Low-Energy Nuclear Structure Lecture 3: ‘Probing’ Wavefunctions

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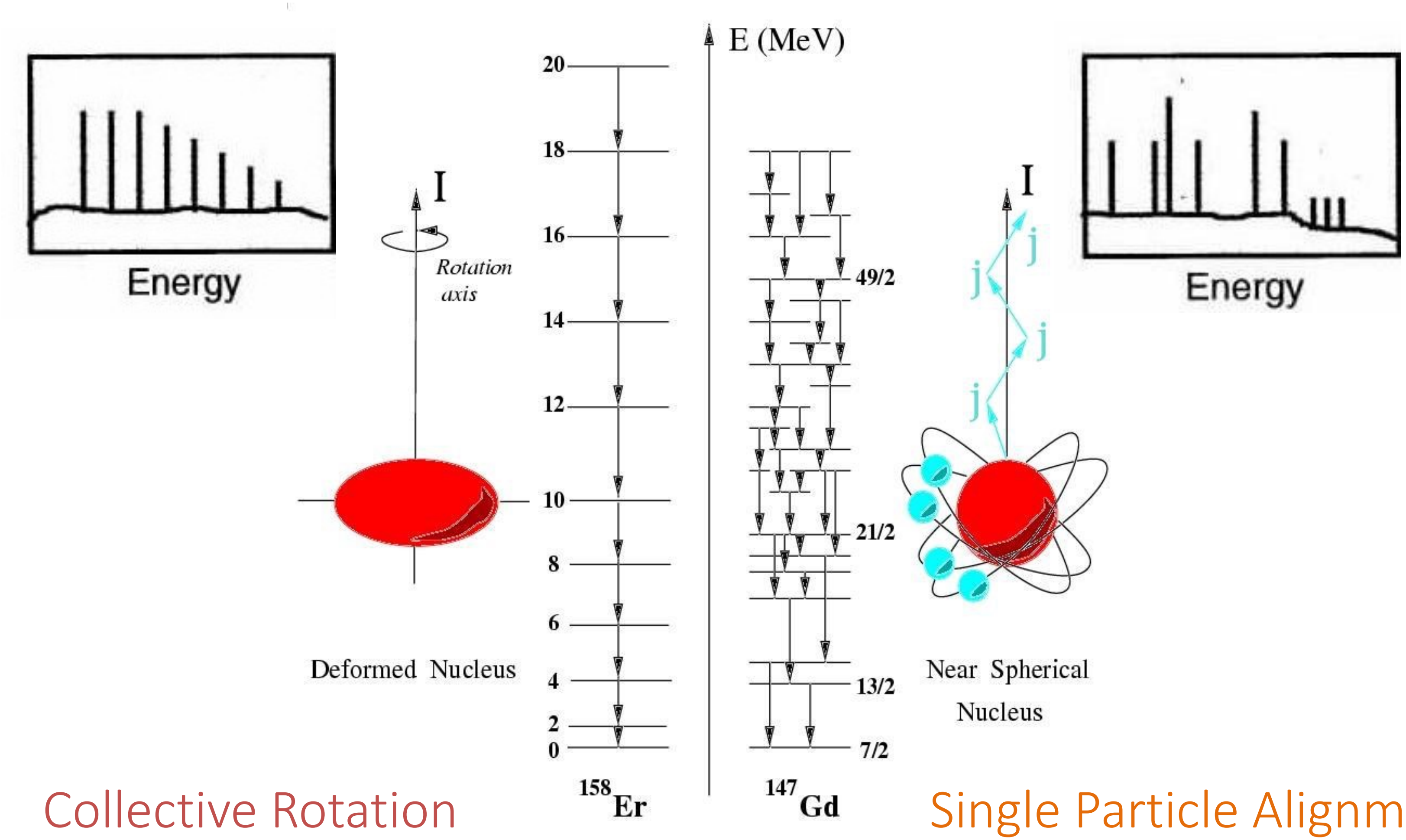
The (Third) Plan

- Investigating level schemes
  - Decay spectroscopy
- Details of nuclear wavefunctions
  - Single particle ‘occupancies’ and spectroscopy with nuclear reactions
  - Excited state lifetimes and transition probabilities
- Example – planning an experiment
  - What, where, why?

Level schemes – collective vs. single particle

Level Schemes Contain Structural Information



Collective Rotation

Single Particle Alignment

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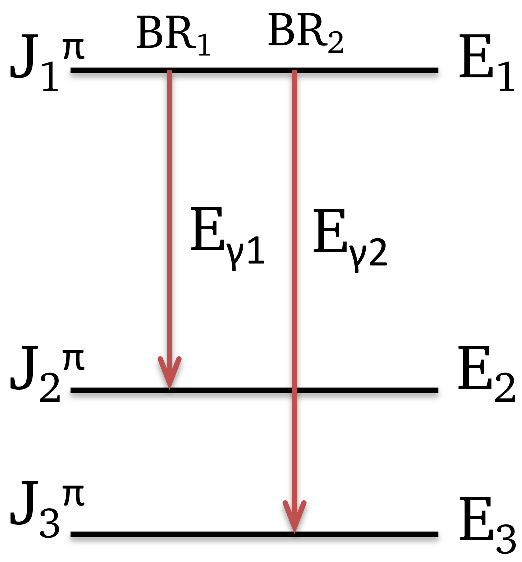

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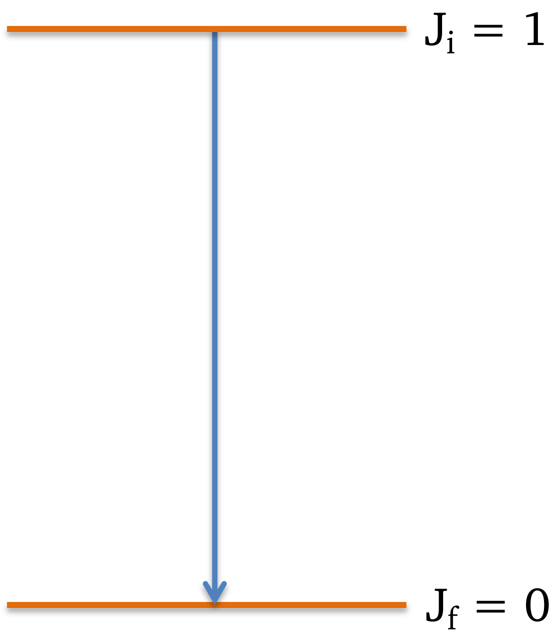
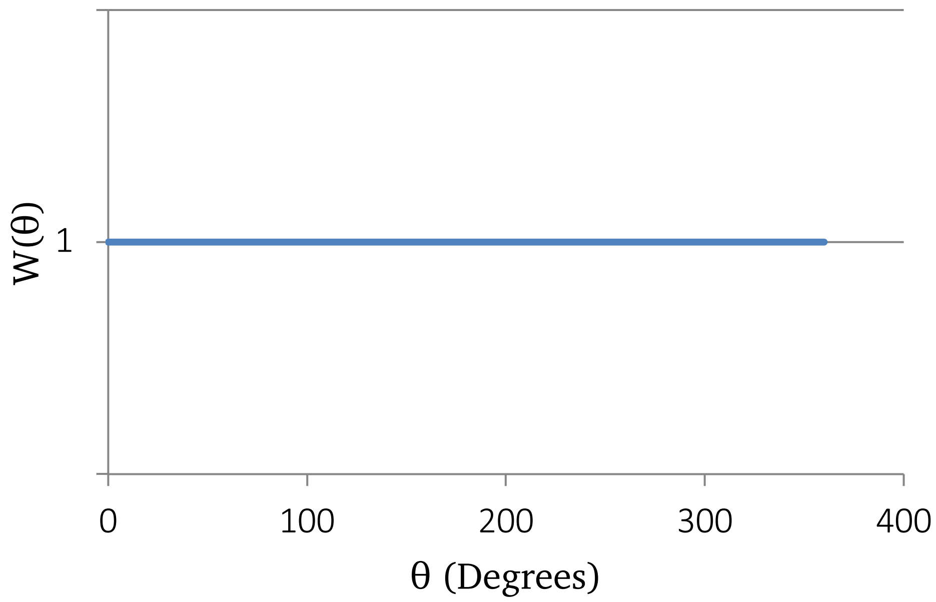
- Knowledge of J_i and J_f limit the multipolarity (L) of gamma-ray transitions
- To measure multipole order (L) we can measure angular distributions
- To determine E vs. M we need to measure polarization of the transition

Gamma-Ray Angular Distributions

- Angular distribution of a gamma-ray depends on the values of m_i and m_f





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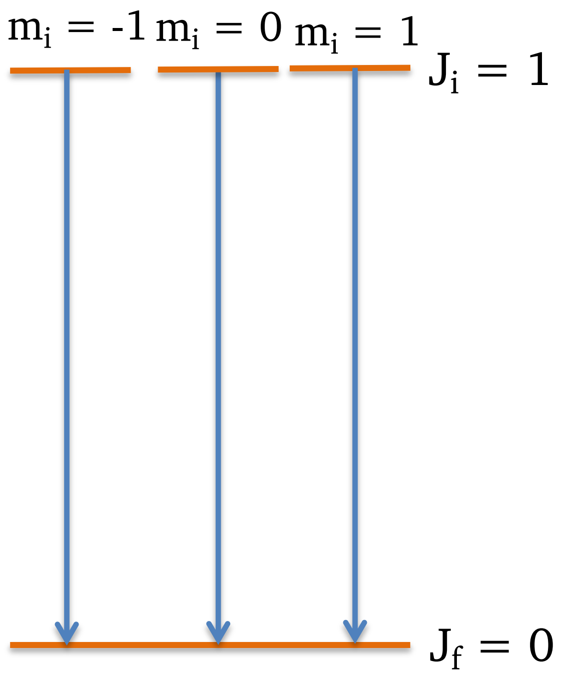
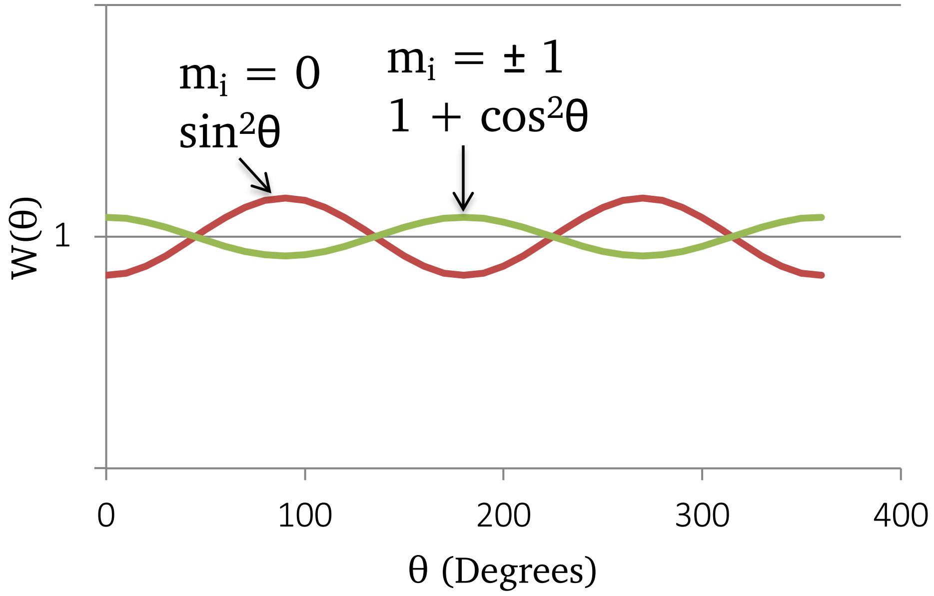
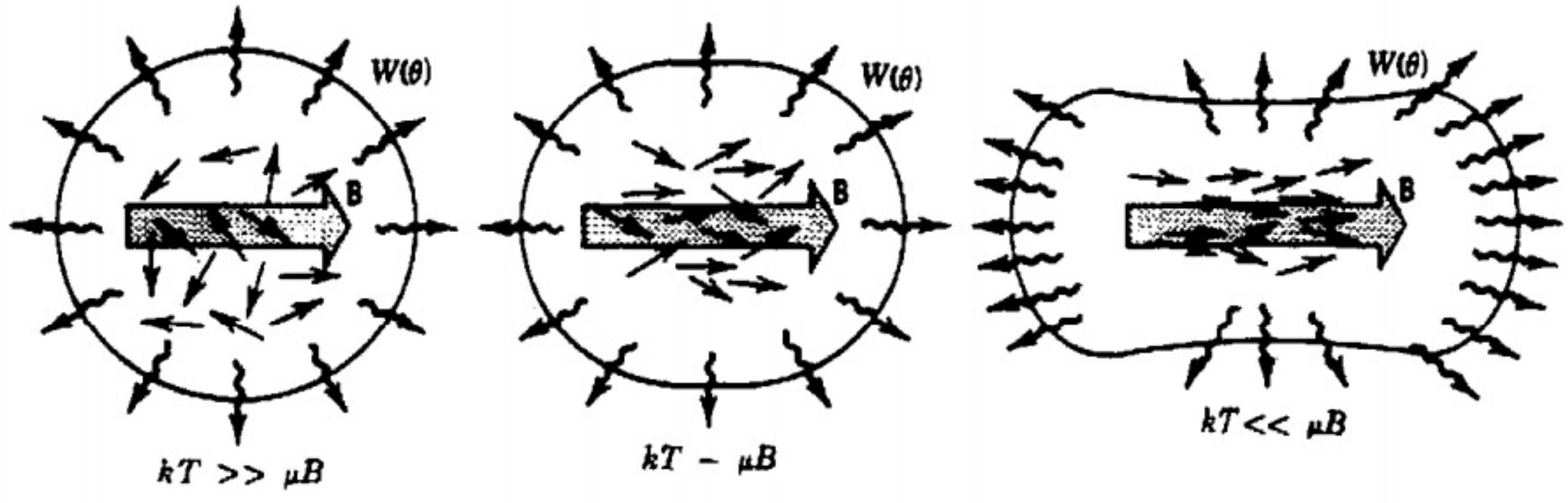




Gamma-Ray Angular Distributions

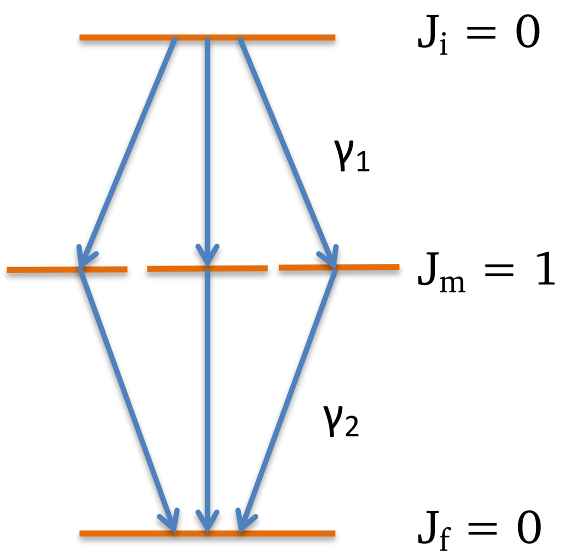
- If we produce unequal populations $p(m_i)$ angular distributions $W(\theta)$ will be non-constant

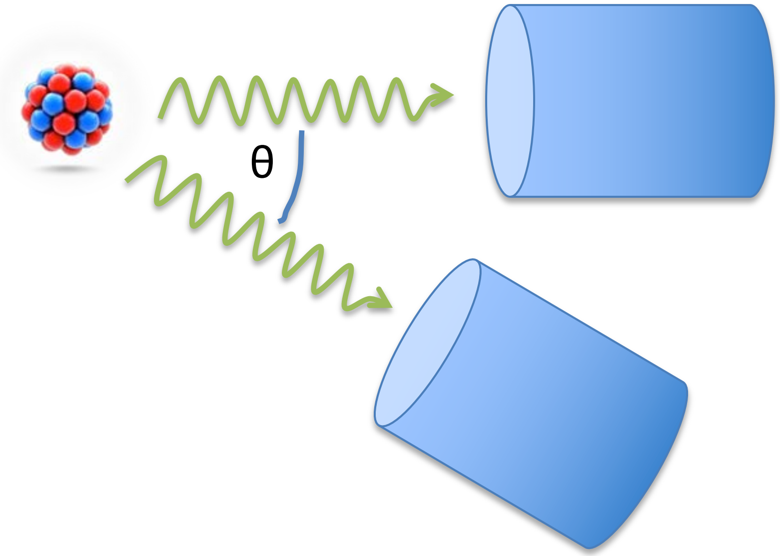
Nuclear Orientation



Gamma-Ray Angular Correlations

- Observation of a previous radiation selects an unequal mixture of populations $p(m_i)$

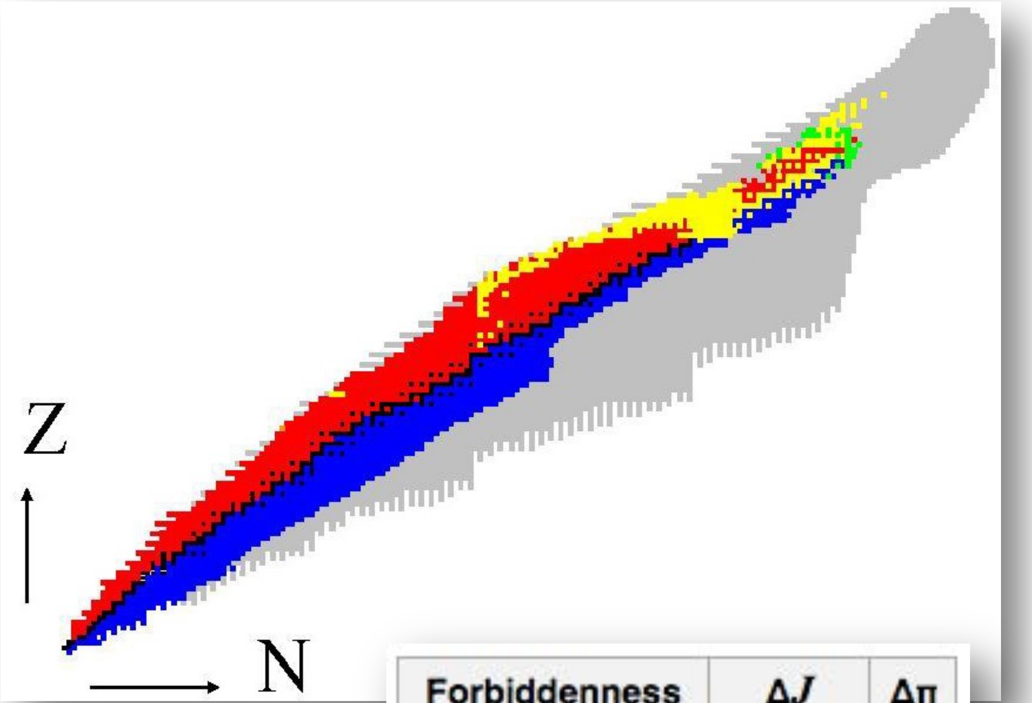




- First gamma defines z-axis -- $\theta_1 = 0$
- $p(m_m) = 0$ for $m_m = 0$
- Distribution of γ_2 relative to γ_1 is $m = \pm 1$ --> $m = 0$
- $W(\theta)$ --> $1 + \cos^2\theta$

Back to β decay…

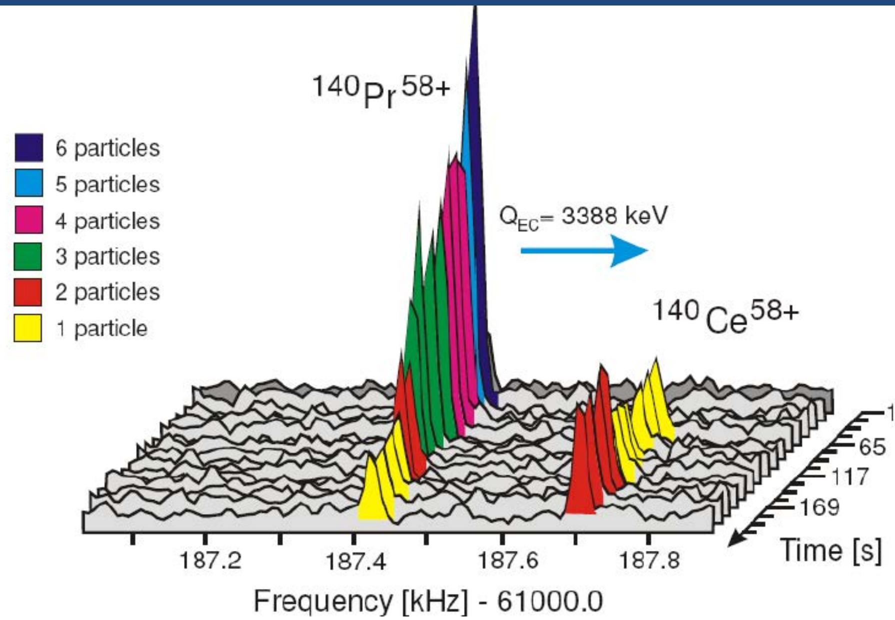
- The majority of nuclides on the chart decay via β^+ or β^- decay
  - $n \dashrightarrow p + \beta^- + \nu_e$
  - $p \dashrightarrow n + \beta^+ + \bar{\nu}_e$
- We can consider β-decay (and other decays) as a tool to populate excited states in daughter nuclei, but with a unique selectivity

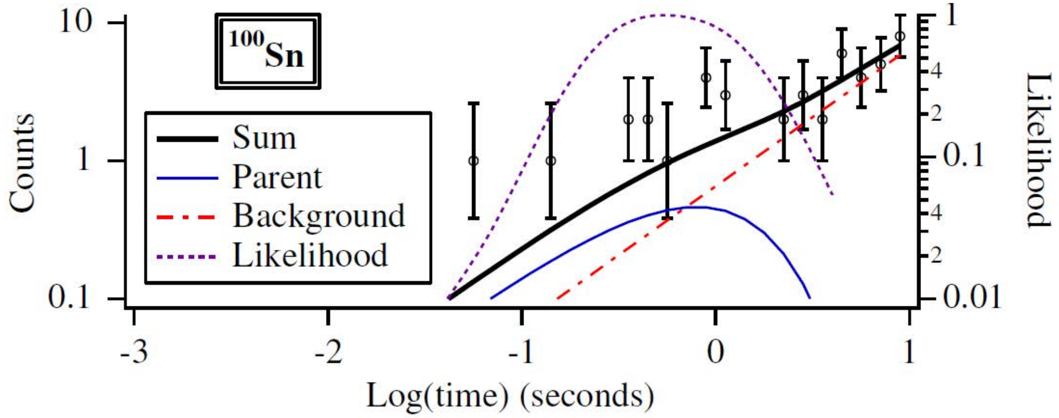


Forbiddenness	ΔJ	Δπ
Superallowed	$0^+ \rightarrow 0^+$	no
Allowed	0, 1	no
First forbidden	0, 1, 2	yes
Second forbidden	1, 2, 3	no
Third forbidden	2, 3, 4	yes

β-decay half-lives

- even with the most limited statistics, half-lives can be extracted
- the equations of exponential decay are well known and can be applied using statistical techniques such as maximum likelihood to obtain half-lives from tens of observed decays

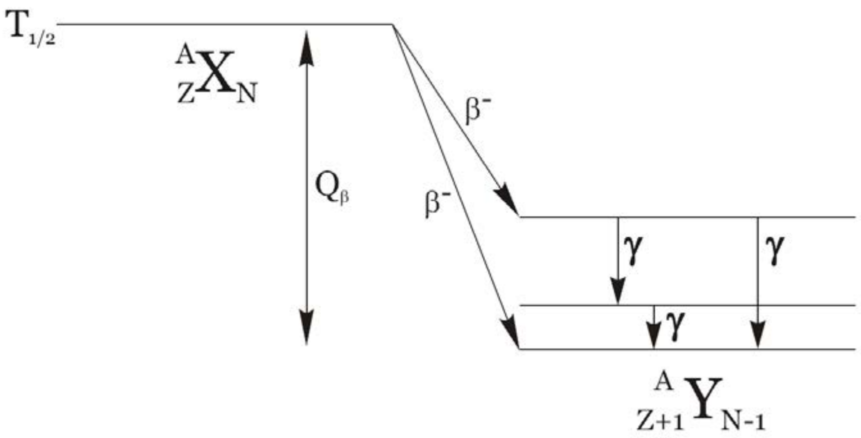




D. Bazin et al., PRL 101, 252501 (2008).
F. Bosch et al., Int. J. Mass Spectr. 251, 212 (2006).

Implantation β decay spectroscopy

β-Delayed Gamma Spectroscopy

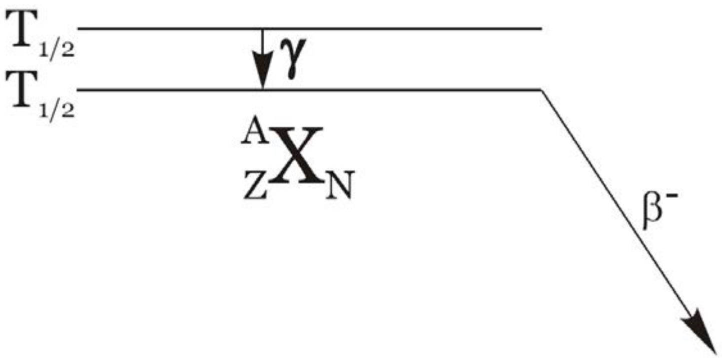


• gamma rays following decay events provide information on low-level structure of daughter nuclei

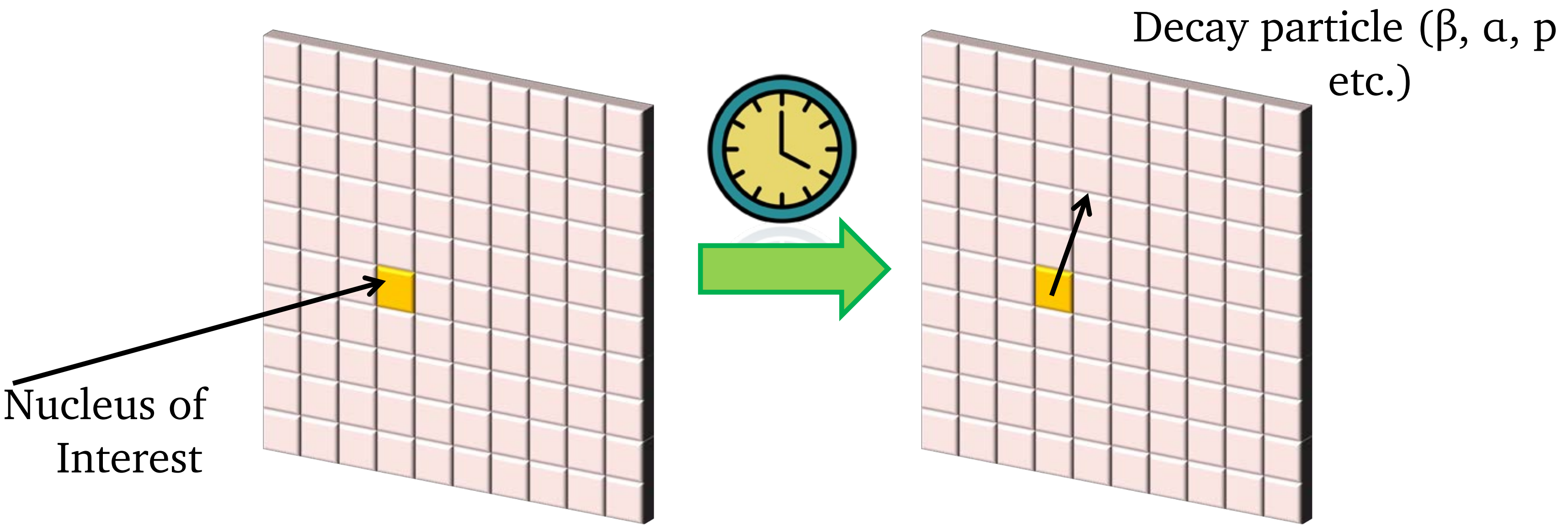
Isomeric Decay

• depending on the production mechanism, nuclei may be produced in long-lived excited states (isomeric states)

• a TAC for implantation-gamma provides the possibility for isomer lifetime determination, if you look for gammas following an implantation

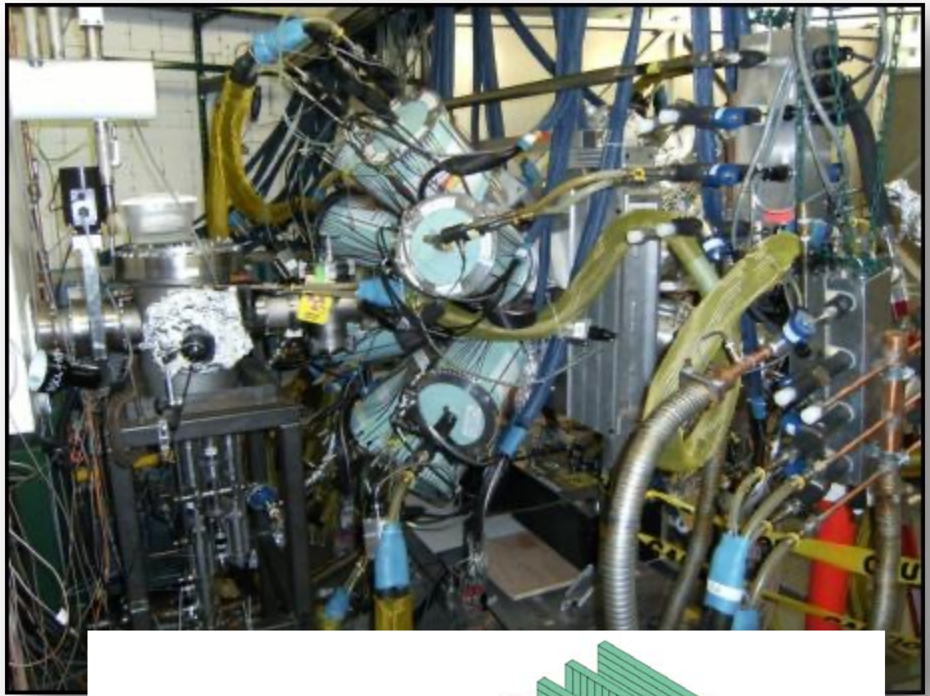


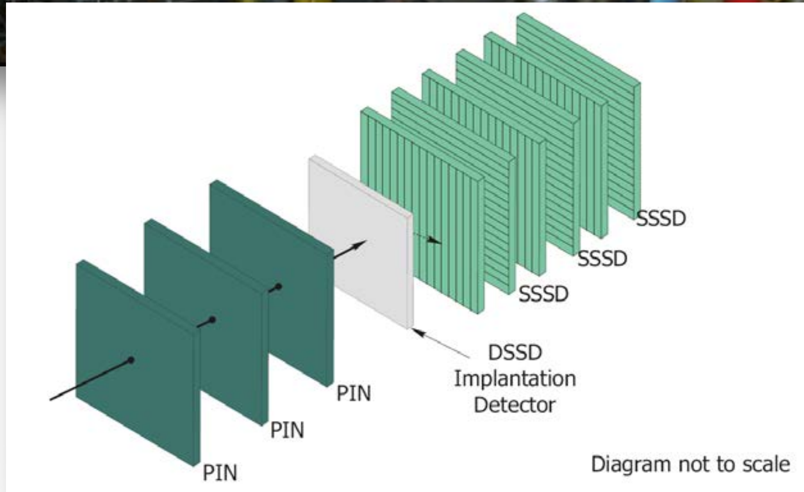
Implant-decay correlation technique

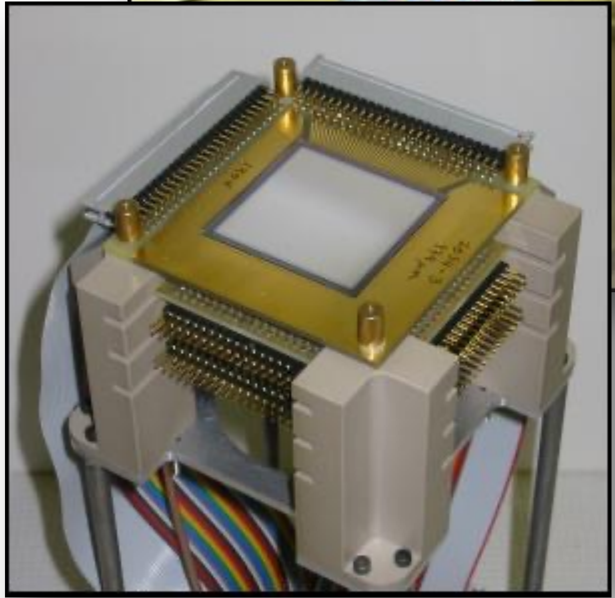


The use of highly-segmented detectors (usually Si) allows temporal and spatial correlations between implanted nuclei, and their subsequent decays → detect the implant and the decay to obtain half-lives and information on levels in the daughter relative to the parent ground state

β-decay spectroscopy set-up: NSCL

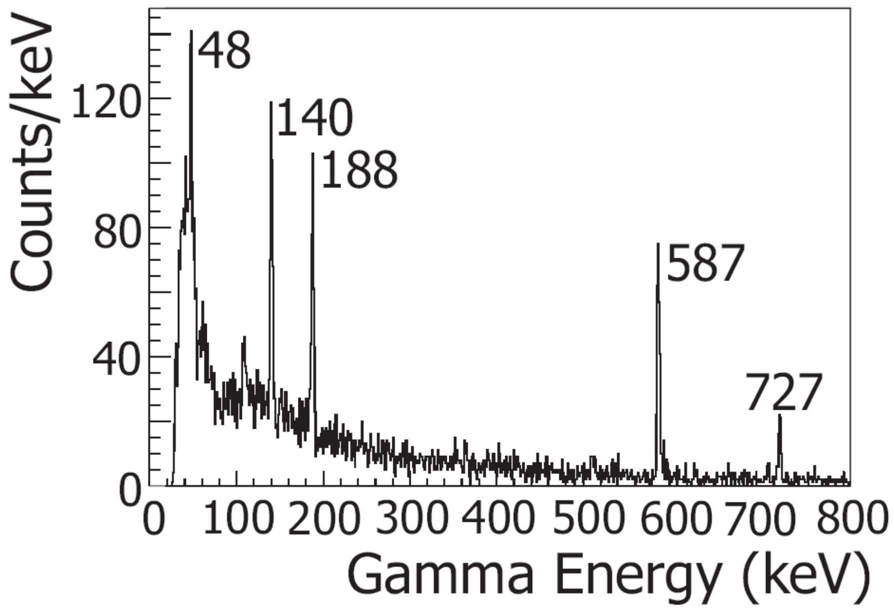


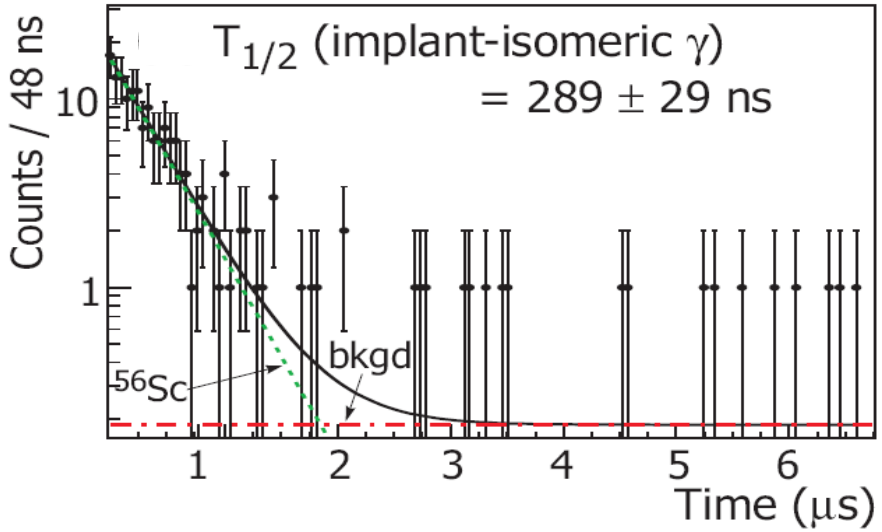




β-decay spectroscopy: complex example

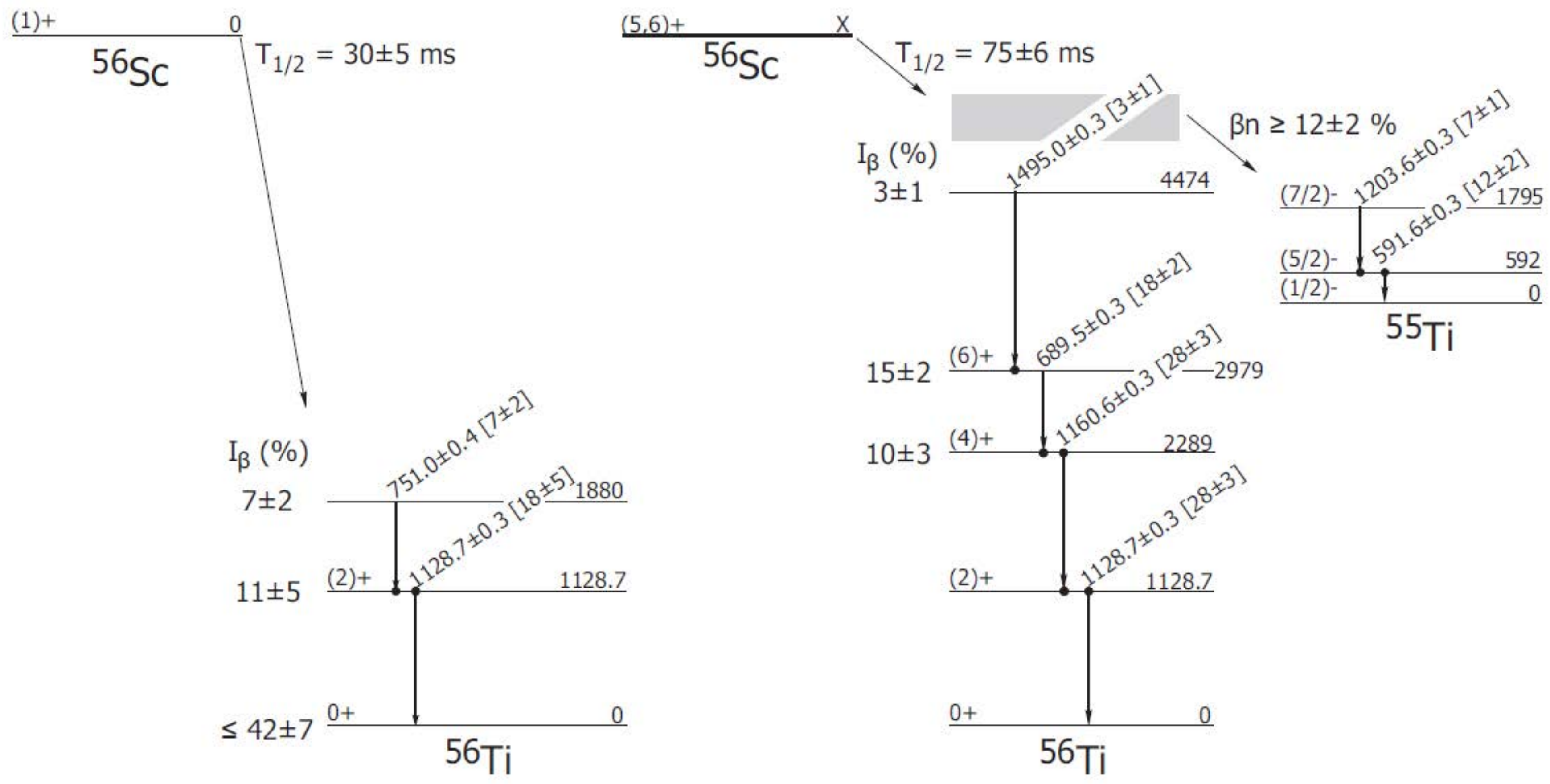
A. Look at the gamma-rays in coincidence with the nucleus of interest (^{56}Sc) implantations – by fitting half-lives of the isomer, and through gamma-gamma correlations, build up a level scheme, and can get relative spin-parities for the states in ^{56}Sc



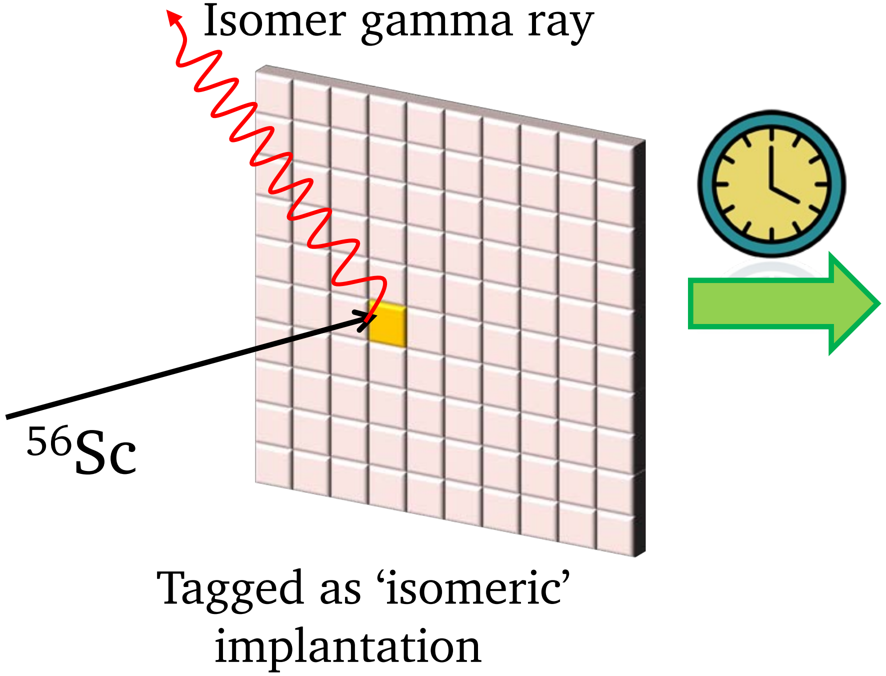


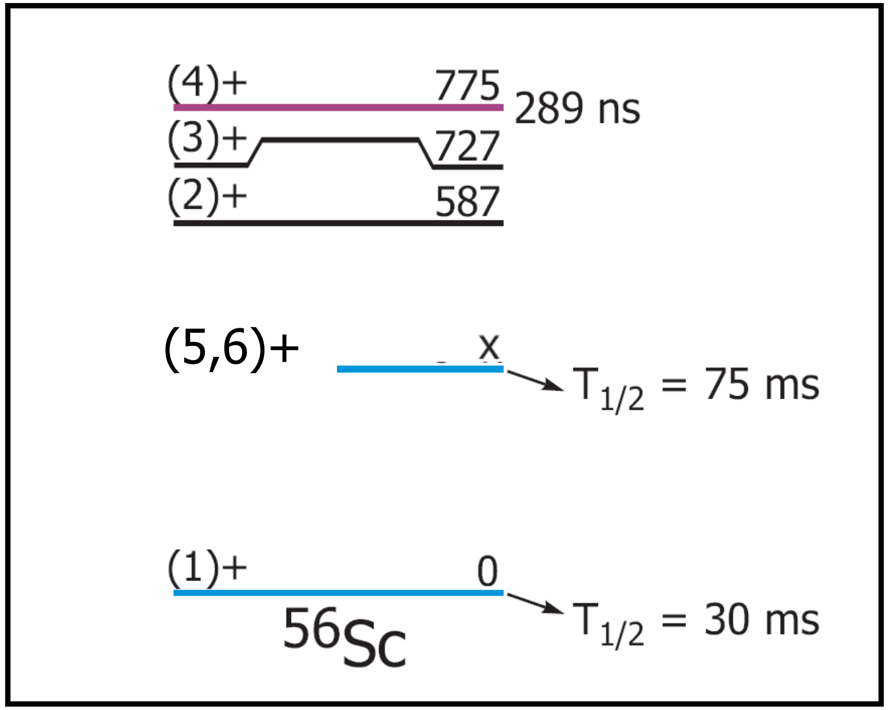
HLC et al., PRC 82, 014311 (2010).

β-decay spectroscopy: complex example



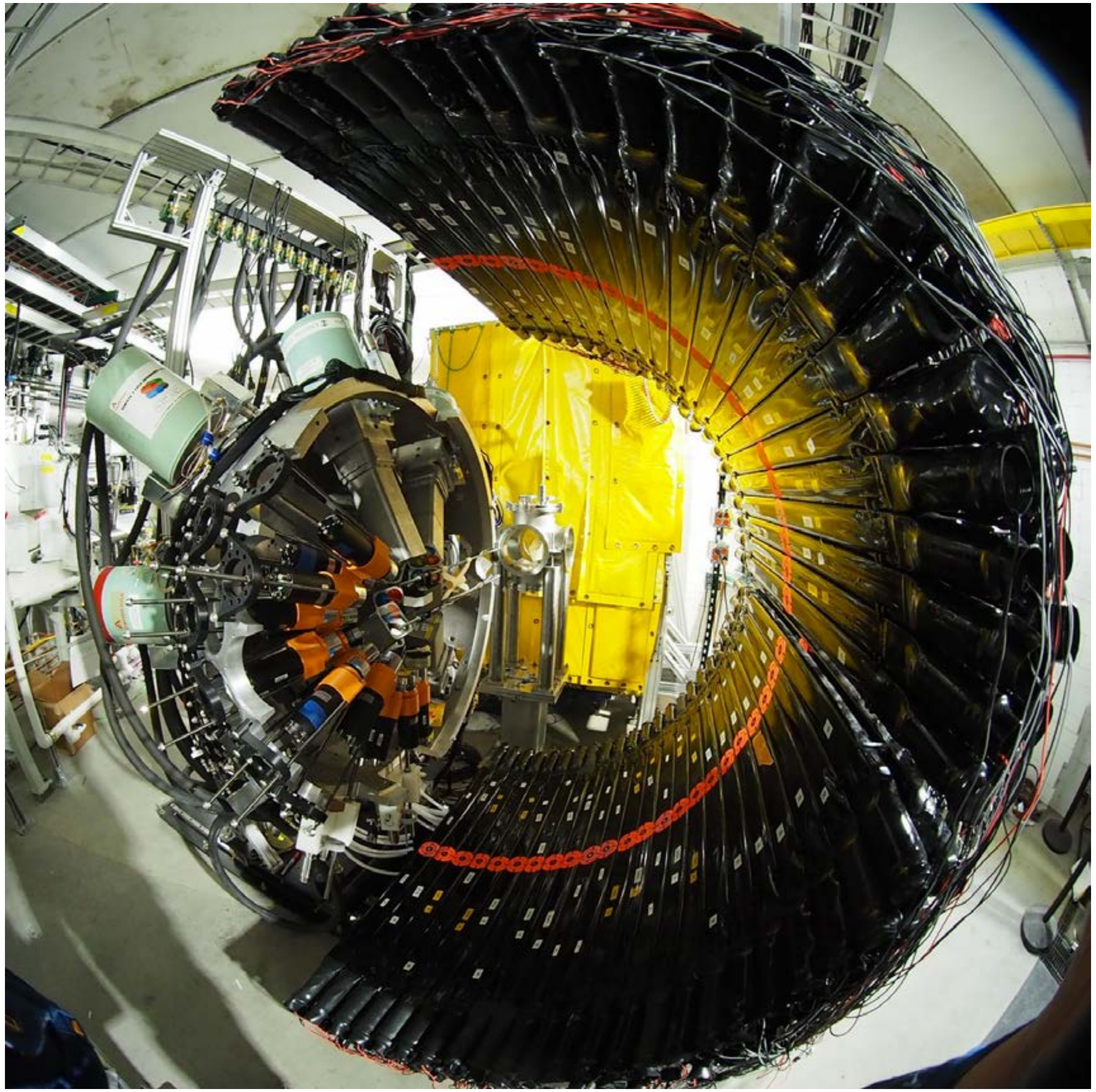
β-decay spectroscopy: complex example

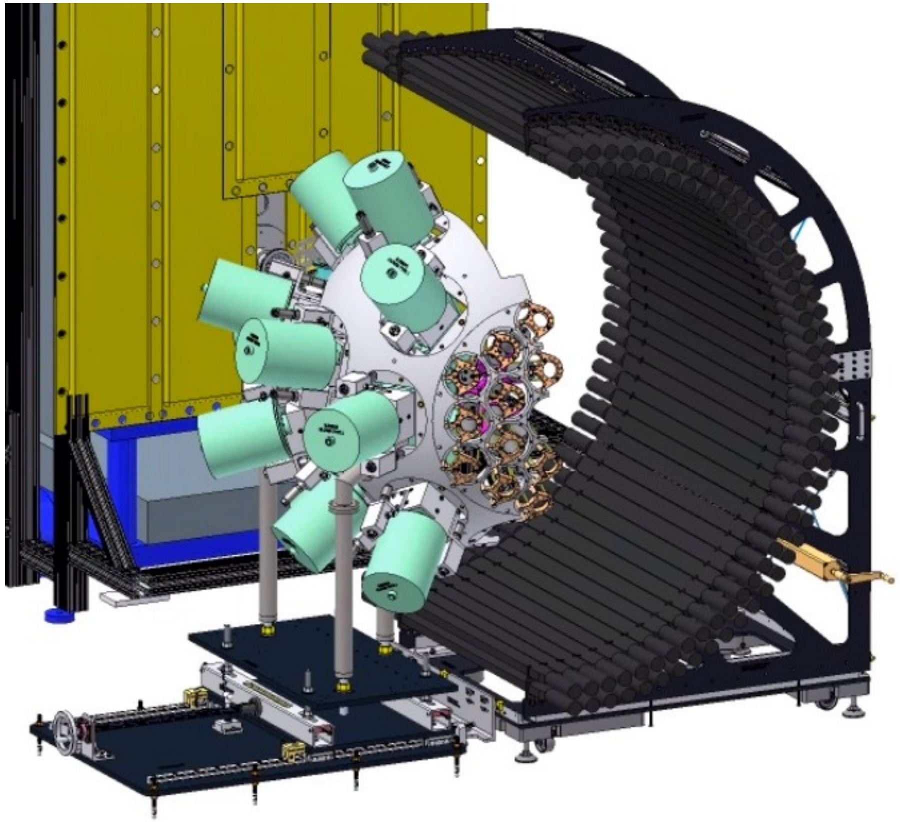




D. Gate on implantations that came in coincidence with isomer gamma-rays and look at half-life → determine which state the isomer populates, and fix the spin/parity

β-decay spectroscopy at FRIB

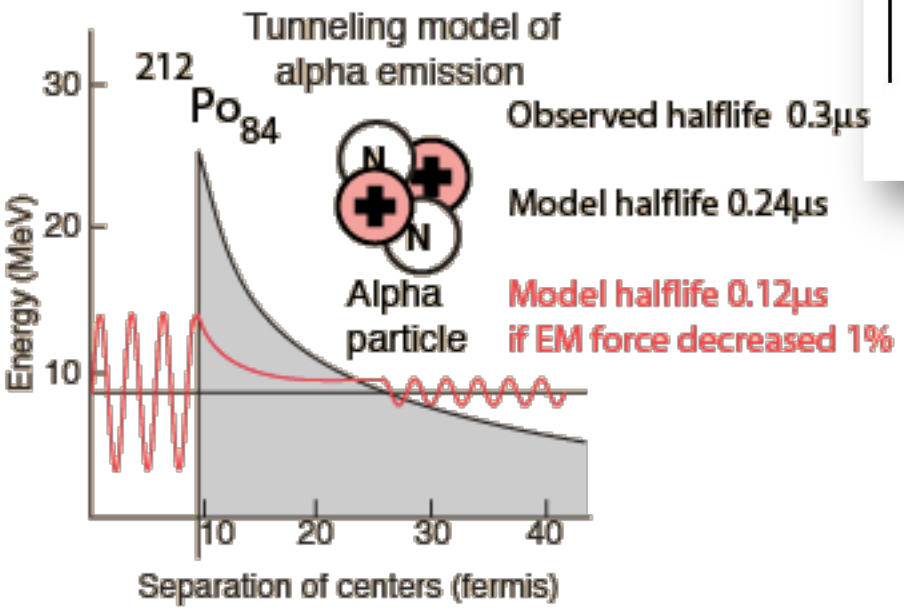


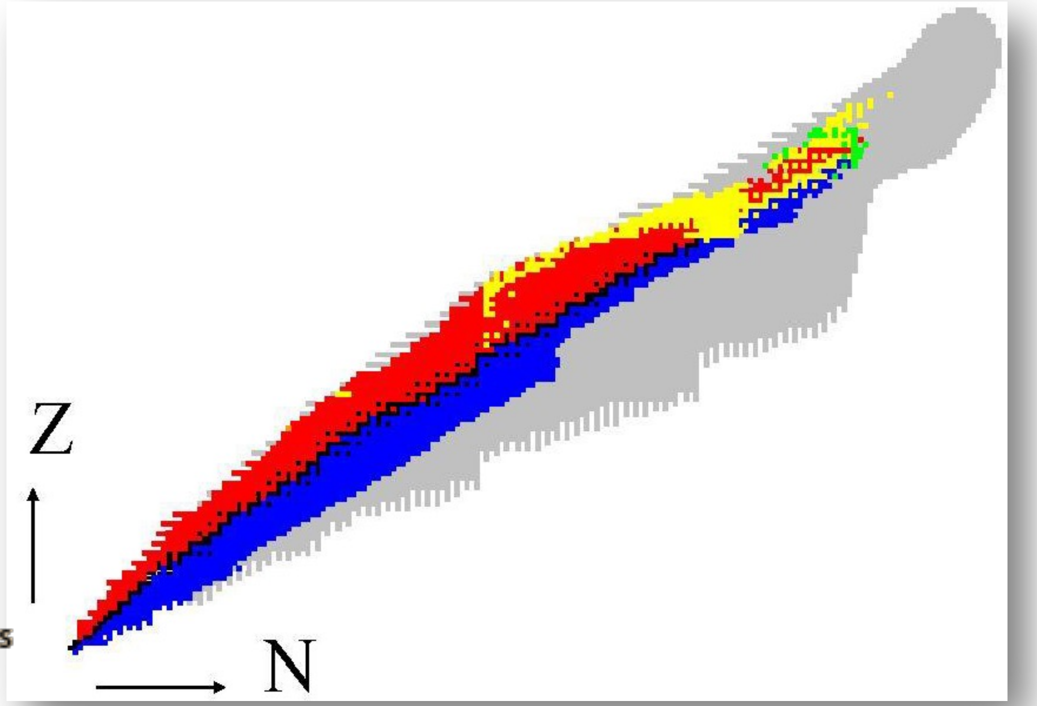


The FRIB Decay Station initiator (FDSi) is being led by ORNL and UTK, and includes (in addition to HPGe), fast timing scintillators, neutron-detection and possibilities for TAS measurements

Alpha decay

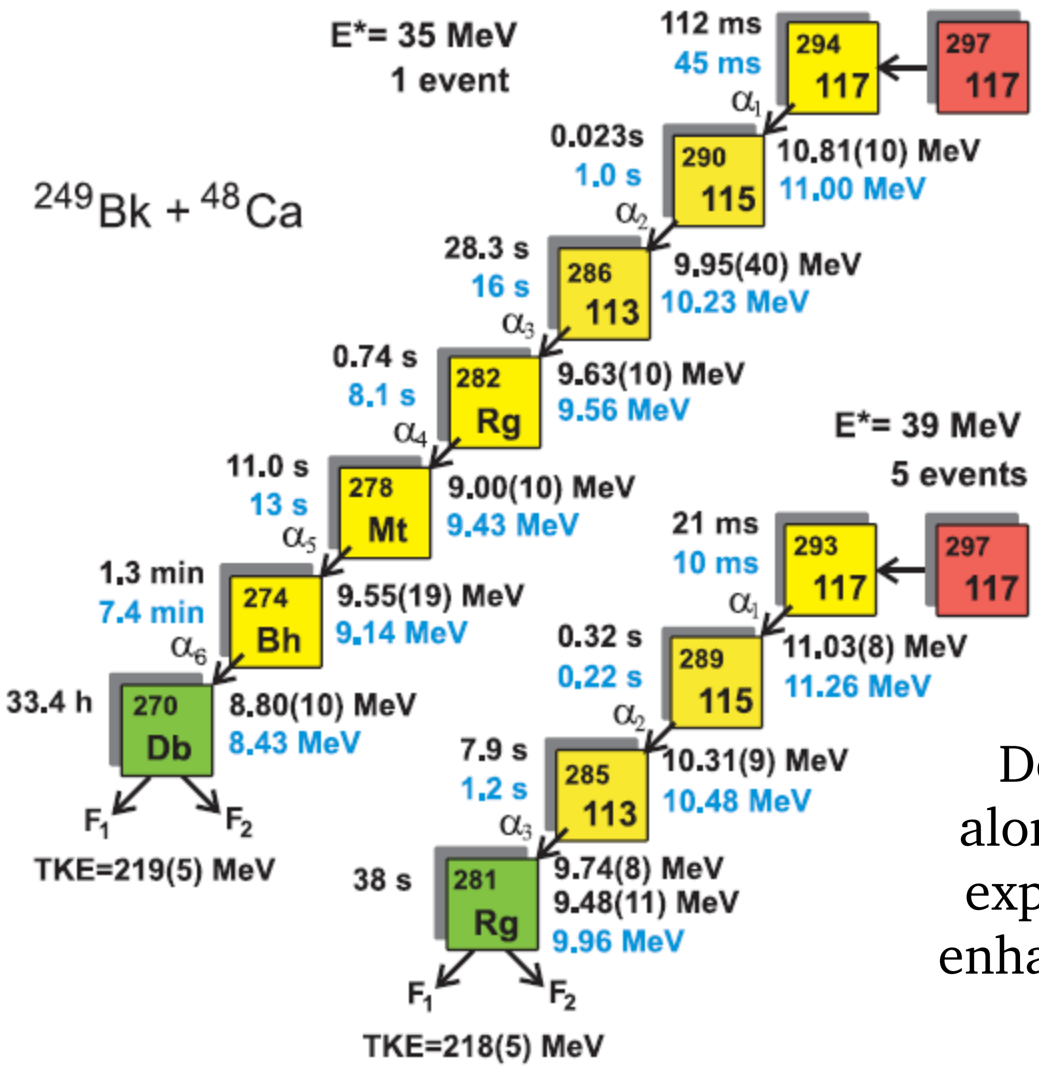
- α decay occurs only in heavier systems on the nuclear chart
- Alpha decay however probes different aspects of the nuclear forces





- Different selectivity in the process --> favour low L alpha emission

Alpha decay – heavy element structure



Yu. Ts. Oganessian et al., PRL 104, 142502 (2010).

The heaviest nuclei decay via emission of ‘heavy’ particles – alpha decay – or by spontaneous fission

Since alphas and fission products are relatively easy to detect, even a single nucleus can provide significant information

Decay properties of element 117 alone, from only 6 events, provide experimental evidence supporting enhanced stability beyond Z = 111

The heaviest nuclei – patience required!

Selected for a Viewpoint in *Physics*
PRL **104**, 142502 (2010) PHYSICAL REVIEW LETTERS week ending 9 APRIL 2010

Synthesis of a New Element with Atomic Number Z = 117

Yu. Ts. Oganessian,[1,*] F. Sh. Abdullin,[1] P. D. Bailey,[2] D. E. Benker,[2] M. E. Bennett,[3] S. N. Dmitriev,[1] J. G. Ezold,[2] J. H. Hamilton,[4] R. A. Henderson,[5] M. G. Itkis,[1] Yu. V. Lobanov,[1] A. N. Mezentsev,[1] K. J. Moody,[5] S. L. Nelson,[5] A. N. Polyakov,[1] C. E. Porter,[2] A. V. Ramayya,[4] F. D. Riley,[2] J. B. Roberto,[2] M. A. Ryabinin,[6] K. P. Rykaczewski,[2] R. N. Sagaidak,[1] D. A. Shaughnessy,[5] I. V. Shirokovsky,[1] M. A. Stoyer,[5] V. G. Subbotin,[1] R. Sudowe,[3] A. M. Sukhov,[1] Yu. S. Tsyganov,[1] V. K. Utyonkov,[1] A. A. Voinov,[1] G. K. Vostokin,[1] and P. A. Wilk[5]

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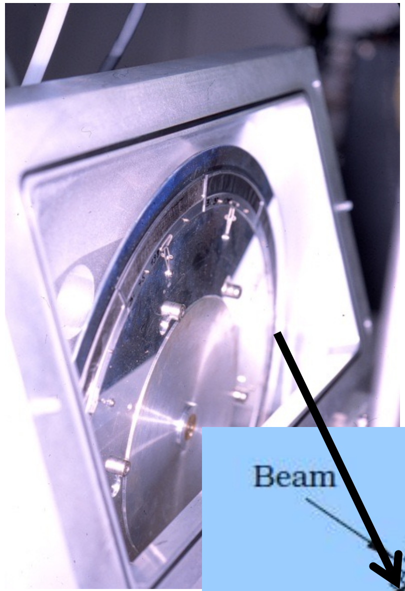
[2]*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[3]*University of Nevada Las Vegas, Las Vegas, Nevada 89154, USA*
[4]*Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA*
[5]*Lawrence Livemore National Laboratory, Livermore, California 94551, USA*
[6]*Research Institute of Atomic Reactors, RU-433510 Dimitrovgrad, Russian Federation*

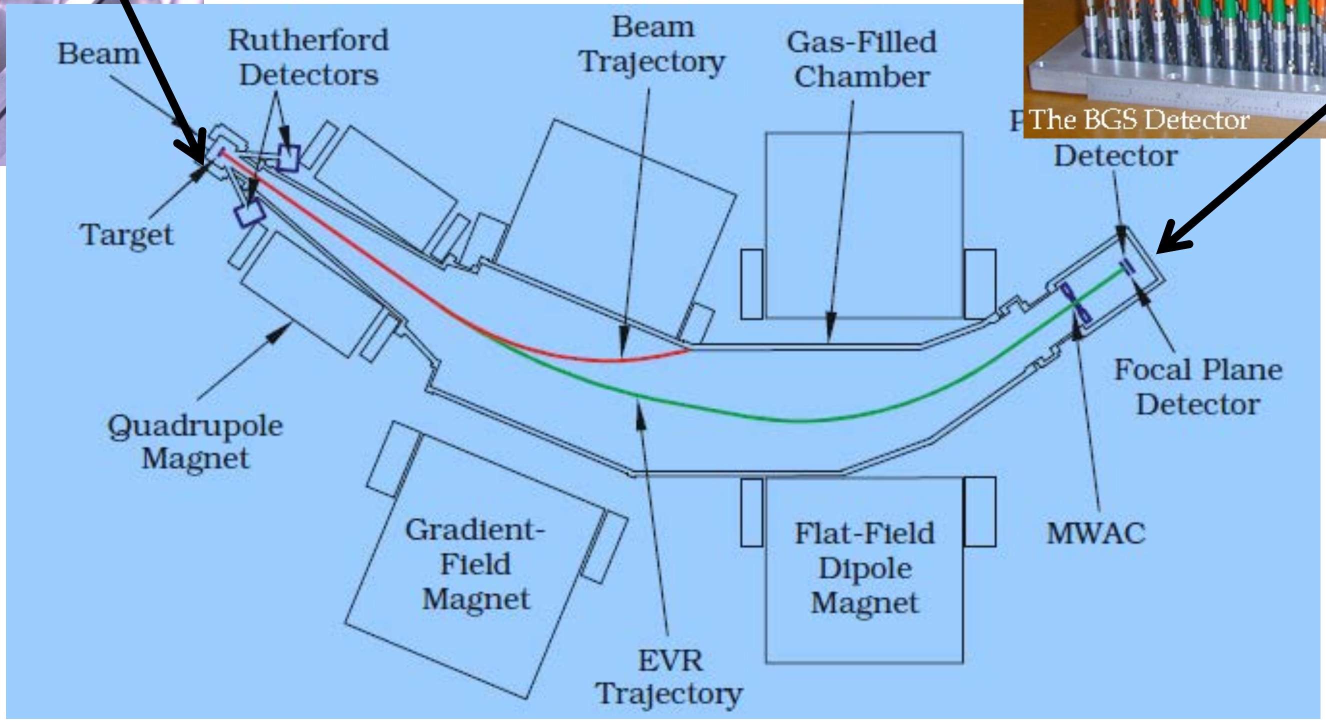
(Received 15 March 2010; published 9 April 2010)

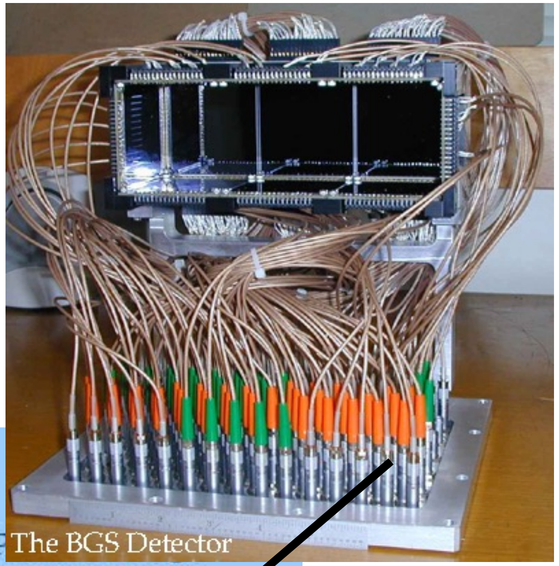
Experiment ran for 70 days, ^{48}Ca at 7 x 10^{12} ions/second on ^{249}Bk
→ 5 observed decay chains for ^{293}117 and 1 for ^{294}117,
corresponding to cross-sections of 0.5pb and 1.1pb

Spectroscopy of heavy elements

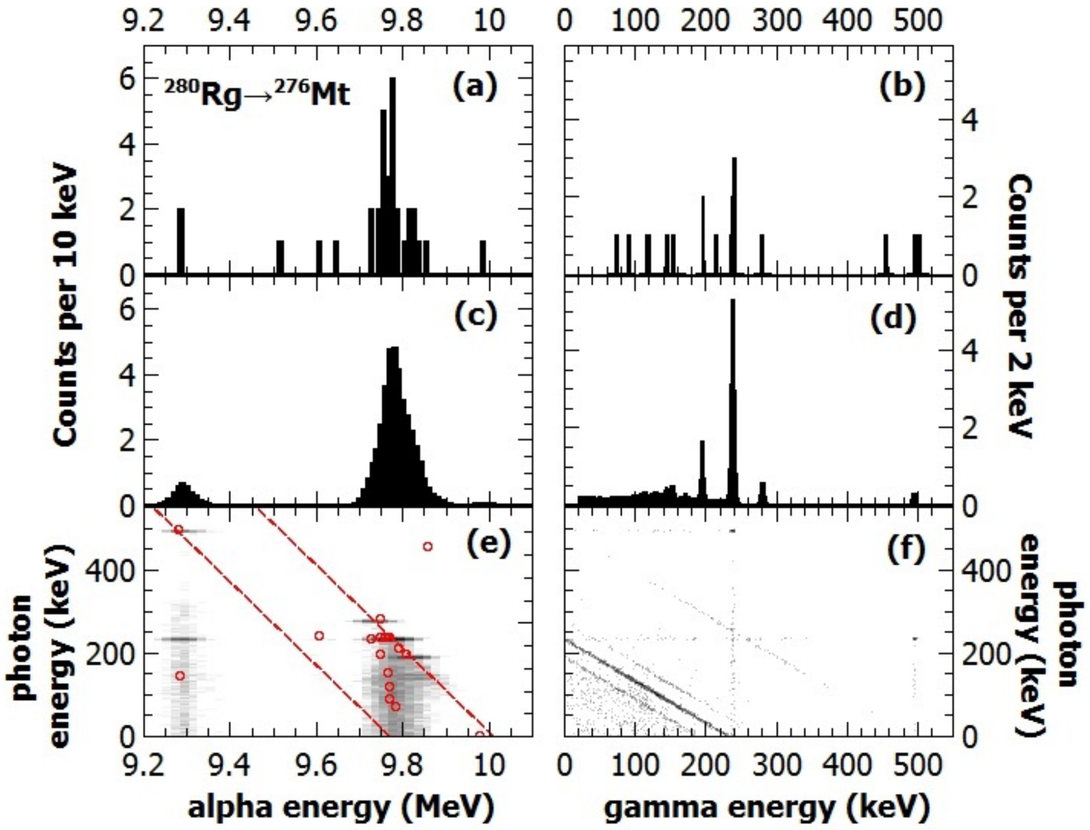


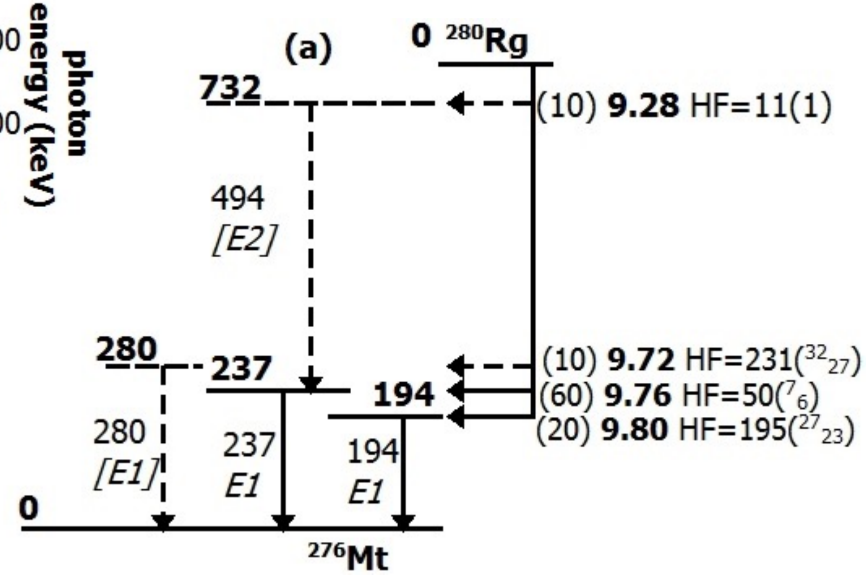
Efficiency is critical!! Gas-filled separators 'collect' charge states, high efficiency separation, and Si box-type arrays provide high efficiency for detecting residues.





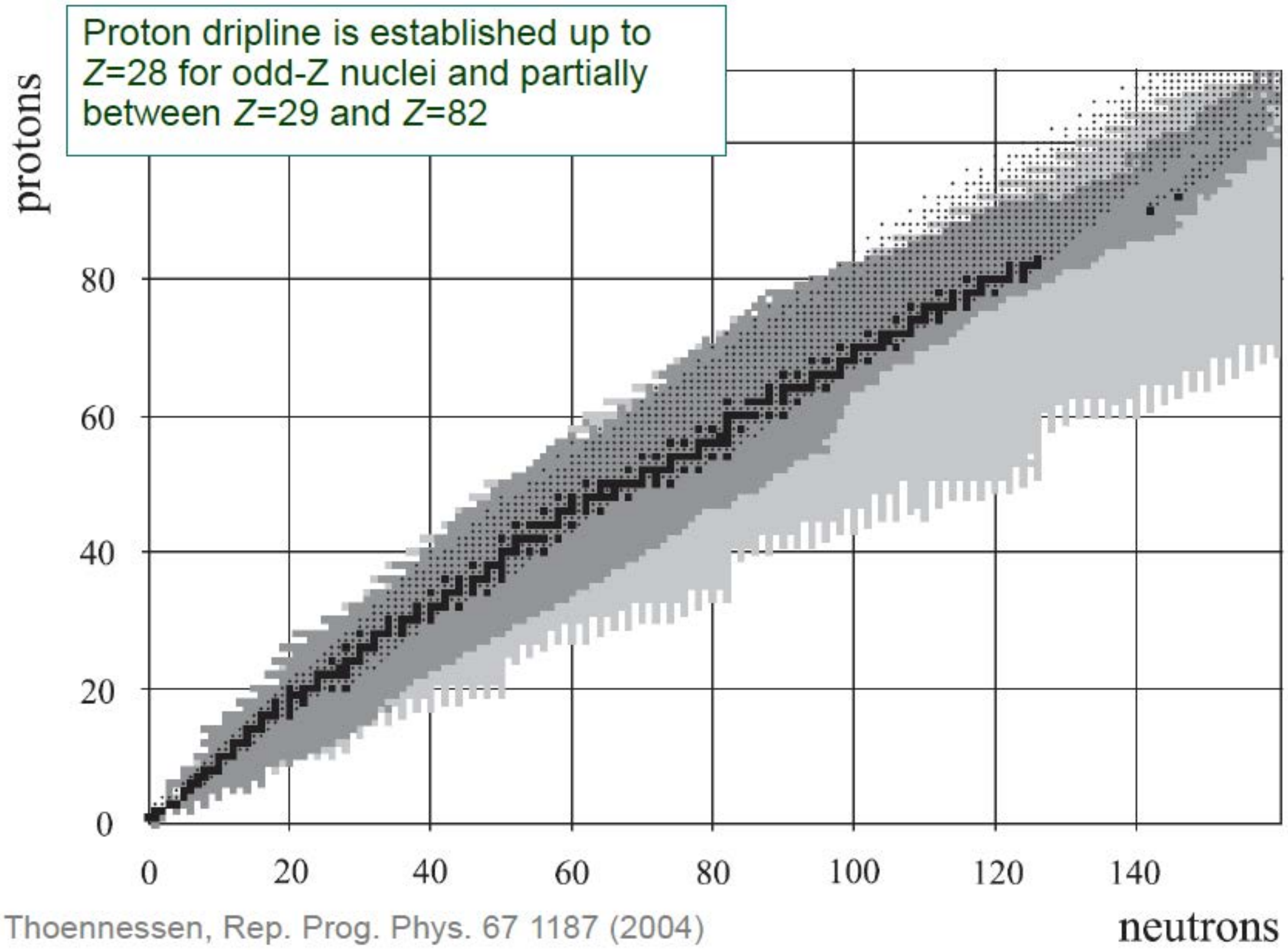
Spectroscopy from element 115





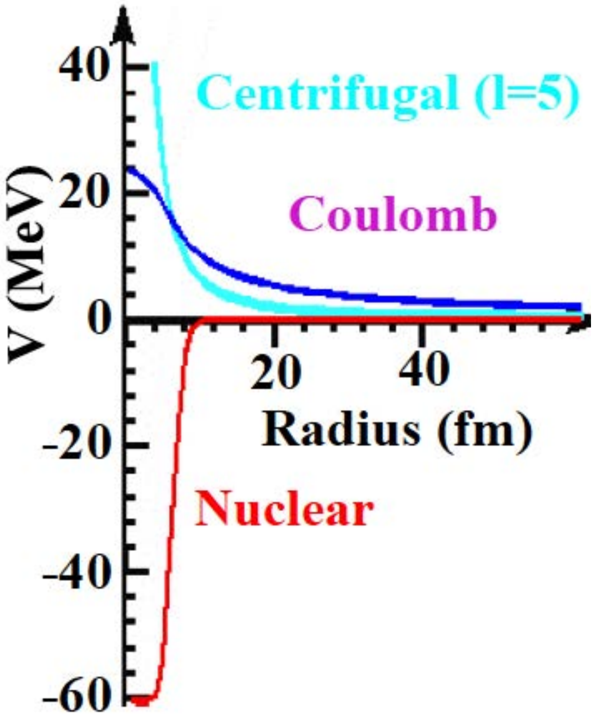
J. M. Gates et al.

Proton dripline

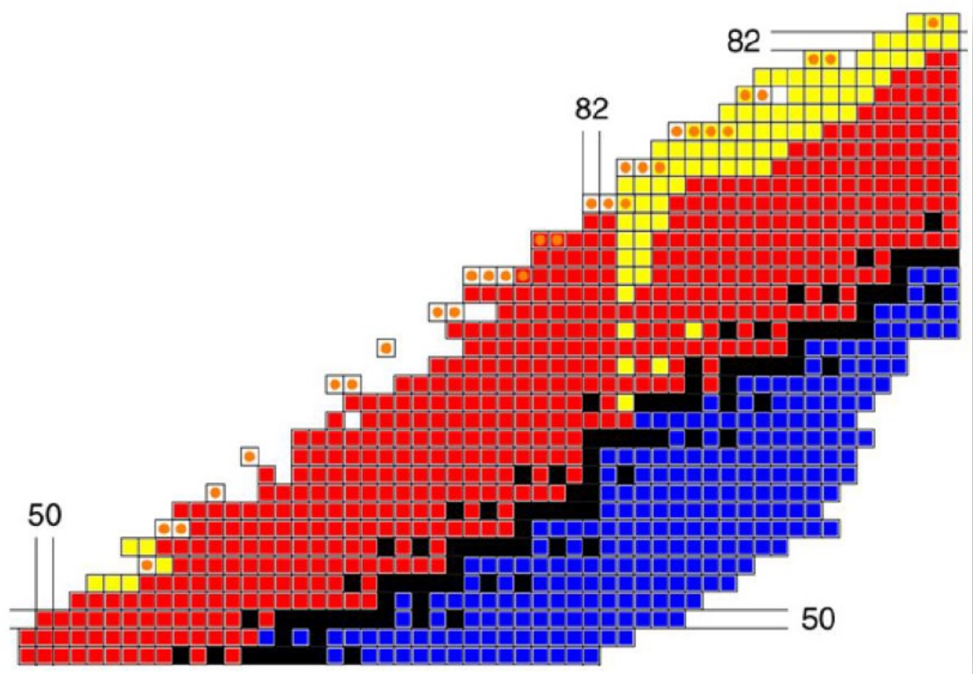


Thoennessen, Rep. Prog. Phys. 67 1187 (2004)

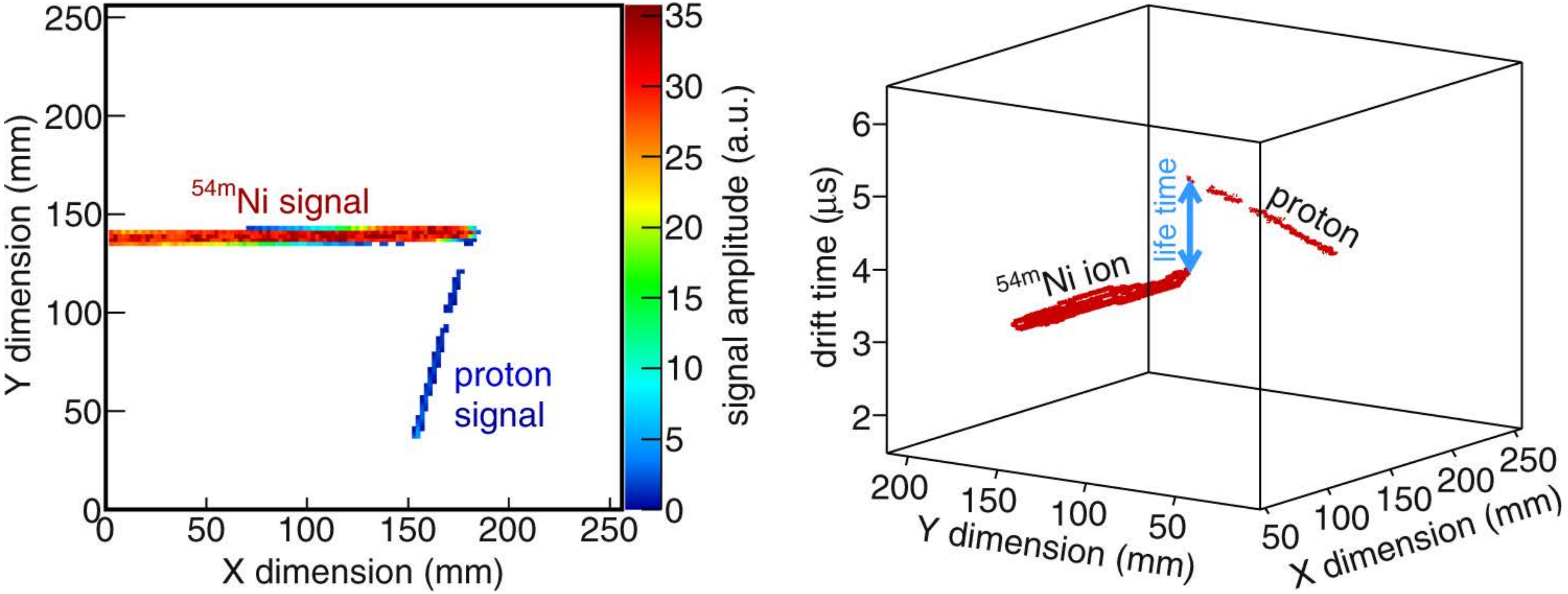
Proton decay



- Even when the Q value for proton removal becomes positive, proton emission is hindered due to the Coulomb (and centrifugal) barriers --> radioactivity



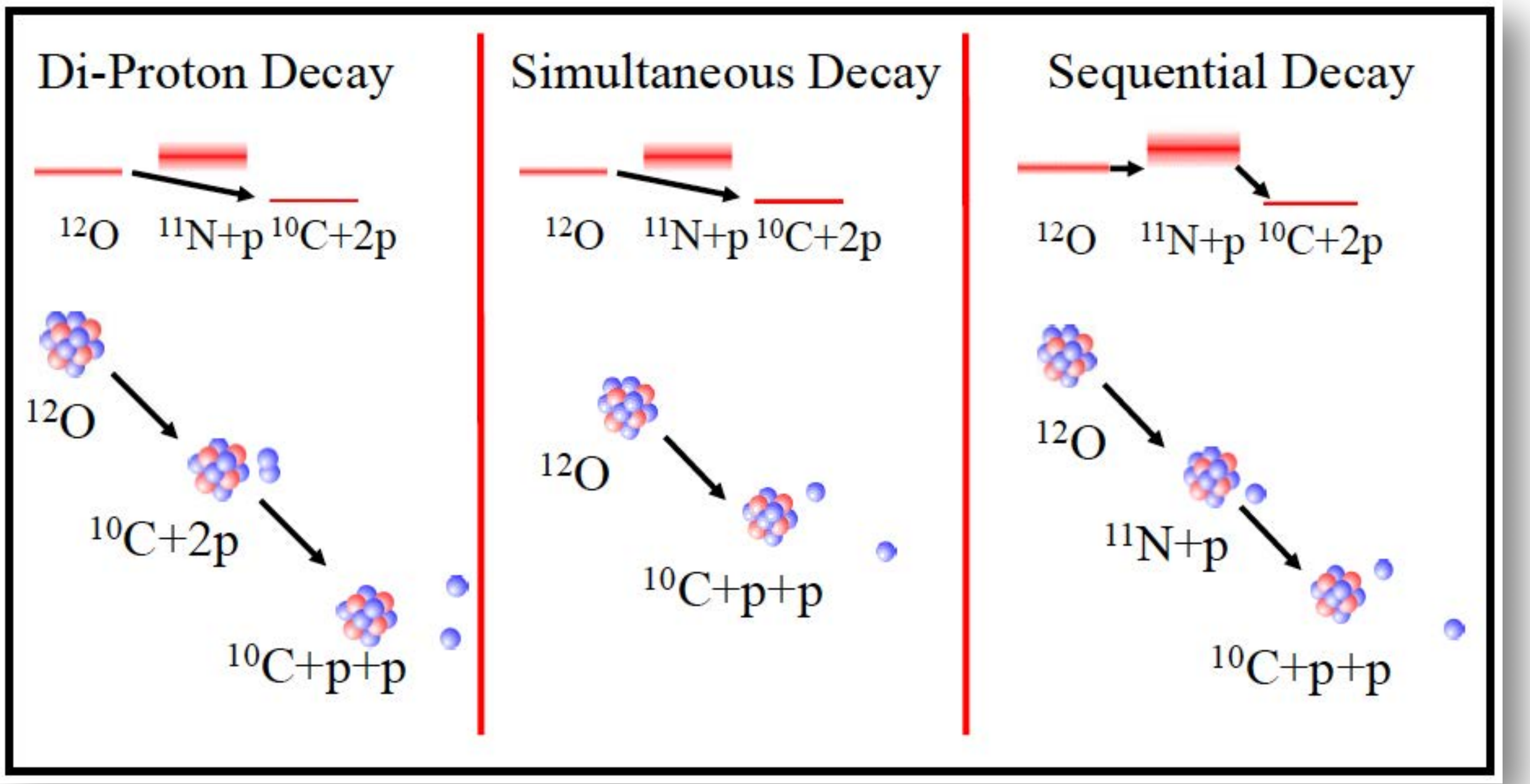
Proton emission branches in ^{54m}Ni



- A recent experiment with the ACTAR TPC measured proton decay from isomeric states in ^{54}Ni
- Data were reproduced reasonably well with shell-model calculations for the initial and final state wavefunctions and a barrier penetration model for the proton emission

J. Giovinazzo *et al.*, Nature Communications **12**, 4805 (2021).

2p decay



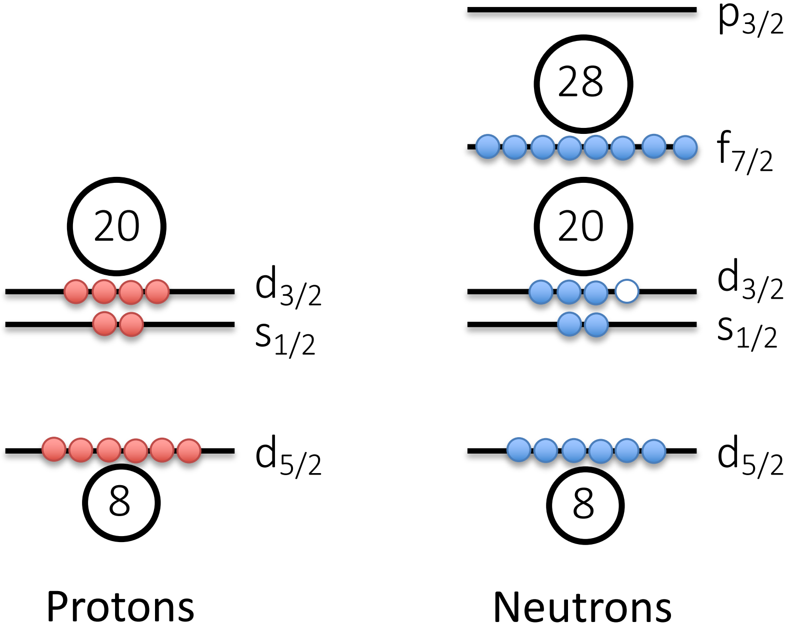
Di-Proton Decay
Simultaneous Decay
Sequential Decay
12O
11N+p
10C+2p
12O
11N+p
10C+2p
12O
11N+p
10C+2p
12O
10C+2p
10C+p+p
12O
10C+p+p
12O
11N+p
10C+p+p

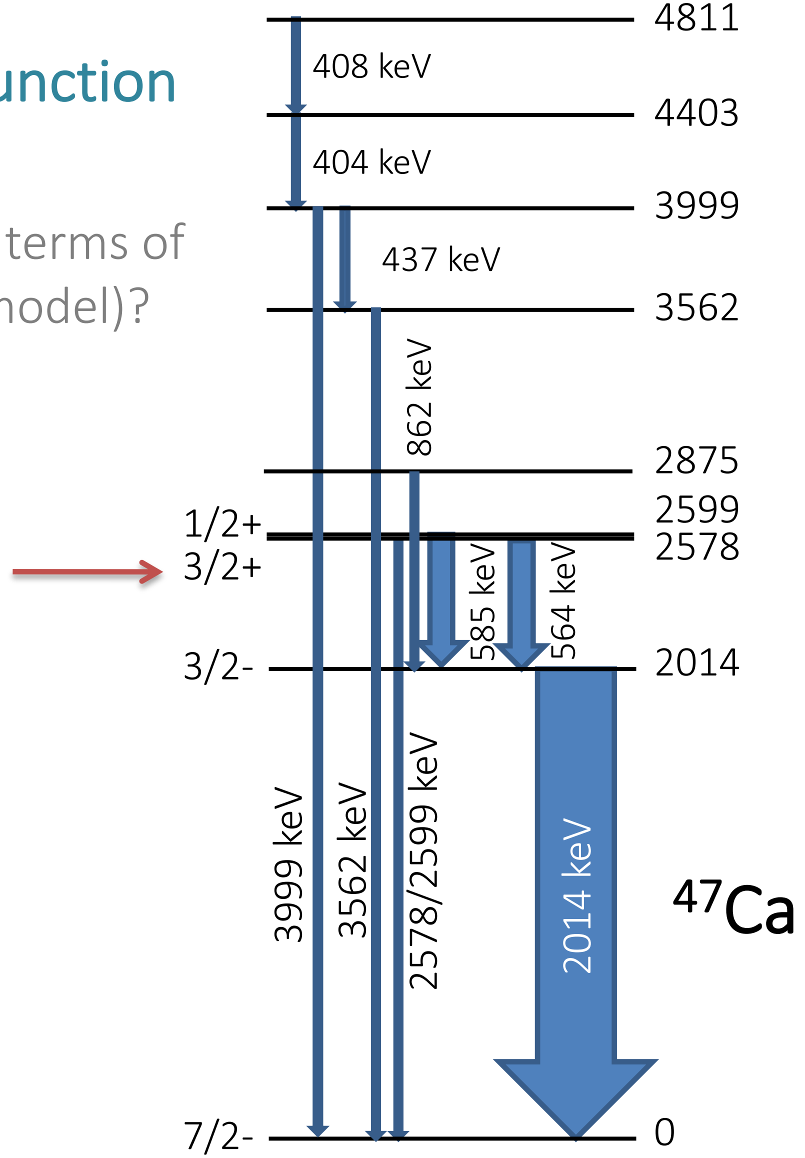
Probing wavefunctions

Beyond excitation energies and spins?

Can we probe the details of the wavefunction 'directly'?

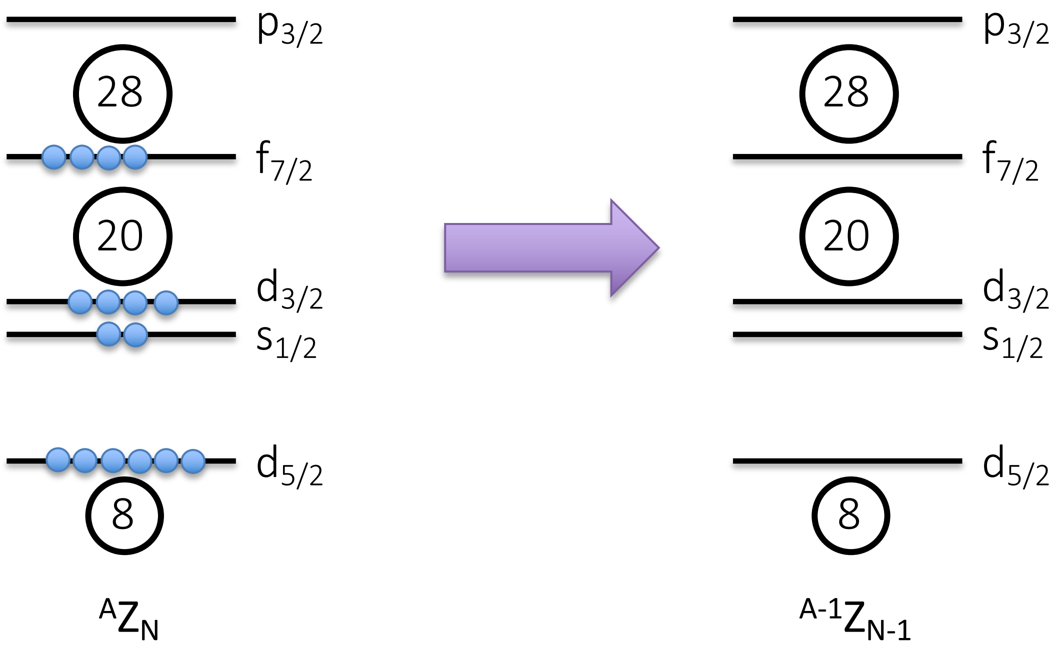
Is there a way to tell where the particles are in terms of single-particle states (even within a specific model)?





Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout



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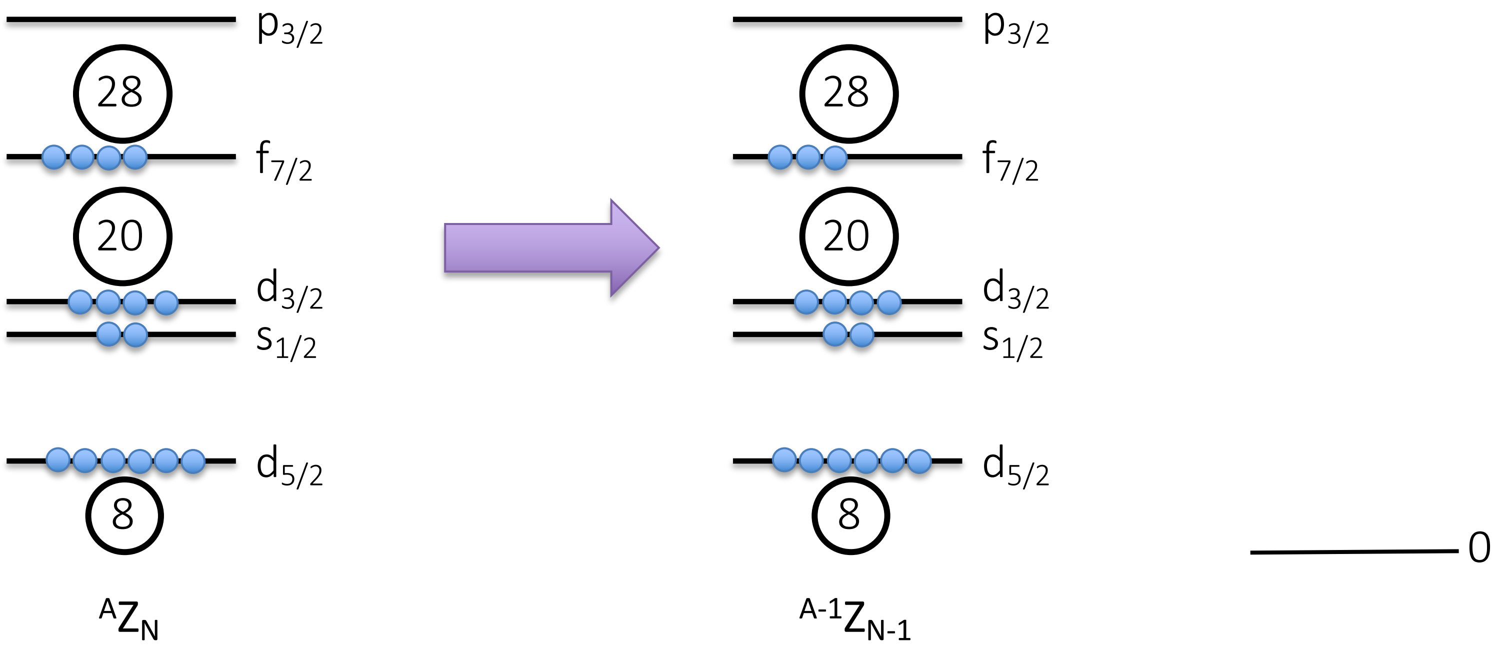

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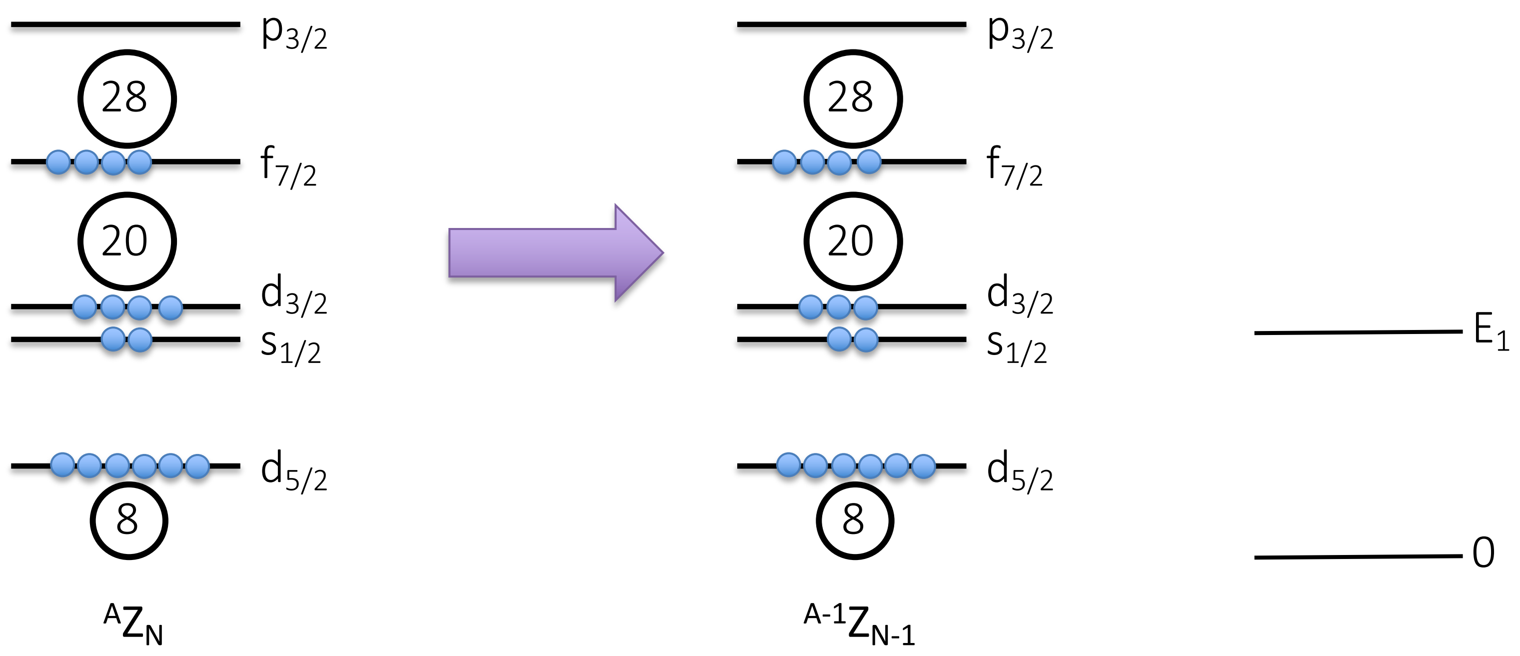

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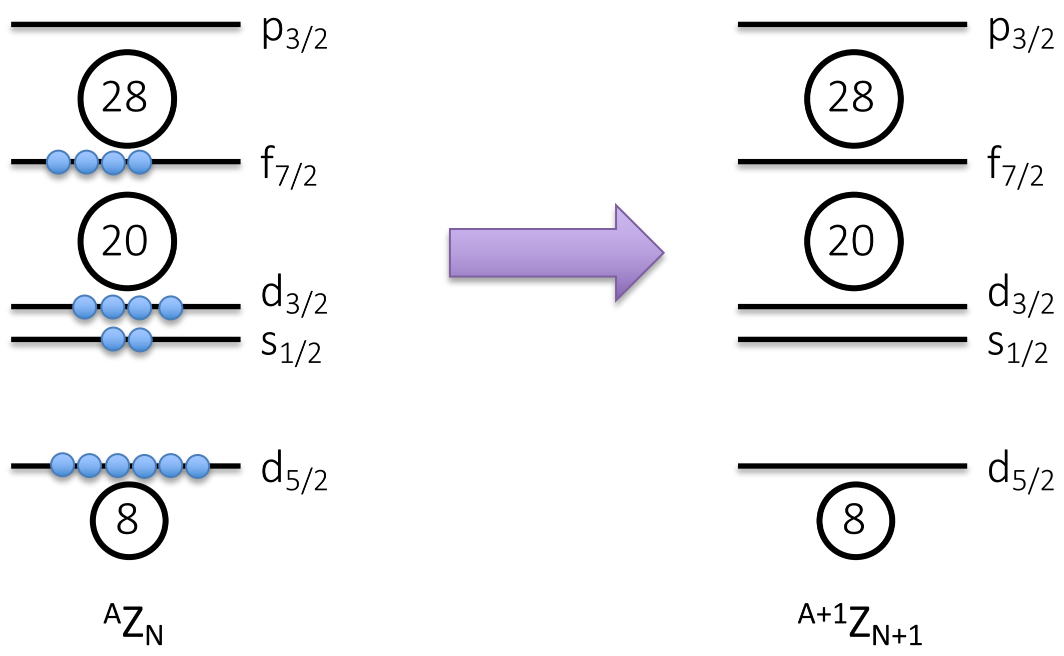
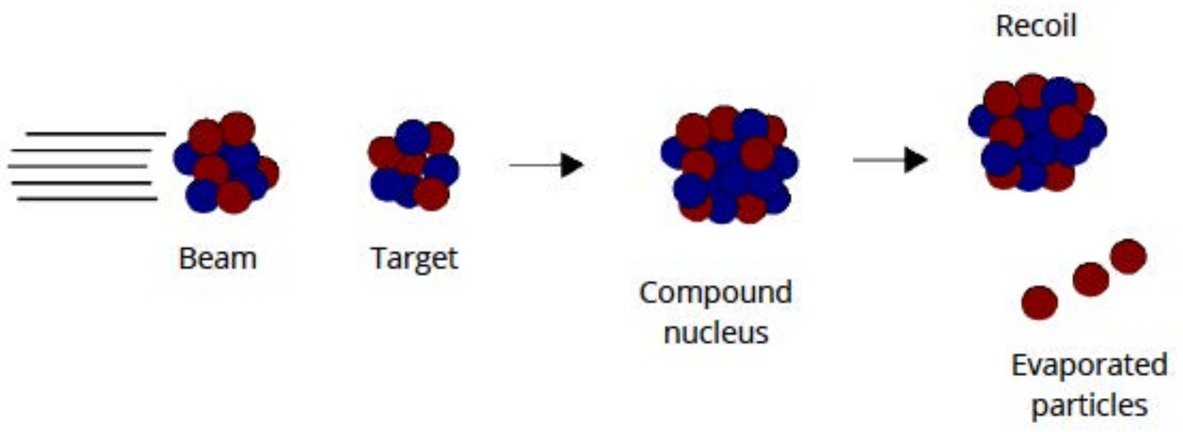


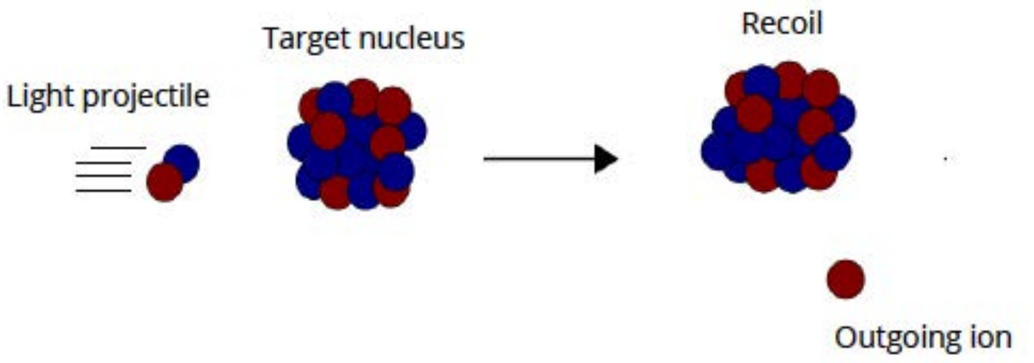
Selectivity of the reaction mechanism

- Knockout / nucleon removal
- Fusion – evaporation
- Transfer
- Deep inelastic
- Scattering (elastic / inelastic)
- Capture

Fusion evaporation vs. direct transfer

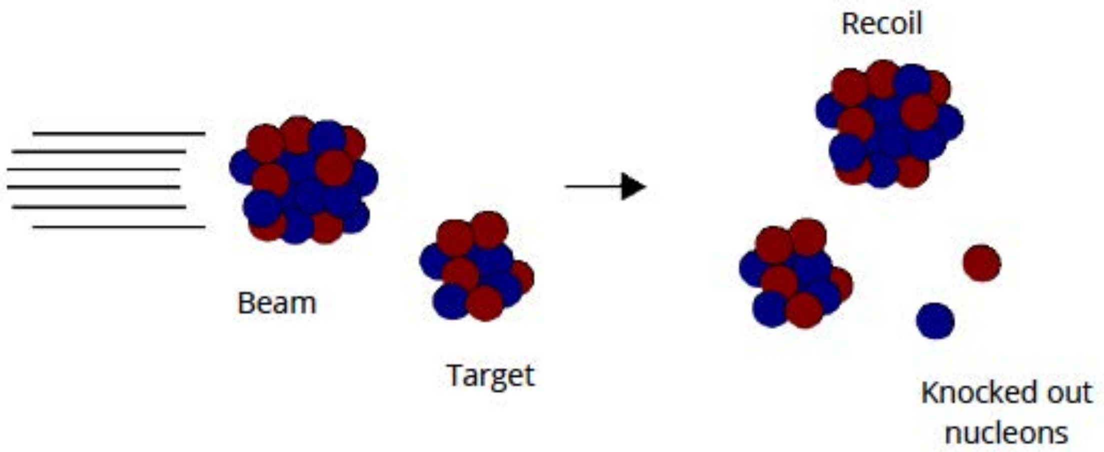


- A + b = C -> D + X
  - $^{12}C(^{18}O,3n)^{27}Si^*$
- Compound system has NO memory of its formation
- Evaporated particle energies give excitation energies of final states

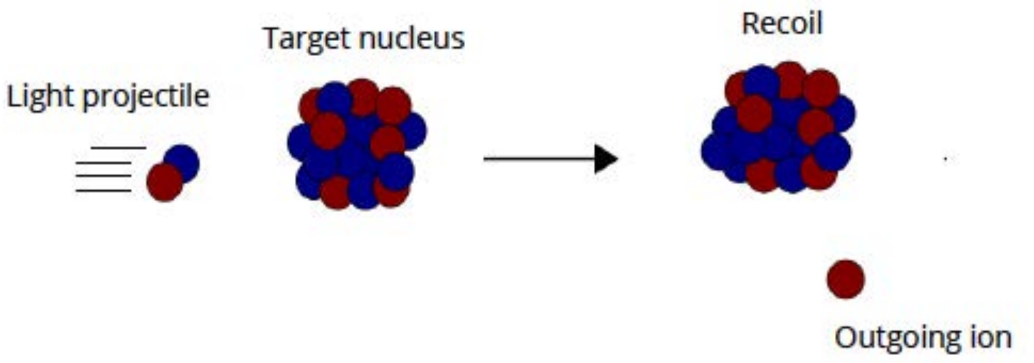


- Two-body A(b,c)D
  - $^{16}O(d,p)^{17}O^*$
- Outgoing particle DO retain knowledge of transferred particles

Knockout reaction vs. direct transfer



- A + b = c – Xn - Xp
  - $^{9}Be(^{44}S,-1p1n)^{42}P^{*}$
- Momentum distribution of recoil reflects orbital momentum transfer



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Transfer reactions

Single-nucleon

[e.g., (d,p), (^{3}He,d), (α,t)]

- Single-particle states

Two-nucleon

[e.g., (t,p), (^{3}He,p), (α,d)]

- Pair transfer (2n, d, etc.)

Charge exchange

[e.g., (p,n), (^{3}He,t), (t,^{3}He)]

- Gamow Teller Strengths
- Isobaric analog states

Surrogate reactions

[e.g., (^{6}Li,d), (^{7}Li,t), (d,n)]

- Mimics the analogous particle transfer

Heavy Ion

[e.g., (^{13}C,^{12}C), (^{12}C,^{10}Be), (^{14}C,^{10}C)]

- Highly selective
- Exploratory

Transfer reactions: measured quantities

- Momenta and angles of outgoing light particles [or heavy-ion recoils]

Reaction: A(b,c)D

[e.g., $^{208}Pb(^{3}He,d)^{209}Bi$]

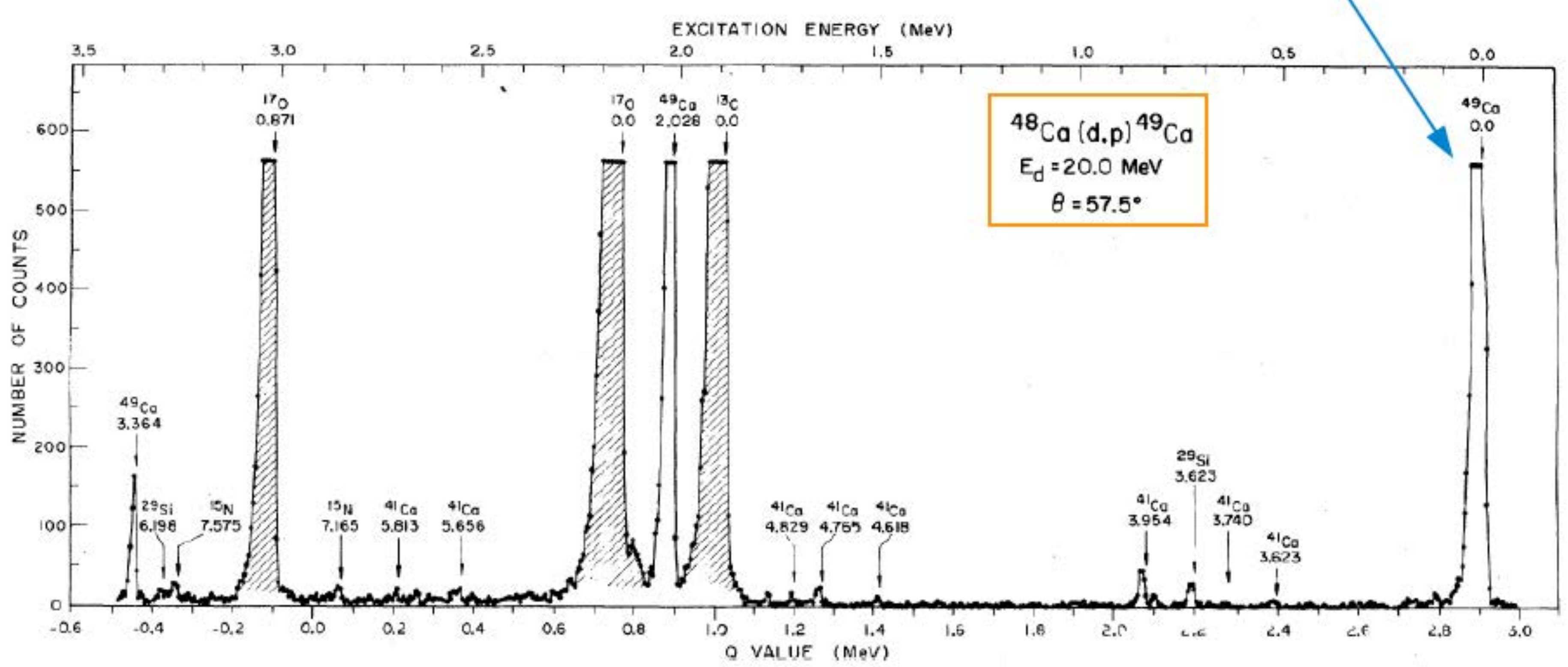
$$\mathrm{BE_D} = \mathrm{M_D} + \mathrm{E^*_D} = \sqrt{\mathrm{M}_c^2 + \mathrm{E}_{cm}^2 - 2 \cdot \mathrm{E}_{cm} \cdot \mathrm{E}_c'}$$

$$\mathrm{E}_c' = \mathrm{f}(\mathrm{E}_c, \theta_c)$$

$$\mathrm{Q} = (\mathrm{BE}_c + \mathrm{BE_D}) - (\mathrm{BE_A} + \mathrm{BE}_b)$$

Transfer reactions: measured quantities

[Q(g.s) = +2.92 MeV]

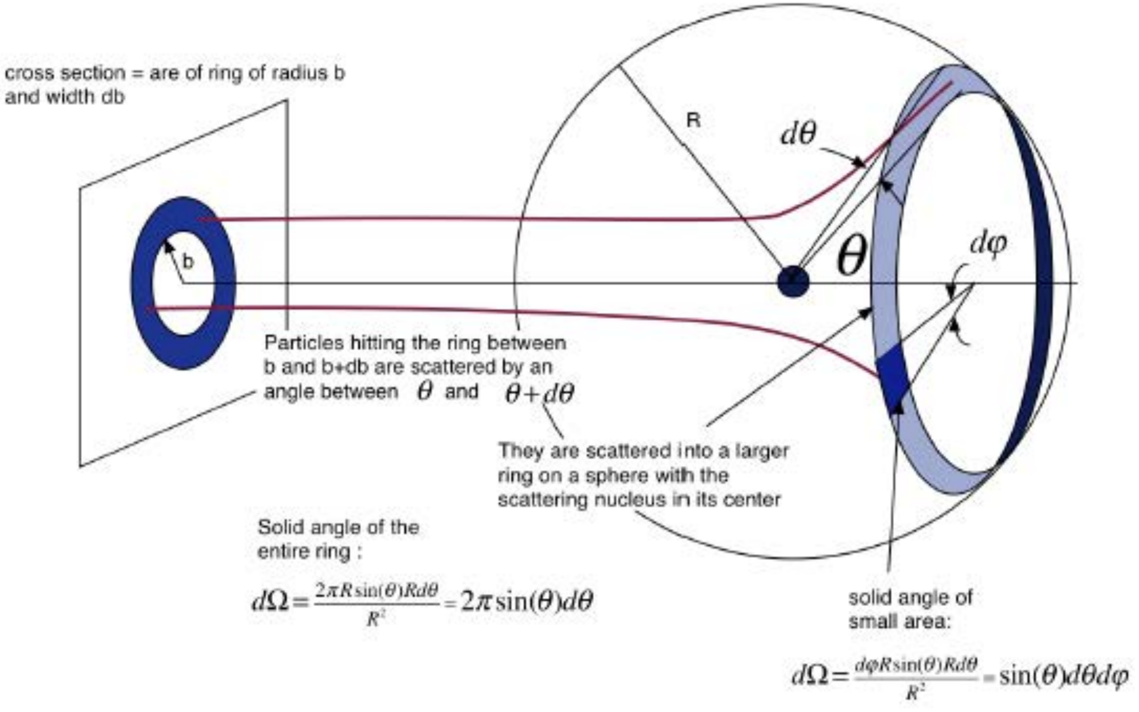


$$Q = (BE_c + BE_D) - (BE_A + BE_b)$$

Transfer reactions: measured quantities

Cross sections – Yields as a function of angle

[differential cross section: millibarns per ster radians (mb/sr)]



Rutherford Scattering

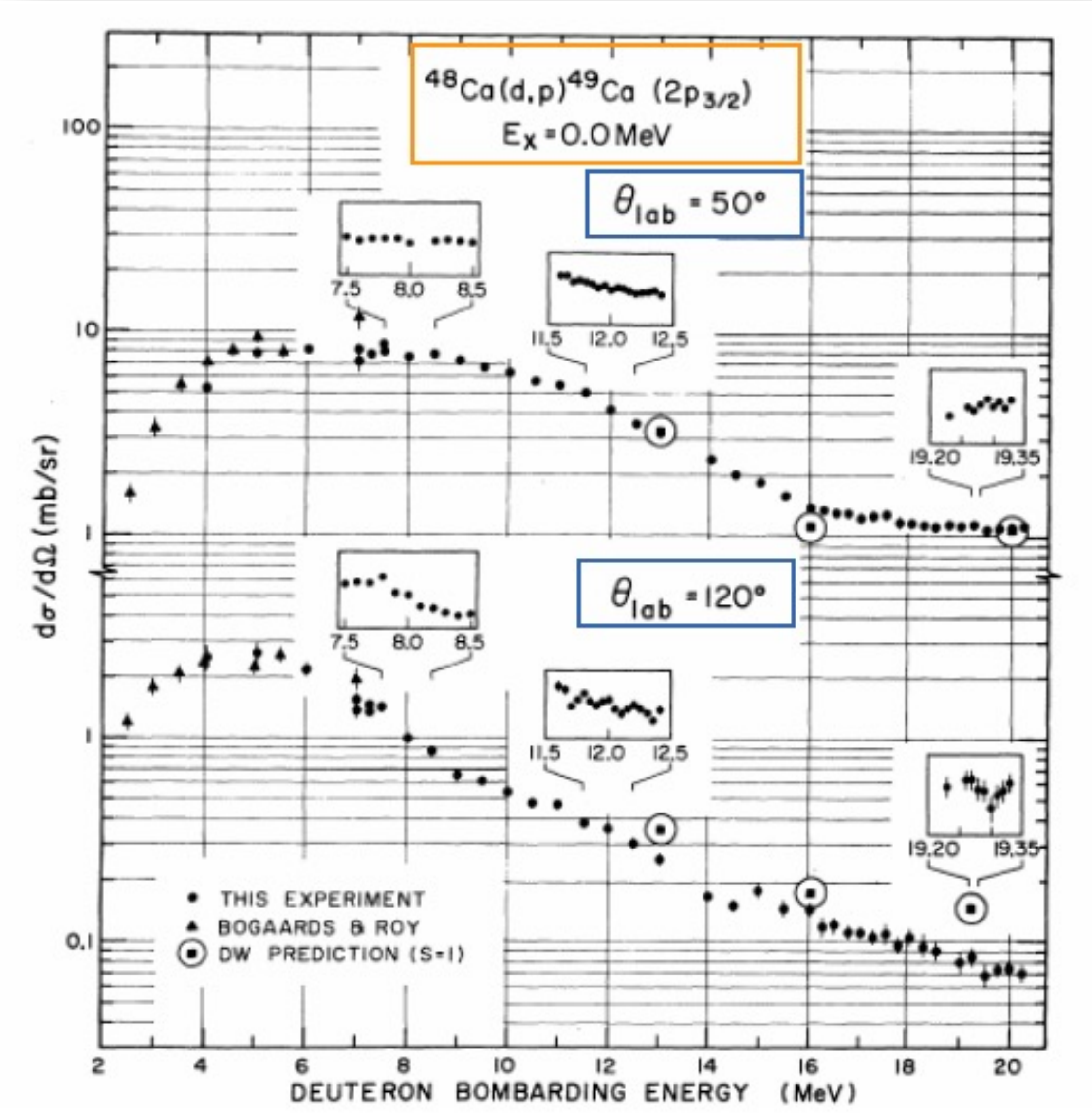
[V = Coulomb]

$$\frac{d\sigma}{d\Omega} = \frac{(zZe^2)^2}{(4\pi\varepsilon_0)^2 (4E_{kin})^2} \frac{1}{\sin^4\left(\theta/2\right)}$$

Transfer Reaction

[V = Nuclear + Coulomb]

Cross section vs. incident beam energy



Transfer reactions: extracted quantities

Sensitivity of the differential cross sections to orbital angular momenta (*l*) of transferred nucleon(s)

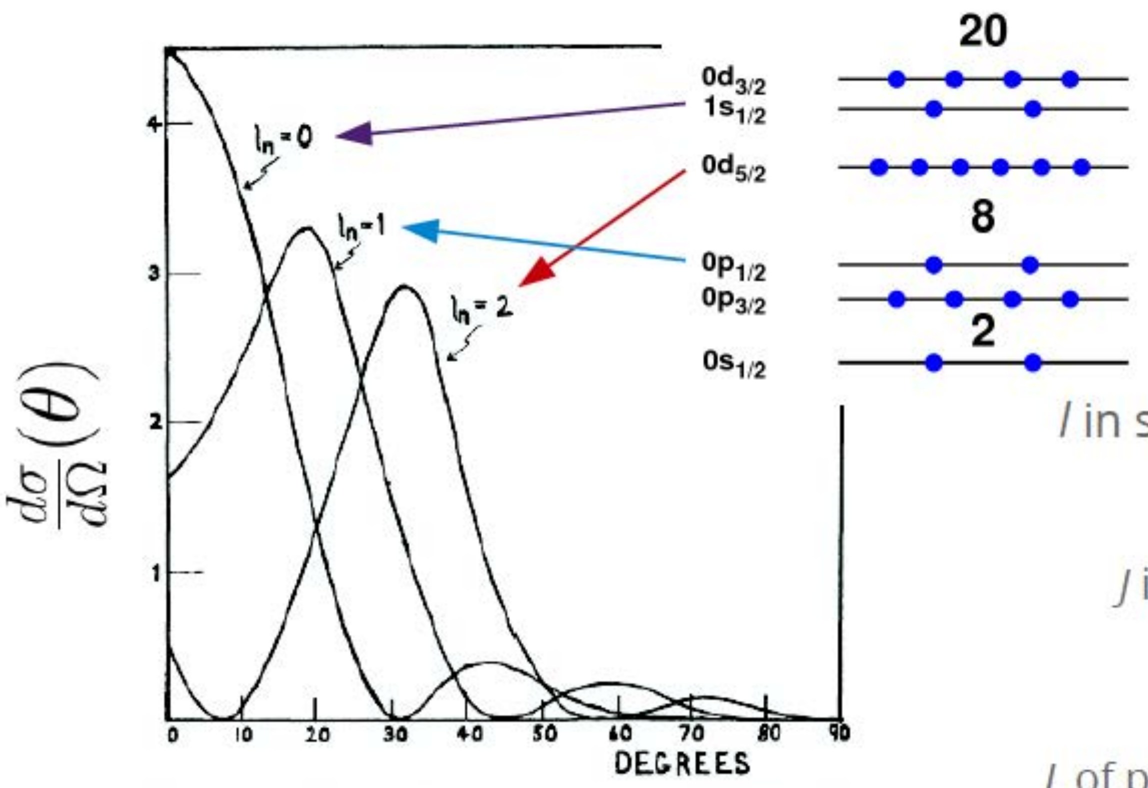


FIG. 1. Theoretical angular distributions for (d, p) and (d, n) reactions for different angular momentum transfers to the initial nucleus.

l in single-particle transfer
[e.g., (d,p)]

J if incoming particle is polarized
[analyzing power]

L of pair in two-particle transfer
[e.g., (t,p)]

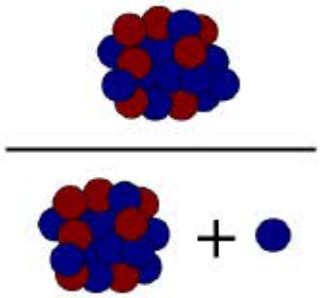
Transfer reaction: extracted quantities

Experimental spectroscopic factor

[Relative values are typically reliable (<25%)]

[absolute values can be tricky (>30%)!]

S_{lj} =

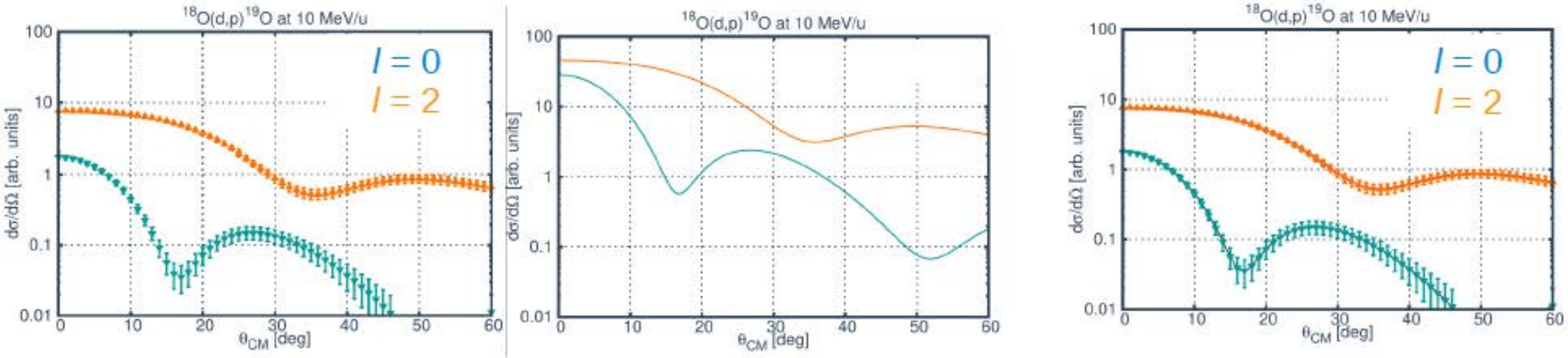


Final state

Initial state + nucleon

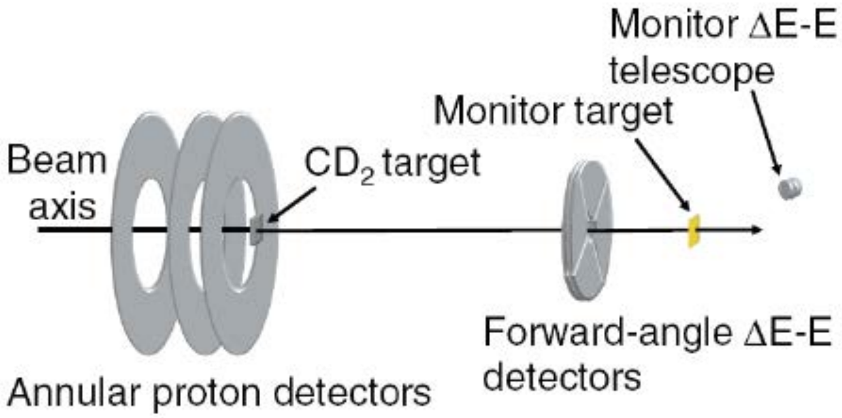
$$\left.\frac{d\sigma}{d\Omega}\right|_{Measured} = g S_{lj} \left.\frac{d\sigma}{d\Omega}\right|_{DWBA}$$

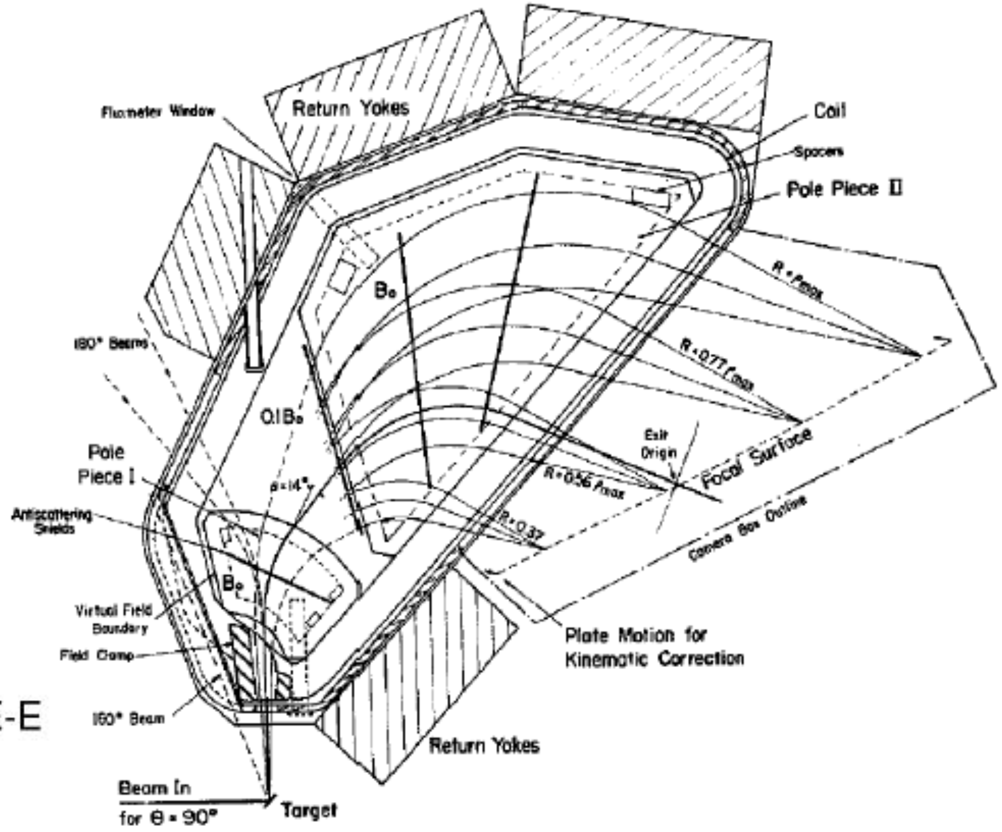
- Statistical factor → g
- Amount of overlap between initial and final states — Spectroscopic Factor → S_{lj}
- Calculated cross section for "pure" single-particle like state → $\left.\frac{d\sigma}{d\Omega}\right|_{DWBA}$



Low-energy transfer experiments

Detection systems depend on kinematics of the reaction
--> 'normal kinematics' with a light beam on a heavy target – spectrographs can analyze the light outgoing particle
--> 'inverse kinematics' with a heavy beam on a light target – detect the light outgoing particle, or analyze the beam-like particle





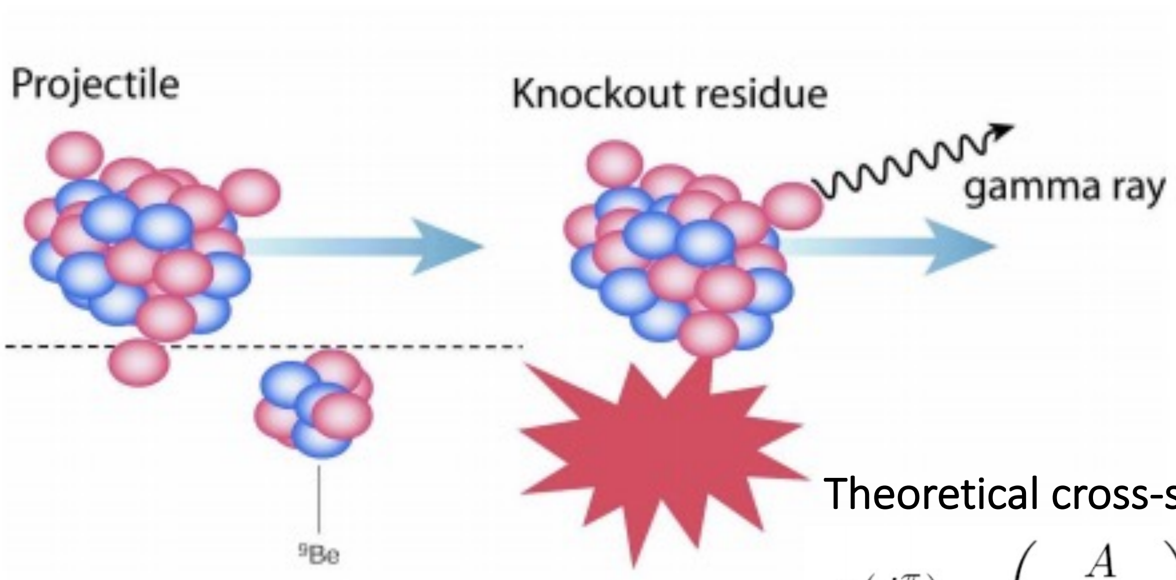
Nucleon knockout reactions

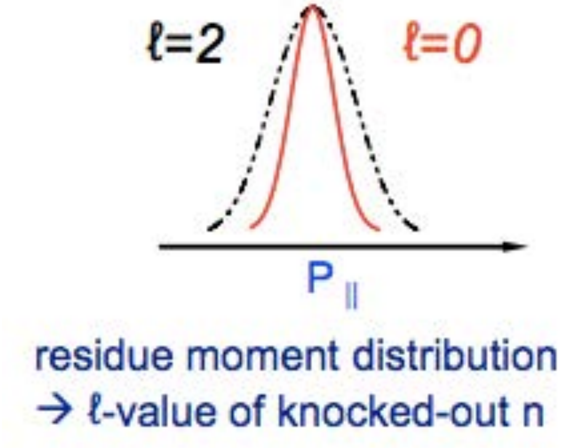
Intermediate energy beams (> 50 MeV/nucleon)

- Sudden approximation + eikonal approach for reaction theory

Spectroscopic strengths --> exclusive cross-sections

- Populated states in A-1 residue provide detailed measure of beam structure





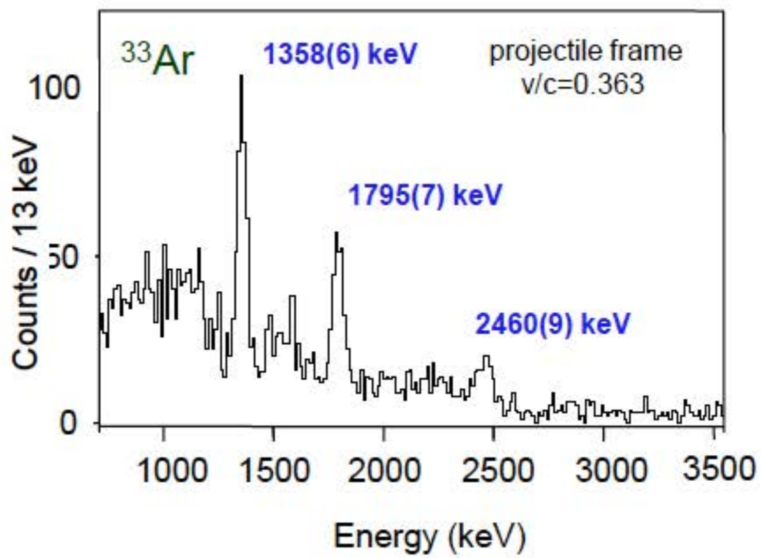
Theoretical cross-section

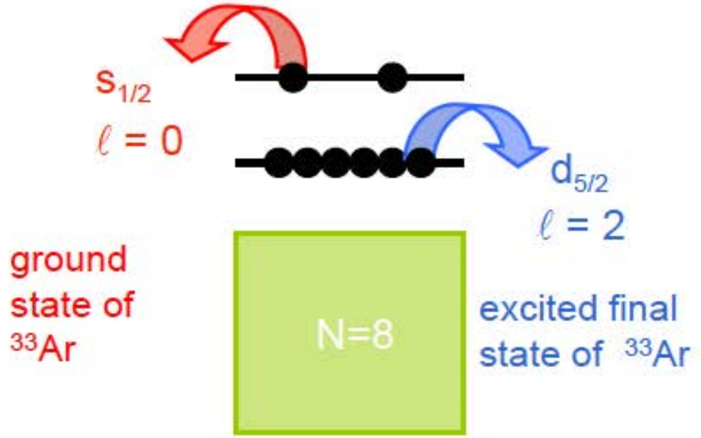
Reaction theory

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2 S(j^\pi)\sigma_{sp}(j, S_N + E_x[j^\pi])$$

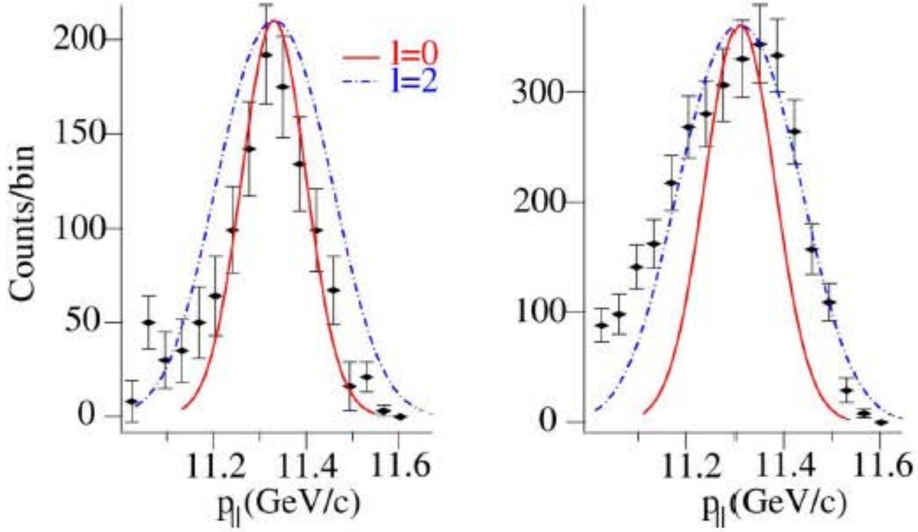
Structure theory

Neutron knockout – $^{9}Be(^{34}Ar, ^{33}Ar)X$





	BR (%)	σ_{exp} (mb)	C^2S_{exp}
$1/2^+$	30.2(46)	4.7(9)	0.38(6)
$3/2^+$	20.2(44)	3.2(8)	0.36(9)
$5/2^+$	31.7(31)	4.9(7)	0.56(8)
$(5/2^+)$	17.9(30)	2.8(6)	>0.34(7)



A. Gade et al., PRC 69, 034311 (2004).

Excited state lifetimes

Lifetimes and transition probabilities

Transition probability for gamma-decay relates strongly to specific nuclear matrix elements --> provide a stringent test of theoretical wavefunctions

Consider the case of the first 2+ states in even-even nuclei

$$\tau_\gamma = 40.81 \times 10^{13} E^{-5} [B(E2)\uparrow/\mathrm{e}^2\mathrm{b}^2]^{-1}$$

$$B(E2 : J_i \to J_f) = \frac{1}{2J_i + 1} \langle \psi_f || E2 || \psi_i \rangle^2$$

Lifetimes are of order ps --> how do we measure these lifetimes?

Recoil-distance (plunger) method

The lifetime of excited states in the range of 10-100s of ps can be measured by populating the state via Coulomb excitation or knock-out reactions, and observing the Doppler-shift of the decay gamma-ray.

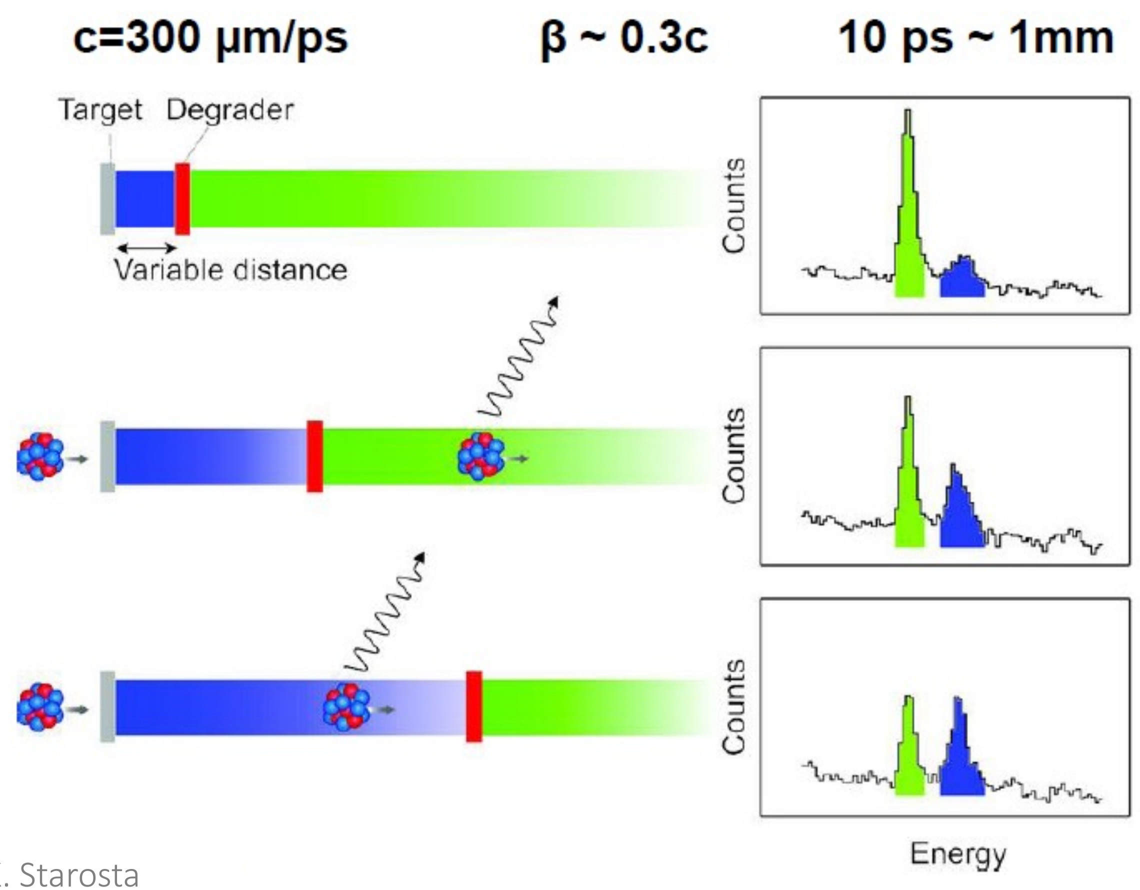
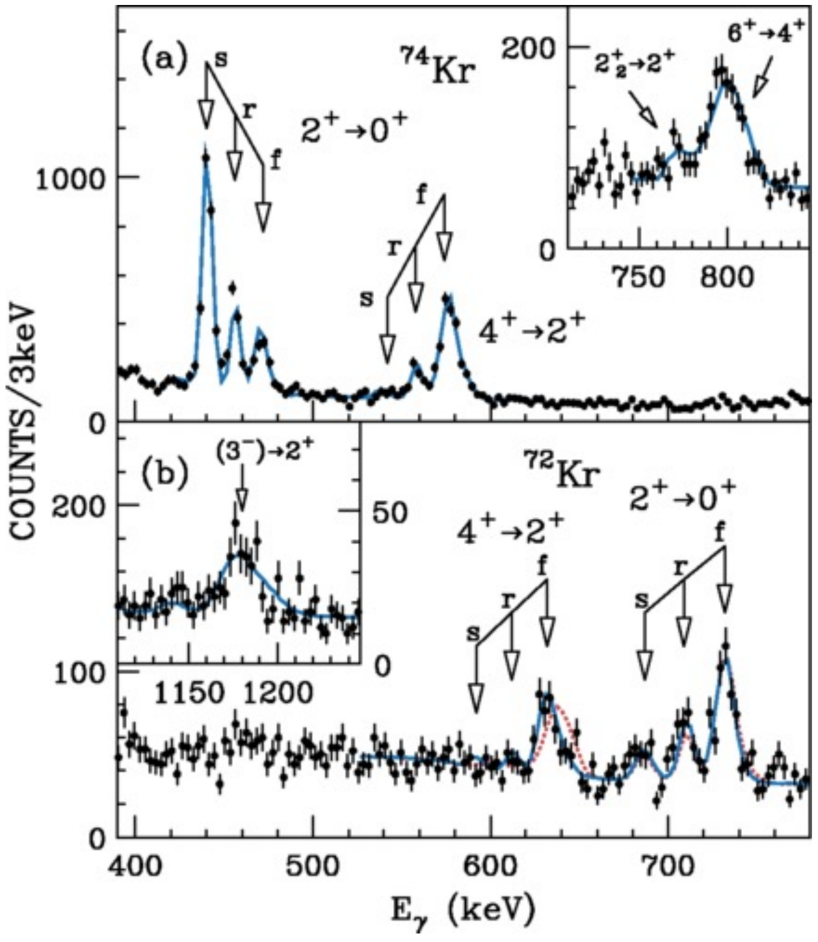
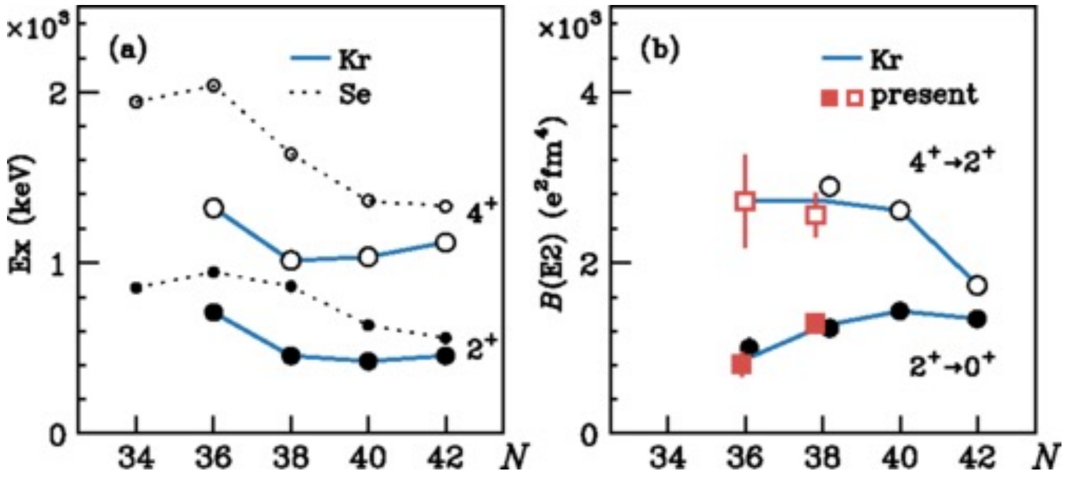


Figure: Adapted from K. Starosta

Lifetime in ^{72,74}Kr





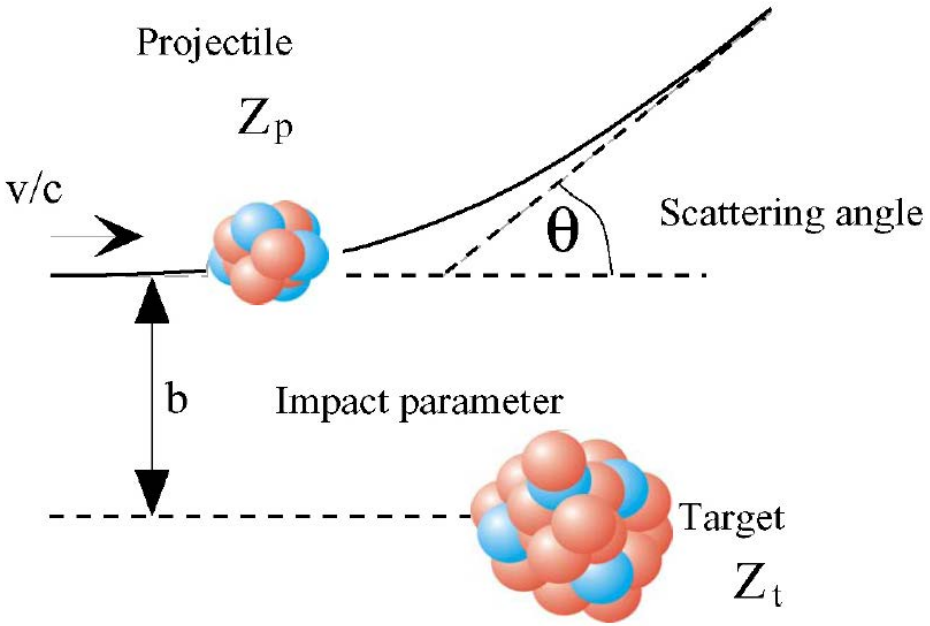
Lifetimes are related to the reduced transition probabilities B(E2), which are an indicator for collectivity in the nuclear structure.

Here, the irregular behaviour for the 4+ and 2+ states suggest a rapid shape evolution in ^{72}Kr

H. Iwasaki *et al*., Phys. Rev. Lett. **112**, 142502 (2014).

Coulomb excitation

Collectivity: B(E2) from excitation probability

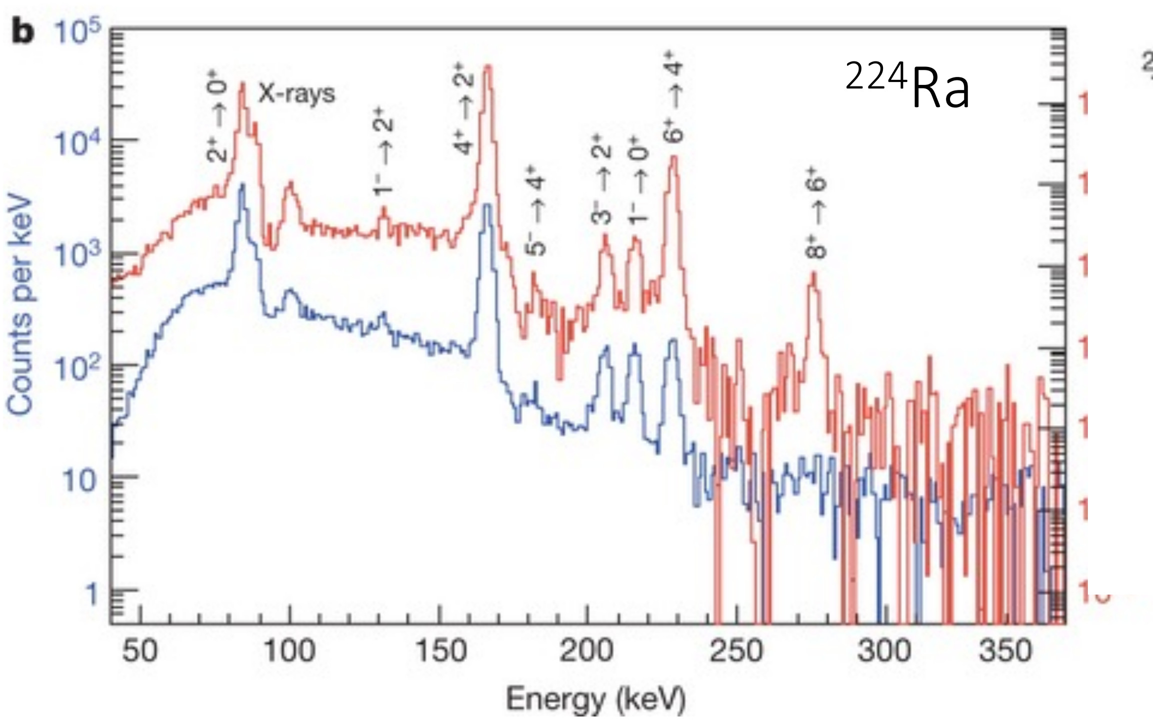


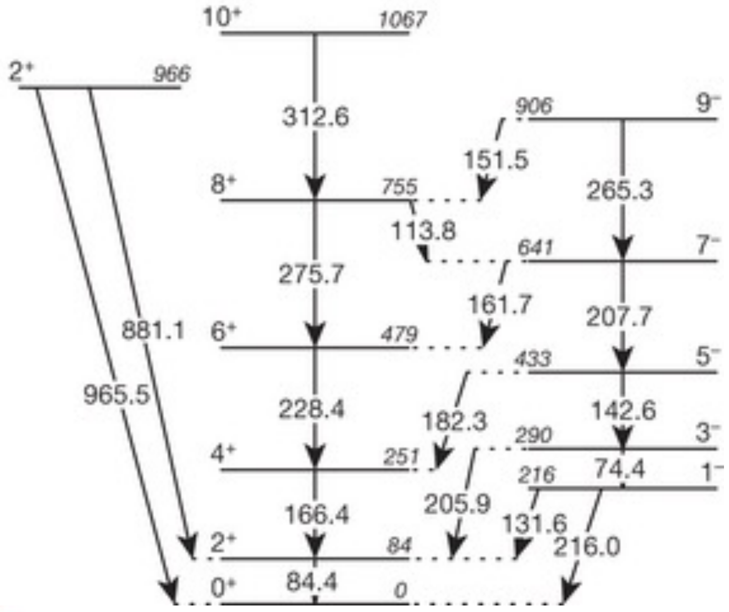
Coulomb excitation:

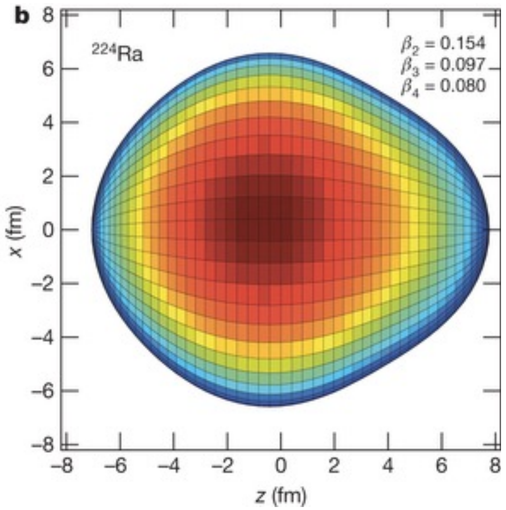
- purely Coulomb interaction causes excitation of the nucleus of interest
- well described interaction, and cross-section relates to transition matrix element, i.e. B(E2) for 0+ --> 2+ in even-even nuclei.

$$\sigma_{\pi\lambda} \approx \left(\frac{Z_{\text{pro}}e^2}{\hbar c}\right)^2 \frac{\pi}{e^2 b_{\min}^{2\lambda-2}} B(\pi\lambda, 0 \to \lambda) \begin{cases} 1/(\lambda-1) & \text{for } \lambda \geqslant 2 \\ 2\ln(b_a/b_{\min}) & \text{for } \lambda = 1 \end{cases}$$

Pear shaped nuclei and atomic EDM





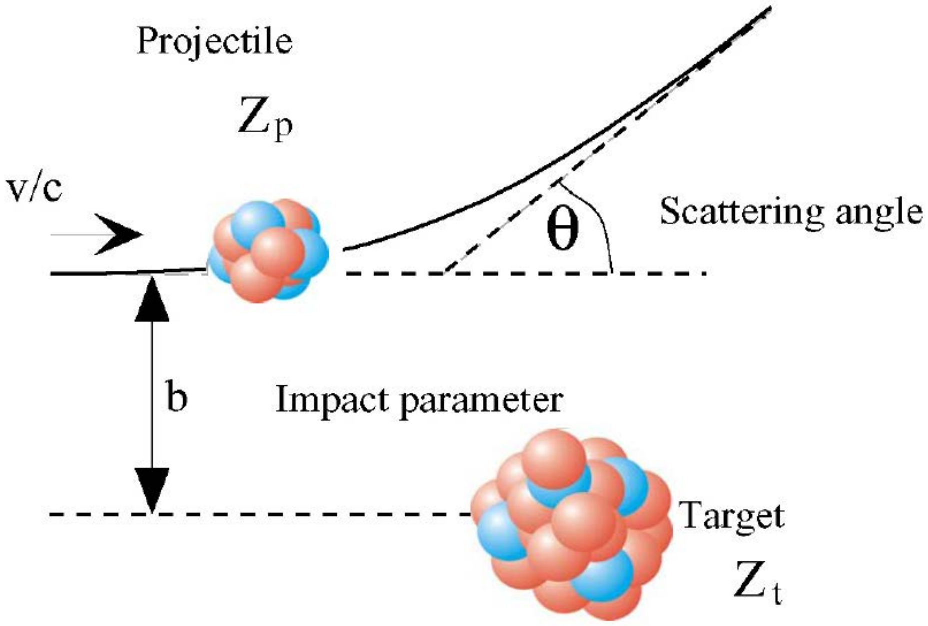


$$\langle I' || \mathrm{E}\lambda || I \rangle = \sqrt{(2I'+1)(2\lambda+1)/16\pi}\,(I'0\lambda0|I0)Q_\lambda$$

L. P. Gaffney *et al.*, Nature **497**, 199 (2013).

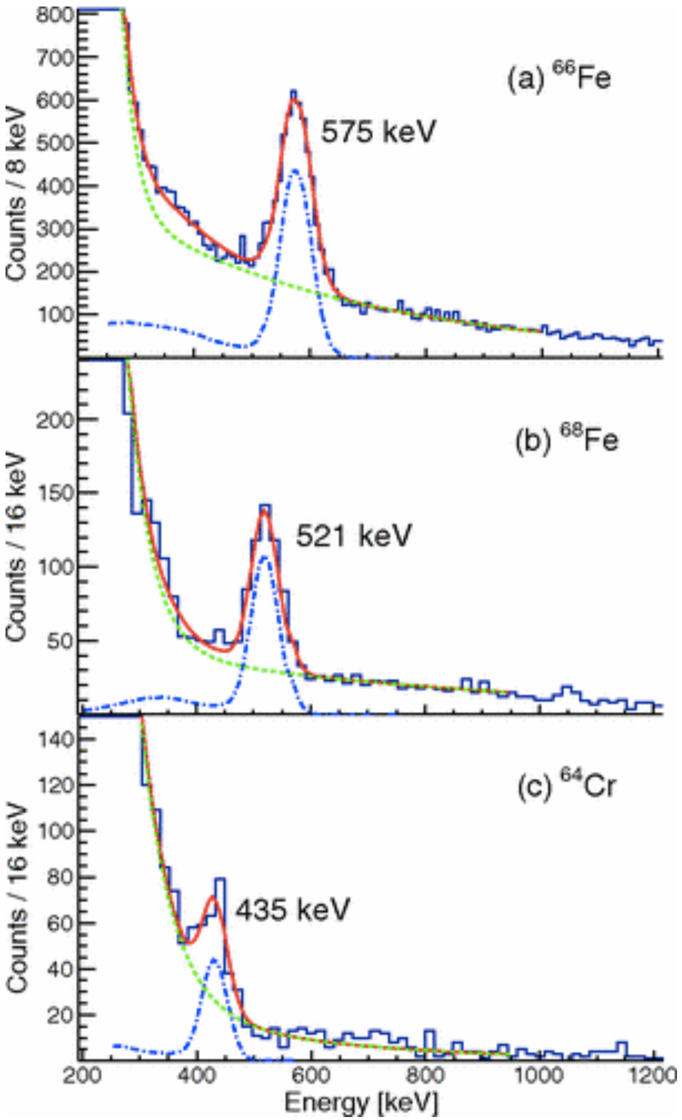
Intermediate-energy Coulex

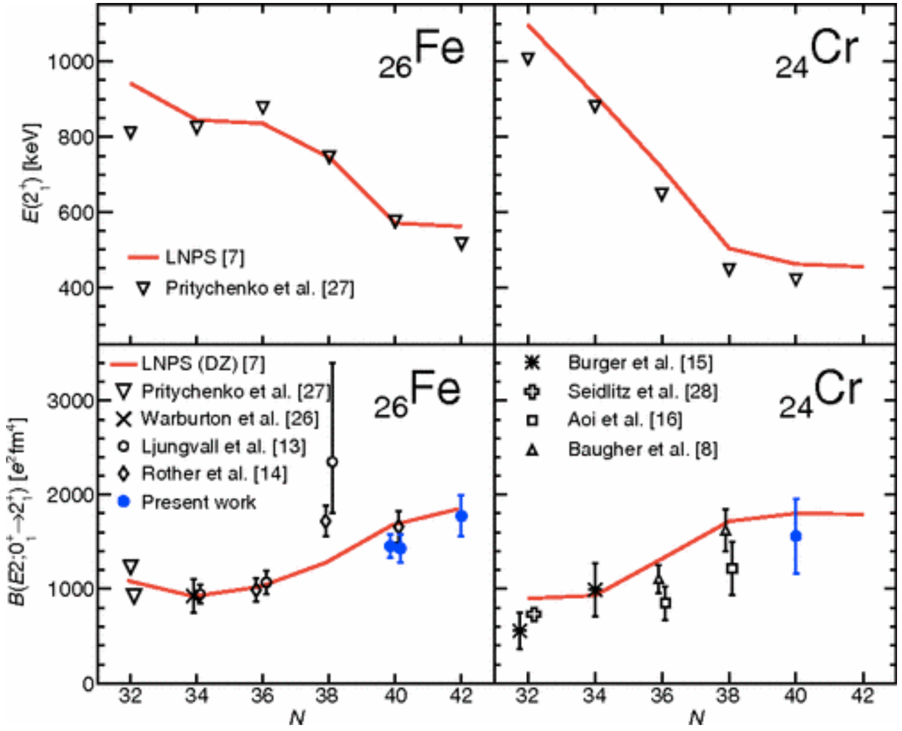
- In conventional (low-energy) Coulomb excitation, bombarding energies are well below the Coulomb barrier
- At high energies (~100 MeV/A), nuclear contribution can be significant for small impact parameters, but for $b > R_{int}$ Coulomb dominates



- At a given beam velocity, b relates to the scattering angle θ, so restricting analysis to forward scattering angles ensures ‘safe’ Coulex

Neutron-rich Fe and Cr





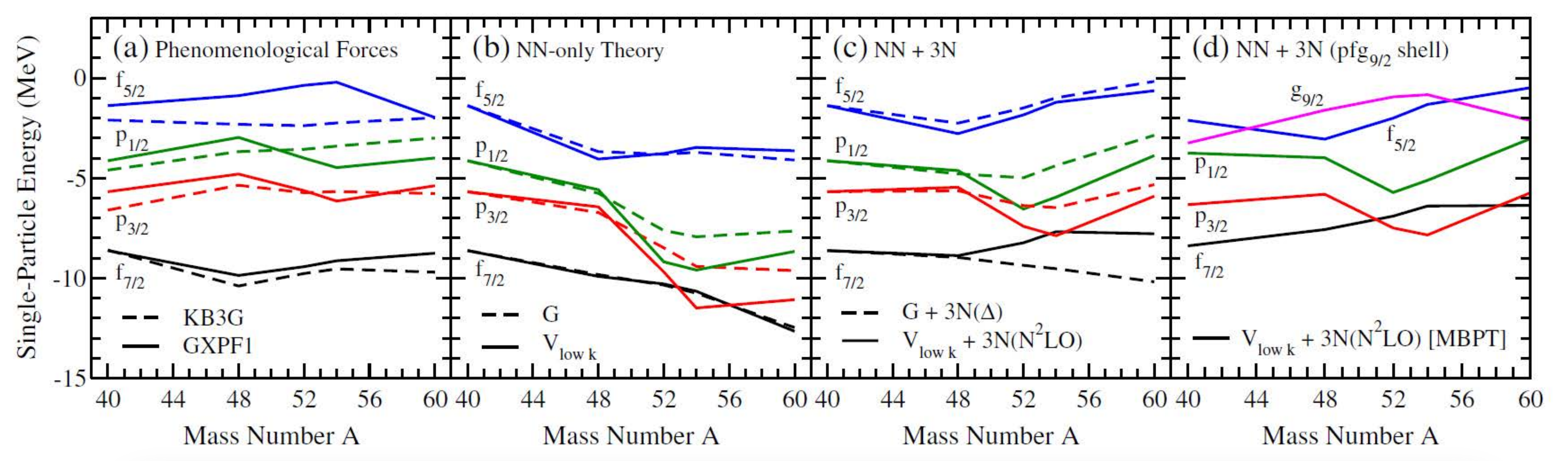
HLC *et al.*, Phys. Rev. Lett. **110**, 242701 (2013).

And what have I skipped?

- ‘Exotic’ decay modes
  - 1p and 2p decay at the proton dripline
  - Neutron decay --> recent sequential 2n decay at NSCL
- Resonance spectroscopy – properties of unbound states (beyond the proton and neutron driplines)
- Reactions for spectroscopy and more --> deep inelastic reactions, multi-nucleon transfer, charge-exchange, etc.
- And much, much more…

Example: Designing an experiment to access the physics

We read this theory paper…



	$^{50}Ca_{gs} \rightarrow {}^{49}Ca$ SF $\frac{1}{2J_1+1}$											
	$\frac{3}{2}^-_{gs}$	$\frac{3}{2}^-_1$	$\frac{7}{2}^-_1$	$\frac{7}{2}^-_2$	$\frac{7}{2}^-_3$	$\frac{7}{2}^-_4$	$\frac{5}{2}^-_1$	$\frac{5}{2}^-_2$	$\frac{1}{2}^-_1$	$\frac{1}{2}^-_2$	$\frac{9}{2}^+_1$	$\frac{9}{2}^+_2$
GXPF1	1.73	0.03	7.71	0.00	0.00	0.01	0.00	0.06	0.17	0.00	-	-
(SR)	(1.82)		(7.90)				(0.09)		(0.19)		-	
pf NN+3N	1.57	0.23	4.55	2.03	0.02	0.21	0.03	0.10	0.35	0.01	-	-
(SR)	(1.95)		(7.31)				(0.30)		(0.44)		-	
pfg$_{9/2}$ NN+3N	1.65	0.09	4.54	1.18	0.00	0.03	0.10	0.01	0.20	0.00	1.26	0.05
(SR)	(1.81)		(6.09)				(0.20)		(0.24)		(1.66)	

J.D. Holt et al., J. Phys. G: Nucl. Part. Phys. **39**, 085111 (2012).
J.D. Holt, J. Menendez, A. Schwenk, private communication.

Can we inform this physics question?

- Theory tells us there is a difference in spectroscopic factor for removal of neutrons in ^{50}Ca to states in ^{49}Ca
  - Is this observable? Can we design a measurement to test the different predictions? What could we do? What would our experiment observables be?
  - Where could we do this type of experiment? What facility could we use? What type of equipment?
  - What exactly would we **measure**? How would we have to interpret the data? Do we need theory to interpret the data?