Lecture 3 – ν cross sections

1) $\nu - e$ elastic scattering

2) $\nu_e - n$ / $\bar{\nu}_e\, p$ absorption

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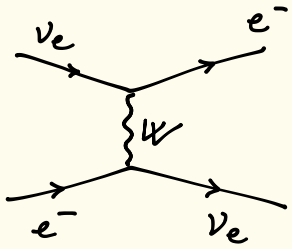
1) $\overset{(-)}{\nu}_\alpha + e^- \longrightarrow \overset{(-)}{\nu}_\alpha + e^-$

<u>elastic</u> : energy/momentum redistribution ; no threshold

Limit to tree-level (lowest order in coupling)

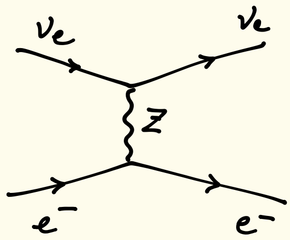
Feynman diagrams:

$\nu_e + e^- \longrightarrow \nu_e + e^-$



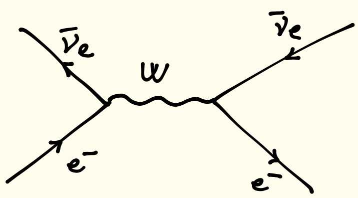
Charged current

+



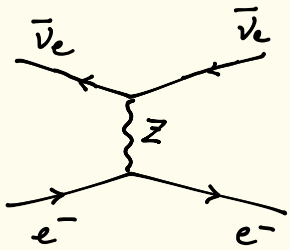
Neutral current

$\bar{\nu}_e + e^- \longrightarrow \bar{\nu}_e + e^-$



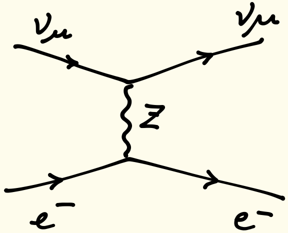
Charged current

+



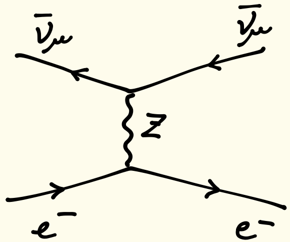
Neutral current

$$\nu_{\mu,\tau} + e^- \longrightarrow \nu_{\mu,\tau} + e^-$$



Neutral Current

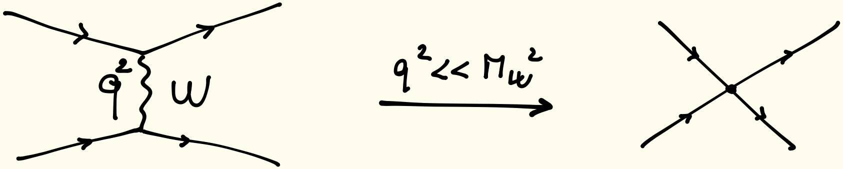
$$\bar{\nu}_{\mu,\tau} + e^- \longrightarrow \bar{\nu}_{\mu,\tau} + e^-$$



Neutral Current

Cross section calculation: $\nu_e + e^- \longrightarrow e^- + \nu_e$

Low energy ($q^2 \ll M_W^2$): four-fermion interaction:



$$G_F = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2}$$

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \Big\{ \underbrace{\left[\bar{\nu}_e \gamma^\rho (1-\gamma^5) e \right] \left[\bar{e} \gamma_\rho (1-\gamma^5) \nu_e \right]}_{CC}$$

$$+ \underbrace{\left[\bar{\nu}_e \gamma^\rho (1-\gamma^5) \nu_e \right] \left[\bar{e} \gamma_\rho (g_V^e - g_A^e \gamma^5) e \right]}_{NC} \Big\}.$$

$$\begin{cases} g_V^e = -\frac{1}{2} + 2\sin^2\theta_W \\ g_A^e = -\frac{1}{2} \end{cases}$$

$$\begin{cases} g_V^f = I_3^f - 2q_f \sin^2\theta_W \\ g_A^f = I_3^f \end{cases}$$

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \Big\{ \underbrace{\left[\bar{\nu}_e \gamma^\rho (1-\gamma^5) e \right] \left[\bar{e} \gamma_\rho (1-\gamma^5) \nu_e \right]}_{CC}$$

$$+ \underbrace{\left[\bar{\nu}_e \gamma^\rho (1-\gamma^5) \nu_e \right] \left[\bar{e} \gamma_\rho (g_V^e - g_A^e \gamma^5) e \right]}_{NC} \Big\}$$

After Fierz transformation:

$$\boxed{\mathcal{L} = -\frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e \gamma^\rho (1-\gamma^5) \nu_e \right] \left[\bar{e} \gamma_\rho \left((1+g_V^e) - (1+g_A^e) \gamma^5 \right) e \right]}$$

4-momenta, Mandelstam invariants

$p_{\nu i} + p_{ei} = p_{\nu f} + p_{ef}$ (i: initial, f: final)

$$
\begin{cases}
s = (p_{\nu i} + p_{ei})^2 = (p_{\nu f} + p_{ef})^2 \\
t = (p_{\nu i} - p_{\nu f})^2 = (p_{ef} - p_{ei})^2 = q^2 = -Q^2 \\
u = (p_{\nu i} - p_{ef})^2 = (p_{\nu f} - p_{ei})^2
\end{cases}
$$

- Center of mass frame: $\boxed{\sigma \propto G_F^2 \, s}$ (dimensional argument)

↳ True in all frames due to invariance!

In e^- rest frame: $s \simeq 2 m_e E_\nu$

Steps of cross section calculation:

1. Write $-i\mathcal{M}$:

$$-i\mathcal{M} = -i\frac{G_F}{\sqrt{2}}\left[\bar{u}(p_{\nu f})\gamma^{\rho}(1-\gamma^5)u(p_{\nu i})\,\bar{u}(p_{ef})\gamma_{\rho}\left((1+g_V^e)-(1+g_V^e)\gamma^5\right)u(p_{ei})\right]$$

2. Calculate spin-average: $\frac{1}{2}\sum_{s_f}\sum_{s_i}|\mathcal{M}|^2$

3. Simplify, using traces

4. Calculate σ:

$$\boxed{d\sigma = \frac{1}{2}\sum_{s_f}\sum_{s_i}|\mathcal{M}|^2\frac{(2\pi)^4\,\delta^4(p_{\nu i}+p_{ei}-p_{\nu f}-p_{ef})}{4\sqrt{(p_{\nu i}\cdot p_{ei})^2-m_\nu^2 m_e^2}}\frac{d^3p_{ef}}{(2\pi)^3 2E_{ef}}\frac{d^3p_{\nu f}}{(2\pi)^3 2E_{\nu f}}}$$

Result:

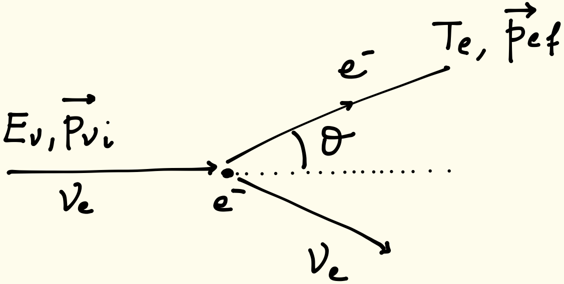
$$\boxed{\frac{d\sigma}{dQ^2} = \frac{G_F^2}{\pi}\left[g_1^2 + g_2^2\left(1 - \frac{Q^2}{2p_{\nu_i}\cdot p_{e_i}}\right)^2 - g_1 g_2 m_e^2 \frac{Q^2}{2(p_{\nu_i}\cdot p_{e_i})^2}\right]}$$

$$g_1^{\nu_e} = 1 + \frac{1}{2}\left(g_V^e + g_A^e\right) = 1/2 + \sin^2\vartheta_W \simeq 0.73$$

$$g_2^{\nu_e} = \frac{1}{2}\left(g_V^e - g_A^e\right) = \sin^2\vartheta_W \simeq 0.23$$

Useful: function of $E, \vec{p}$ of outgoing e^-

Lab frame: $p_{e_i} = (m_e, 0)$



$$\cos\theta = \frac{\vec{p}_{\nu i} \cdot \vec{p}_{ef}}{|\vec{p}_{\nu i}||\vec{p}_{ef}|}$$

Electron kinetic energy: $T_e \equiv E_e - m_e = Q^2/2m_e$

Properties of T_e :

1) From $E, \vec{p}$ conservation :

$$\blacktriangleright \quad T_e = \frac{2 m_e E_\nu^2 \cos^2\vartheta}{(m_e + E_\nu)^2 - E_\nu^2 \cos^2\vartheta} .$$

2) Max for $\cos\vartheta = 1$:

$$\blacktriangleright \quad T_e^{max}(E_\nu) = \frac{2 E_\nu^2}{m_e + 2 E_\nu} < E_\nu$$

Differential cross sections:

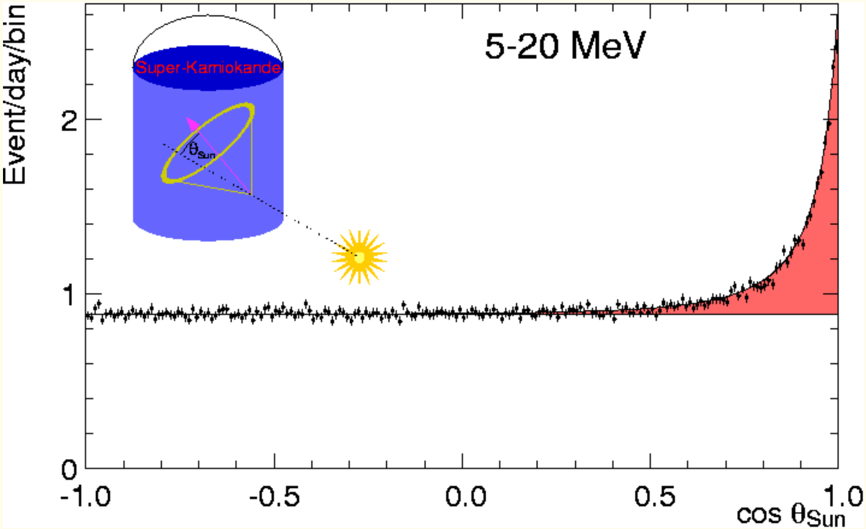
$$\sigma_0 = \frac{2G_F^2 m_e^2}{\pi} \simeq 88.06 \cdot 10^{-46} \ \mathrm{cm}^2$$

$$\boxed{\frac{d\sigma}{dT_e} = \frac{d\sigma}{dQ^2} \cdot \frac{dQ^2}{dT_e} = \frac{\sigma_0}{m_e}\left[g_1^2 + g_2^2\left(1 - \frac{T_e}{E_\nu}\right)^2 - g_1 g_2 \frac{m_e T_e}{E_\nu^2}\right]}$$

$$\boxed{\begin{aligned}\frac{d\sigma}{d\cos\vartheta} &= \frac{d\sigma}{dT_e}\frac{dT_e}{d\cos\vartheta} = \\ &= \sigma_0 \frac{4E_\nu^2 (m_e + E_\nu)^2 \cos\vartheta}{\left[(m_e+E_\nu)^2 - E_\nu^2\cos^2\vartheta\right]^2} \times \Bigg[g_1^2 + g_2^2\left(1 - \frac{2m_e E_\nu \cos^2\vartheta}{(m_e+E_\nu)^2 - E_\nu^2\cos^2\vartheta}\right)^2 \\ &\qquad - g_1 g_2 \frac{2m_e^2\cos^2\vartheta}{(m_e+E_\nu)^2 - E_\nu^2\cos^2\vartheta}\Bigg]\end{aligned}}$$

Pointing to the ν source : $\frac{d\sigma}{d\cos\theta}$ is max. for $\cos\theta = 1$

Example: solar ν in water Cherenkov detector



(from Superkamiokande web page)

Effect of detector threshold: require $T_e \geq T_e^{th}$ (e.g. $T_e^{th} \sim 5\,\mathrm{MeV}$)

$$\sigma(E_\nu, T_e^{th}) = \int_{T_e^{th}}^{T_e^{max}} \left(\frac{d\sigma}{dT_e}\right) dT_e$$

$$\boxed{\sigma(E_\nu, T_e^{th}) = \frac{\sigma_0}{m_e} \left[(g_1^2 + g_2^2)(T_e^{max} - T_e^{th}) - \left(g_2^2 + g_1 g_2 \frac{m_e}{2E_\nu}\right)\left(\frac{T_e^{max\,2} - T_e^{th\,2}}{E_\nu}\right) + \frac{1}{3} g_2^2 \left(\frac{T_e^{max\,3} - T_e^{th\,3}}{E_\nu^2}\right) \right]}$$

Total cross section, for $E_\nu \gg m_e$: $T_e^{th} = 0$, $T_e^{max} \simeq E_\nu$

$$\boxed{\sigma(E_\nu, 0) \simeq \sigma_0 \frac{E_\nu}{m_e} \left(g_1^2 + \frac{1}{3} g_2^2\right) \qquad (E_\nu \gg m_e)}$$

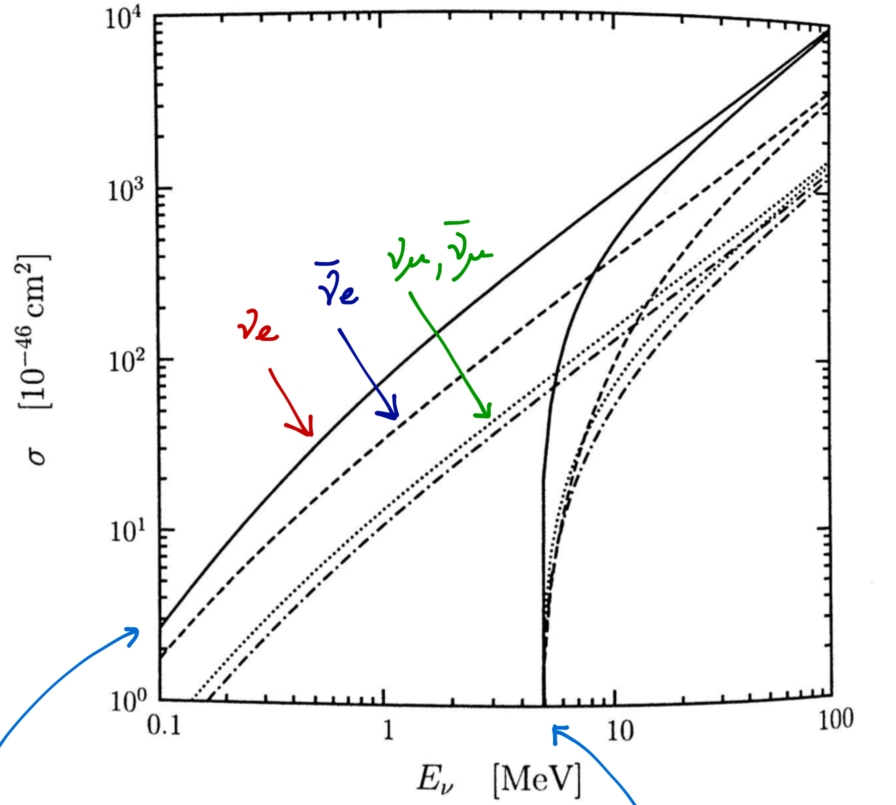
Generalization to other ν species:

- For $\left.\begin{array}{l}\bar{\nu}_e + e^- \longrightarrow e^- + \bar{\nu}_e \\ \overset{(-)}{\nu}_{\mu,\tau} + e \longrightarrow e + \overset{(-)}{\nu}_{\mu,\tau}\end{array}\right\}$ same formalism, different g_1, g_2:

- σ largest for ν_e, due to larger g_1

	g_1	g_2
ν_e	$\frac{1}{2} + \sin^2\theta_W \simeq 0.73$	$\sin^2\theta_W \simeq 0.23$
$\nu_{\mu,\tau}$	$-\frac{1}{2} + \sin^2\theta_W \simeq -0.27$	$\sin^2\theta_W \simeq 0.23$
$\bar{\nu}_e$	$\sin^2\theta_W \simeq 0.23$	$\frac{1}{2} + \sin^2\theta_W \simeq 0.73$
$\bar{\nu}_{\mu,\tau}$	$\sin^2\theta_W \simeq 0.23$	$-\frac{1}{2} + \sin^2\theta_W \simeq -0.27$

(from Kim & Giunti)

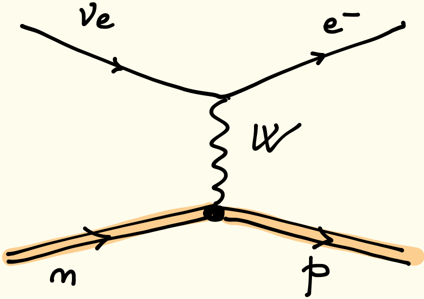


$T_e^{th} = 0$

$T_e^{th} = 4.5$ MeV

2) ν - Nucleon scattering : quasielastic CC reactions

example : $\nu_e + n \longrightarrow p + e^-$



Hadronic vertex: effects of QCD !

Effective treatment: generic model of hadronic current

$$A(\nu_e + n \to p + e^-) = -i\frac{G_F}{\sqrt{2}} V_{ud}\, \bar{u}(p_e)\gamma_\rho(1-\gamma_5)u(p_\nu)$$

Hadronic current $\longrightarrow$

$$\boxed{\begin{aligned} &\times\Big\{\bar{u}(p_p)\Big[\gamma^\rho F_1(Q^2) + \frac{i}{2m_N}\sigma^{\rho\eta} q_\eta F_2(Q^2) \\ &-\gamma^\rho\gamma^5 G_A(Q^2) - \frac{q^\rho}{m_N}\gamma^5 G_P(Q^2)\Big] u_n(p_n)\Big\} \end{aligned}}$$

$\longleftarrow$ vector

$\longleftarrow$ axial

form factors:

F_1 (Dirac)

F_2 (Pauli)

G_A (axial)

G_P (pseudoscalar)

$q = p_\nu - p_e = p_p - p_n$,

$Q^2 = -q^2$

$$\left(\sigma^{\mu\nu} \equiv \tfrac{i}{2}[\gamma^\mu, \gamma^\nu]\right)$$

Hadronic current : derivation / explanation

- Hadrons 4-momenta : p_n , p_p

Construct scalar : $2\, p_n \cdot p_p = m_n^2 + m_p^2 - (p_p - p_n)^2 = m_n^2 + m_p^2 + Q^2$

$\hookrightarrow$ form factors must depend on Q^2.

- Most general vector current:

$$J_V^\rho = \bar{u}(p_p)\left[f_1(Q^2)\gamma^\rho + f_2(Q^2)\underbrace{(p_p^\rho + p_n^\rho)} + f_3(Q^2)(\underbrace{p_n^\rho - p_p^\rho}_{-q})\right] u(p_n)$$

From Dirac equation:

$$\bar{u}(p_p)\left[\underbrace{(p_p^\rho + p_n^\rho)} - (\underbrace{m_n + m_p}_{\simeq 2m_N})\gamma^\rho\right] u_n(p_n) = \bar{u}(p_p)\, i\,\sigma^{\rho\eta} q_\eta\, u(p_n)$$

$$\downarrow$$

$$J_V^\rho = \bar{u}(p_p)\left[\gamma^\rho F_1(Q^2) + \frac{i}{2m_N}\sigma^{\rho\eta} q_\eta\, F_2(Q^2) + \frac{q^\rho}{m_N} F_3(Q^2)\right] u(p_n)$$

(assume $m_n = m_p = m_N$)

- Most general vector current:

$$J_V^\rho = \bar{u}(p_p)\left[\gamma^\rho F_1(Q^2) + \frac{i}{2m_N}\sigma^{\rho\eta} q_\eta F_2(Q^2) + \frac{q^\rho}{m_N} F_3(Q^2)\right] u(p_n)$$

- Most general axial current:

$$J_A^\rho = \bar{u}(p_p)\left[\gamma^\rho\gamma^5 G_A(Q^2) + \frac{q^\rho}{m_N}\gamma^5 G_P(Q^2) + \frac{p_p^\rho + p_n^\rho}{m_N}\gamma^5 G_3(Q^2)\right] u(p_n)$$

$$\begin{cases} \text{T-reversal symmetry} \Rightarrow F_i, G_i \text{ are } \underline{\text{real}} \\ \text{Isospin invariance of QCD} \Rightarrow F_3 = G_3 = 0 \quad \text{(verified experimentally)} \end{cases}$$

$\hookrightarrow$ $\underline{F_1(Q^2),\ F_2(Q^2),\ G_A(Q^2),\ G_P(Q^2)}$

Differential cross section:

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 |V_{ud}|^2 m_N^4}{8\pi (p_\nu \cdot p_n)^2} \left[A(Q^2) + B(Q^2) \frac{s-u}{m_N^2} + C(Q^2) \left(\frac{s-u}{m_N^2} \right)^2 \right]$$

$$s = (p_\nu + p_n)^2 \qquad t = (p_\nu - p_e)^2 = q^2 = -Q^2 \qquad u = (p_e - p_n)^2$$

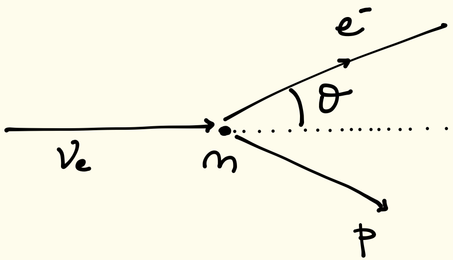
$$\begin{cases} A = \frac{Q^2}{m_N^2} \left\{ \left(1 + \frac{Q^2}{4m_N^2}\right) G_A^2 - \left(1 - \frac{Q^2}{4m_N^2}\right)\left(F_1^2 - \frac{Q^2}{4m_N^2} F_2^2\right) + \frac{Q^2}{m_N^2} F_1 F_2 \right\} \\ B = \frac{Q^2}{m_N^2} G_A (F_1 + F_2) \\ C = \frac{1}{4} \left(G_A^2 + F_1^2 + \frac{Q^2}{4m_N^2} F_2^2 \right) \end{cases}$$

<u>Note:</u>

took limit $m_e/m_N \to 0$

$\hookrightarrow$ G_P terms vanish

Useful: function of $E, \vec{p}$ of outgoing e^-



In lab frame: $\vec{p}_n = 0$

$$
\begin{cases}
s = (p_\nu + p_n)^2 = m_N^2 + 2E_\nu m_N \\
t = (p_e - p_\nu)^2 = -Q^2 = m_e^2 - 2E_\nu(E_e - |\vec{p}_e| \cos\theta) \\
u = (p_e - p_n)^2 = m_N^2 - 2m_N E_\nu + 2E_\nu(E_e - |\vec{p}_e| \cos\theta)
\end{cases}
$$

Final result:

$$\boxed{\begin{gathered}\frac{d\sigma}{d\cos\theta} = -\frac{G_F^2 |V_{ud}|^2 m_N^2}{4\pi} \frac{|\vec{p}_e|}{E_\nu} \left[A(Q^2) + B(Q^2)\frac{s-u}{m_N^2} + C(Q^2)\left(\frac{s-u}{m_N^2}\right)^2 \right] \\ (s-u) = 4 m_N E_\nu - 2E_\nu (E_e - |\vec{p}_e| \cos\theta)\end{gathered}}$$

$$\begin{cases} s = (p_\nu + p_n)^2 = m_N^2 + 2E_\nu m_N \\ t = (p_e - p_\nu)^2 = -Q^2 = m_e^2 - 2E_\nu (E_e - |\vec{p}_e| \cos\theta) \\ u = (p_e - p_n)^2 = m_N^2 - 2 m_N E_\nu + 2E_\nu (E_e - |\vec{p}_e| \cos\theta) \end{cases}$$

Weak charged current form factors

- $F_1(Q^2)$, $F_2(Q^2)$ known from electromagnetic form factors

$$F_1(0) = 1 \,, \qquad F_2(0) = \frac{\mu_p - \mu_n}{\mu_N} - 1 \simeq 3.71$$

- $$G_A(Q^2) = \frac{g_A}{\left(1 + Q^2/m_A^2\right)^2}$$

  $m_A \simeq 1$ GeV "axial mass"

  $g_A \simeq 1.27$

- F.F. precision measurement critical for ν experiments (oscillation, sterile ν searches, etc.)

$\bar{\nu}_e + p \longrightarrow n + e^+$:

- Same formalism, with $B(Q^2) \longrightarrow -B(Q^2)$

- Possibility to measure G_A :

$$\frac{d\sigma(\nu n)}{dQ^2} - \frac{d\sigma(\bar{\nu} p)}{dQ^2} \propto B(Q^2) \propto G_A(Q^2)\left(F_1(Q^2) + F_2(Q^2)\right)$$

Low energy limit : $E_\nu \ll m_N \longrightarrow Q^2/m_N^2 \ll 1$:

$$F_1 \longrightarrow g_V \simeq 1 \qquad G_A \longrightarrow g_A$$

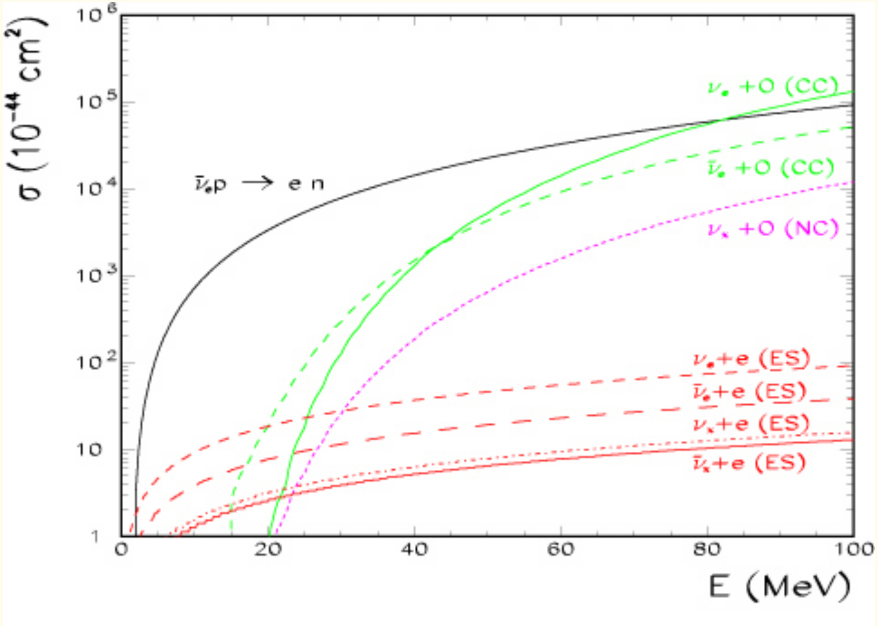
$$\begin{cases} A(Q^2) \simeq \dfrac{2E_\nu\left(E_e - |\vec{p}_e|\cos\theta\right)}{m_N^2}\left(g_A^2 - g_V^2\right) + O\left(\left(E_\nu/m_N\right)^3\right) \\[2ex] B(Q^2)\,\dfrac{s-u}{m_N^2} \simeq O\left(\left(E_\nu/m_N\right)^3\right) \\[2ex] C(Q^2)\left(\dfrac{s-u}{m_N^2}\right)^2 \simeq \dfrac{4E_\nu^2}{m_N^2}\left(g_A^2 + g_V^2\right) + O\left(\left(E_\nu/m_N\right)^3\right) \end{cases}$$

$\longrightarrow$ total cross section:

$$\boxed{\sigma(\nu n) = \sigma(\bar{\nu} p) = \frac{G_F^2 |V_{ud}|^2}{\pi}\left(1 + 3g_A^2\right)E_\nu^2} \simeq 1.6 \cdot 10^{-44}\,\text{cm}^2\left(1 + 3g_A^2\right)\left(E_\nu/\text{MeV}\right)^2$$

Application: $\bar{\nu}_e + p \longrightarrow n + e^+$

- ν from Supernovae, reactors, atmospheric
- Water Cherenkov, liquid scintillator ($C_n H_{2n}$)



Skadhauge and Zukanovich, JCAP 2007

Final words

- Thank you!
- Sorry for typos, etc.
- All questions/feedback, etc:

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