)}) + \frac{Z}{A} \frac{E}{Z}(k_F^{(p)}) = \frac{3}{5} \frac{\hbar^2}{2m} k_F^2 \frac{1}{2} \left[ (1 + \alpha)^{5/3} + (1 - \alpha)^{5/3} \right]
\]

\[= f(\alpha) E_{FG}(k_F).\]
For small asymmetries, $\alpha \approx 0$, the function $f(\alpha)$ can be expanded

$$f(\alpha) = 1 + \frac{5}{9} \alpha^2 + \frac{5}{243} \alpha^4 + \ldots,$$

And thus the equation of state is given by:

$$\frac{E}{A}(\rho, x) = \frac{3}{5} E_{FG}(\rho) + \frac{5}{9} E_{FG}(\rho) \alpha^2 + \ldots = E_{SNM} + \alpha^2 S(\rho) + \alpha^4 S_4(\rho) + \ldots,$$

where $E_{SNM}$ is the energy of symmetric nuclear matter ($\alpha = 0$) and $S(\rho)$ is the symmetry energy given by

$$S(\rho) = \frac{1}{2} \frac{\partial^2}{\partial \alpha^2} [E(\rho, x)]_{\alpha=0} \simeq E_{PNM}(\rho) - E_{SNM}(\rho),$$

and $E_{PNM}$ is the energy of pure neutron matter ($\alpha = 1$).
Around density $\rho_0$ nuclear matter saturates, thus

$$\left. \frac{\partial E_{SNM}(\rho)}{\partial \rho} \right|_{\rho=\rho_0} = 0$$

and we can expand as

$$E_{SNM} = E_0 + \alpha \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \beta \left( \frac{\rho - \rho_0}{\rho_0} \right)^3 + \ldots ,$$

Pure neutron matter instead does not saturate, thus also linear power in $\rho$ is fine.
Symmetry energy

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Then, around $\rho_0$ we can expand:

$$E_{sym} = S_0 + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \ldots ,$$

where $L$ is the slope of the symmetry energy, and $K_{sym}$ is the symmetry compressibility.
Why do we care?

- EOS of neutron matter gives the symmetry energy and its slope.
- Assume that NN is very good - fit scattering data with very high precision.
  
  Three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments.
Neutron matter equation of state

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At nuclear densities neutron matter cannot be modeled as in the dilute regime. Nucleon-nucleon (and three-nucleon) interaction become very important.
Neutron matter at nuclear densities

At nuclear densities neutron matter cannot be modeled as in the dilute regime. Nucleon-nucleon (and three-nucleon) interaction become very important.

Let’s start from Hamiltonians used for nuclei:

How much can we trust the nucleon-nucleon interactions?

In a scattering event with energy $E_{lab}$ two nucleons have

$$k \approx \sqrt{E_{lab} \frac{m}{2}}, \quad \rightarrow k_F$$

that correspond to

$$k_F \rightarrow \rho \approx \frac{(E_{lab} \frac{m}{2})^{3/2}}{2\pi^2}.$$ 

$E_{lab}=150$ MeV corresponds to about 0.12 fm$^{-3}$.

$E_{lab}=350$ MeV to 0.44 fm$^{-3}$.

Argonne potentials useful to study dense matter above $\rho_0=0.16$ fm$^{-3}$, other (soft) interactions not clear.
What is the Symmetry energy?

Assumption from experiments:

\[ E_{SNM}(\rho_0) = -16\text{MeV}, \quad \rho_0 = 0.16\text{fm}^{-3}, \quad E_{sym} = E_{PNM}(\rho_0) + 16 \]

At \( \rho_0 \) we access \( E_{sym} \) by studying PNM
Equation of state of neutron matter using the $AV8' + UIX$ Hamiltonian.

Incidently these can be considered as "extremes" with respect to the measured $E_{sym}$. 
Three-neutron interaction uncertainty:

\[ E_{\text{sym}} = 33.7 \text{ MeV} \]
Neutron matter

Main sources of uncertainties:

- Experimental: $E_{\text{sym}}$
- Theoretical: form of three-neutron interaction not totally understood
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Neutron matter

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- Experimental: $E_{\text{sym}}$
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Which one dominates?
Equation of state of neutron matter using Argonne forces:

\[ E_{\text{sym}} = 35.1 \text{ MeV (AV8'+UIX)} \]
\[ E_{\text{sym}} = 33.7 \text{ MeV} \]
\[ E_{\text{sym}} = 32 \text{ MeV} \]
\[ E_{\text{sym}} = 30.5 \text{ MeV (AV8')} \]

Gandolfi, Carlson, Reddy, PRC (2012)
Symmetry energy

Many experimental efforts to measure $E_{sym}$ (or $S_0$) and its slope $L$:

$$E_{sym} = S_0 + \left( \frac{L}{3} \right) \left( \rho - \rho_0 \right) / \rho_0 + ...$$ from different experiments

Constraints on Symmetry Energy

- Isobaric Analogue States
  - NPA 818, 36 (2009)
- Finite Droplet Range Model
- Pygmy Dipole Resonances
  - PRC 81, 041304 (2010)
- p elastic scattering
  - PRC 82, 044611 (2010)
- HIC: heavy ion collisions;
  - PRL 102, 122701 (2009)
- neutron-star radius

Gráfico con diferentes experimentos:

- $E_{sym} = S_0 + \left( \frac{L}{3} \right) \left( \rho - \rho_0 \right) / \rho_0 + ...

Link to Data

Tsang et al., PRC 86, 015803 (2012)
From the EOS, we can fit the symmetry energy around $\rho_0$ using

$$E_{\text{sym}}(\rho) = E_{\text{sym}} + \frac{L}{3} \rho - 0.16 + \cdots$$

Gandolfi et al., EPJ (2014)

Very weak dependence to the model of 3N force for a given $E_{\text{sym}}$. Knowing $E_{\text{sym}}$ or $L$ useful to constrain 3N! (within this model...)
Neutron matter and the "puzzle" of the three-body force

Note: AV8' + UIX and (almost) AV8' are stiff enough to support observed neutron stars, but AV8' + IL7 too soft. → How to reconcile with nuclei???
Neutron matter at N2LO

EOS of pure neutron matter at N2LO, $R_0=1.0$ fm.


Note: the above (but not all) chiral Hamiltonian able to describe $A=3,4,5$ nuclei and neutron matter reasonably.
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- Low-density neutron matter and cold atoms
- Superfluidity
- Dense matter: free Fermi gas
- Dense matter with interaction
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