Lecture 1 on Hot QCD Matter: Lattice QCD at finite T

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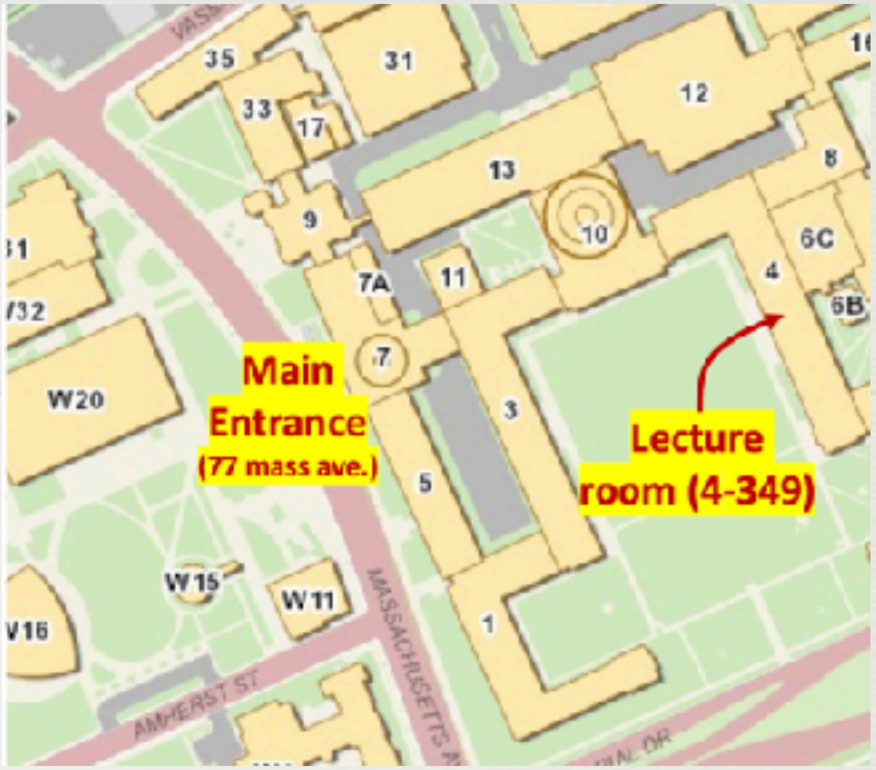
National Nuclear Physics Summer School

MIT 2022

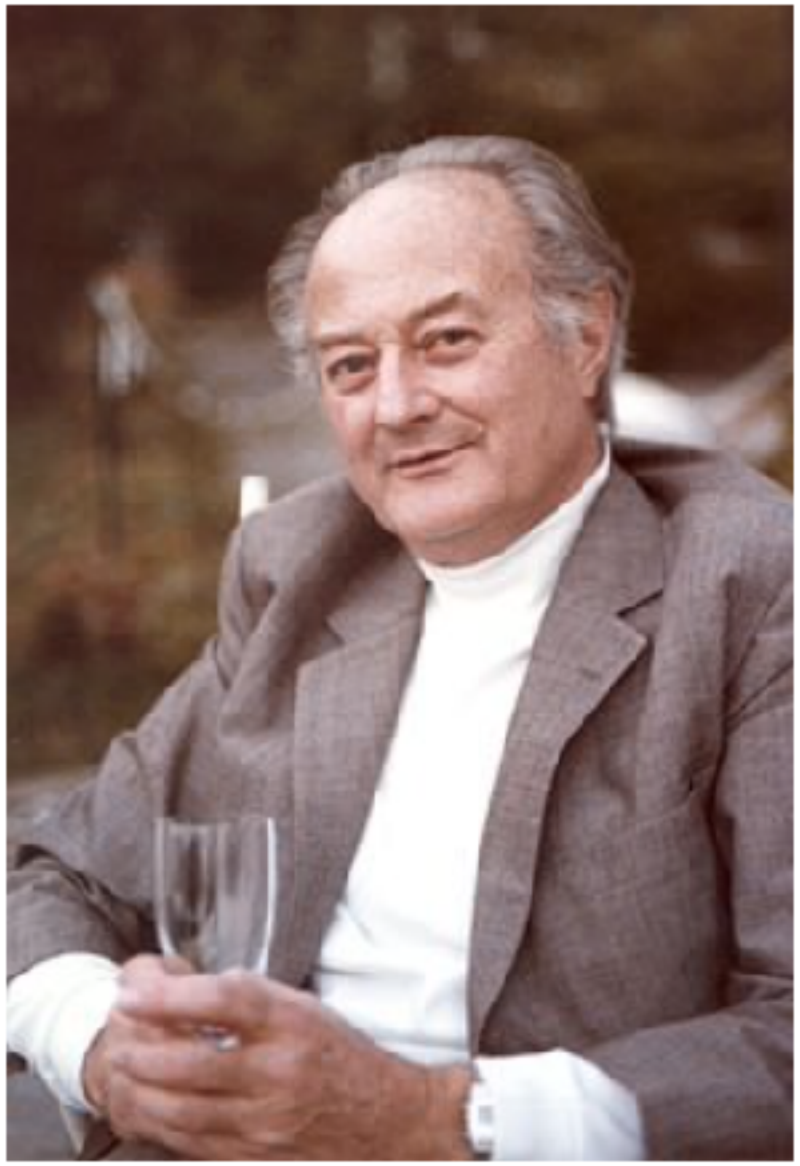
Road Map



- Lecture 1: lattice QCD at finite T, hadron resonance gas, deconfinement
- Lecture 2: Quark Gluon Plasma, heavy-ion collisions, experiment, phenomenology, color glass condensate, relativistic viscous fluid dynamics, jets and heavy flavor
- Lecture 3: Quark Gluon Plasma at large densities, QCD critical point, connections to neutron star mergers

Maximum temperature of matter?

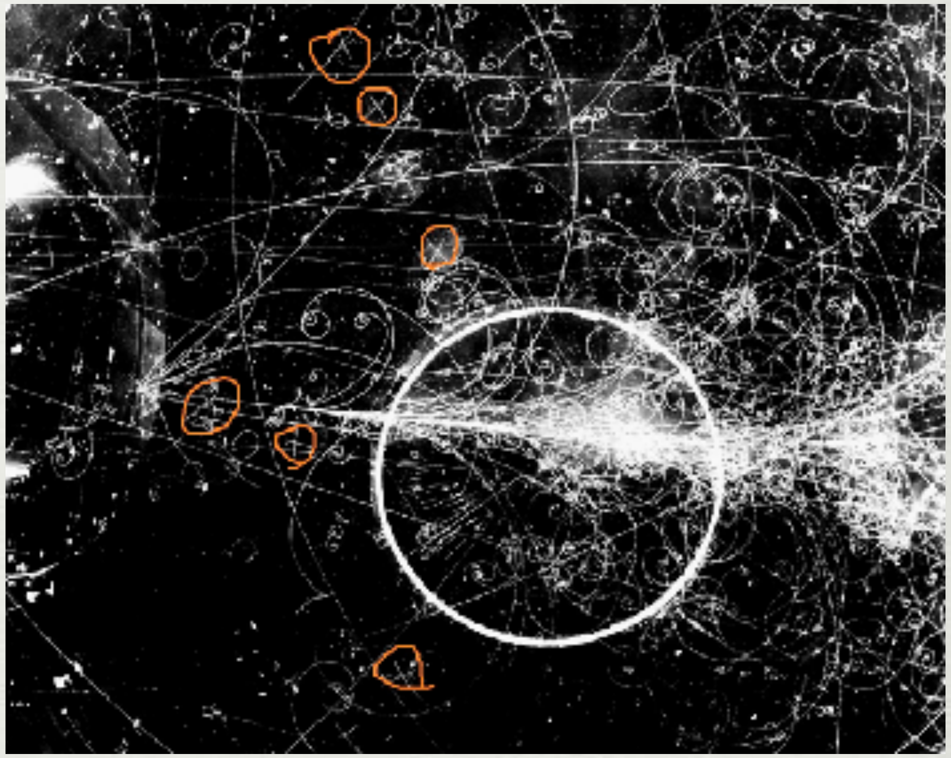
- In the 1960's Hagedorn suggested that matter has a maximum temperature, now known as the Hagedorn Temperature
- Instead of ⇑ temperature, heavier particles are created
- Hagedorn States: "fireballs consist of fireballs, which consist of fireballs..."



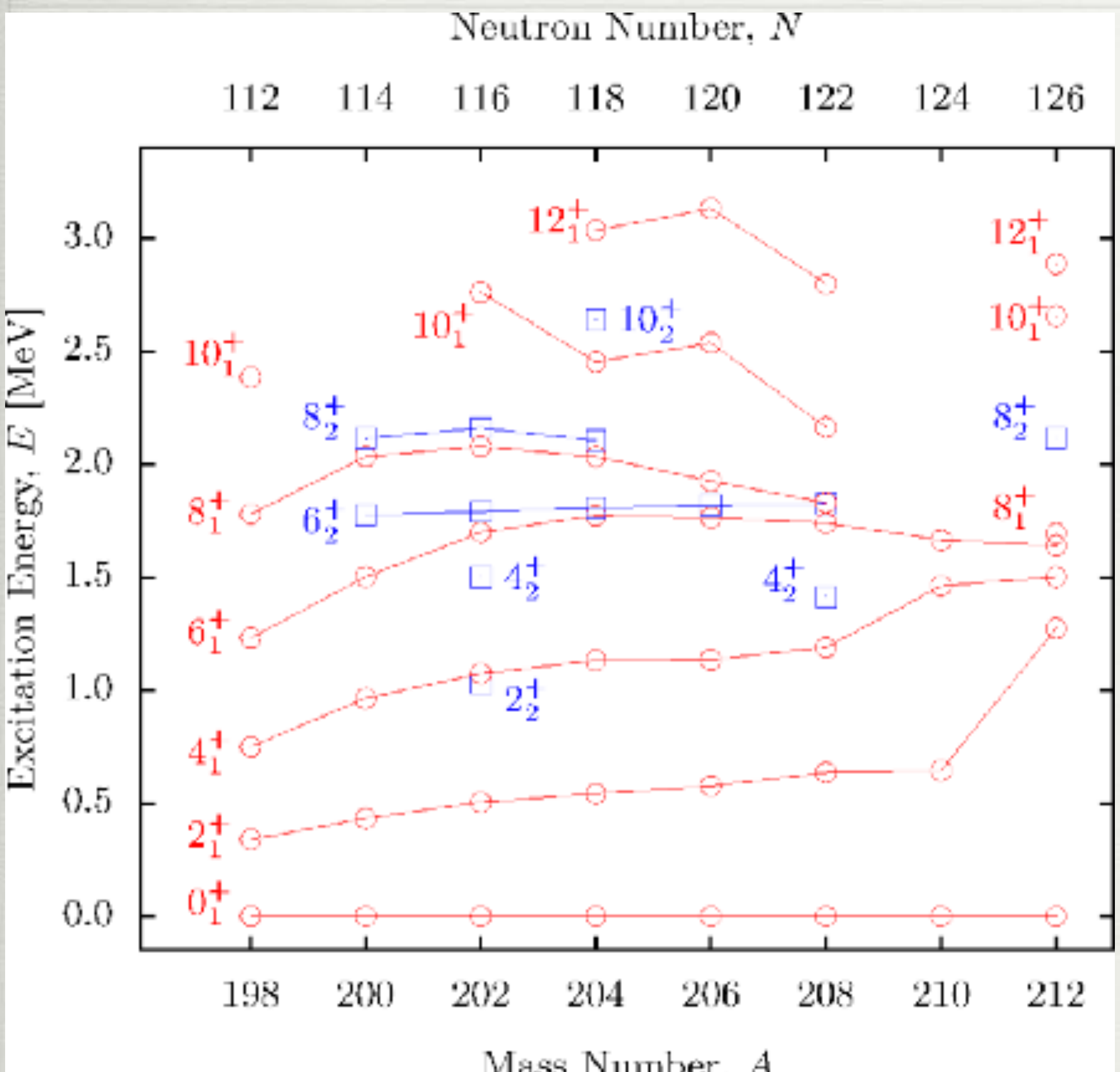
Rolf Hagedorn

Hadrons: “Stable” vs. resonances

- Light mesons: π (stable), ρ, …
- Light baryons: p (super stable), n (stable), N, Δ, …
- Strange mesons: K (stable), …
- $S = \pm 1$ baryons: Λ, Σ (stable), …
- $S = \pm 2$ baryons: Ξ (stable), …
- $S = \pm 3$ baryons: Ω (stable), …



Nuclear excited states vs. resonances



$m_\rho \sim 770 MeV$

$m_\pi \sim 140 MeV$

Mass number ~ 200

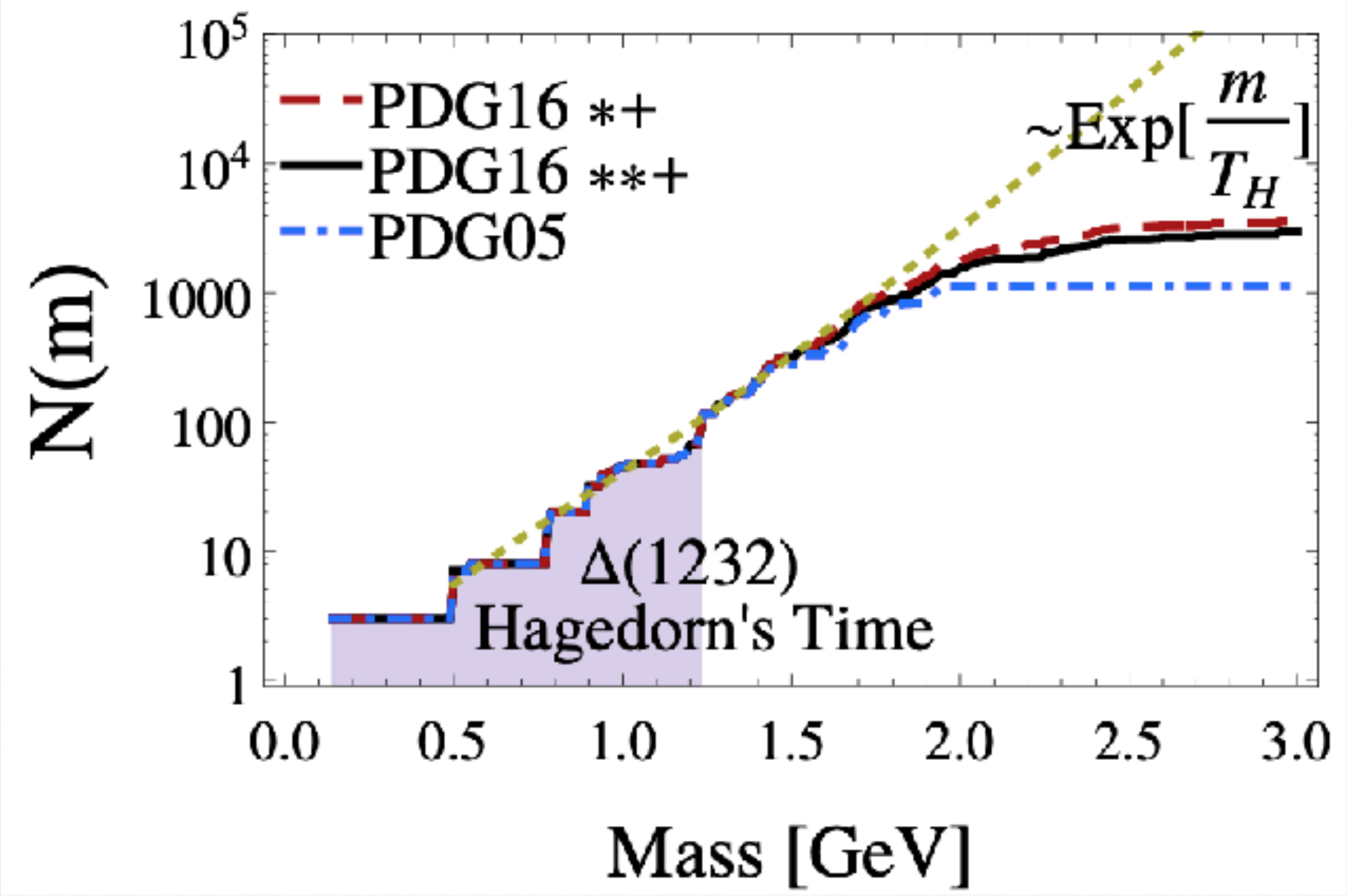
Mass $m_\pi \sim 140$ MeV

Excitation energy $\mathcal{O}(1\ MeV)$

Mass difference $\Delta m \sim 630$ MeV

Limiting temperature: 1960's to today

Count up all known particles, fit to exponential mass spectrum



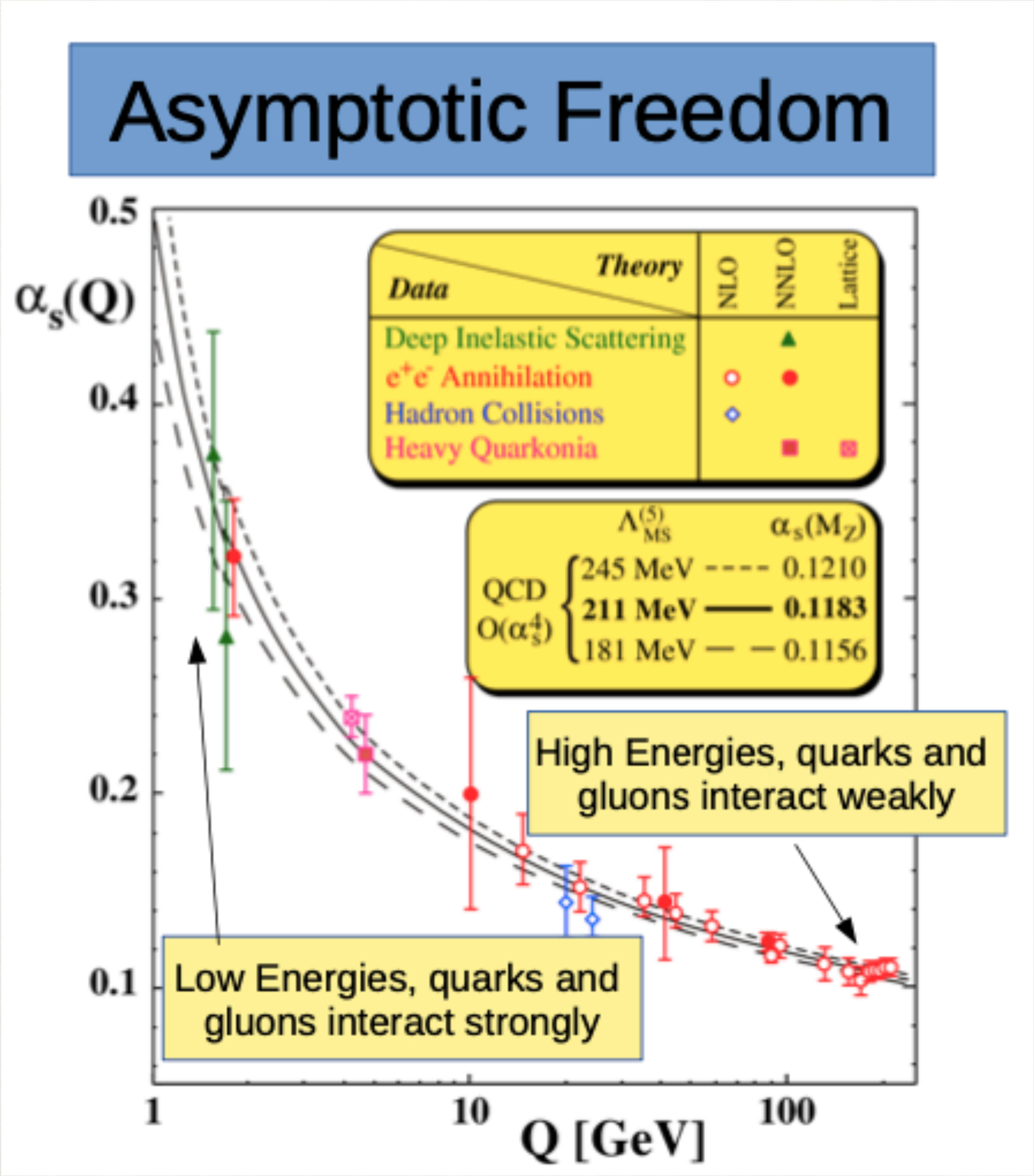
Particle Data Book 2021 gives $T_H \sim 170$ MeV

What happens beyond this temperature?

- Normally, you add energy to the system, the system heats up
- Hagedorn: if there's a limiting temperature, you open new degrees of freedom
- What happens to matter at the highest temperatures we can reach on Earth?

Asymptotic Freedom: hitting QCD matter at high energies

Hadrons

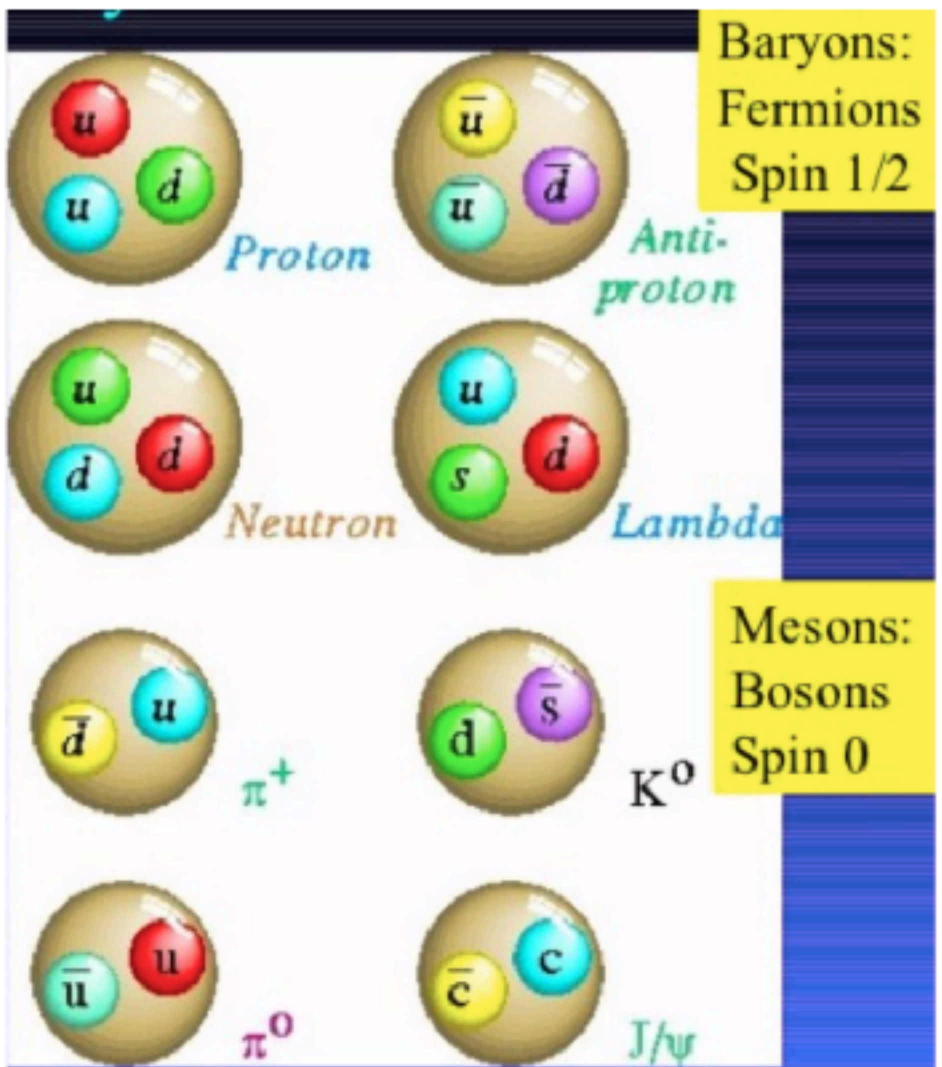


Quarks and gluons

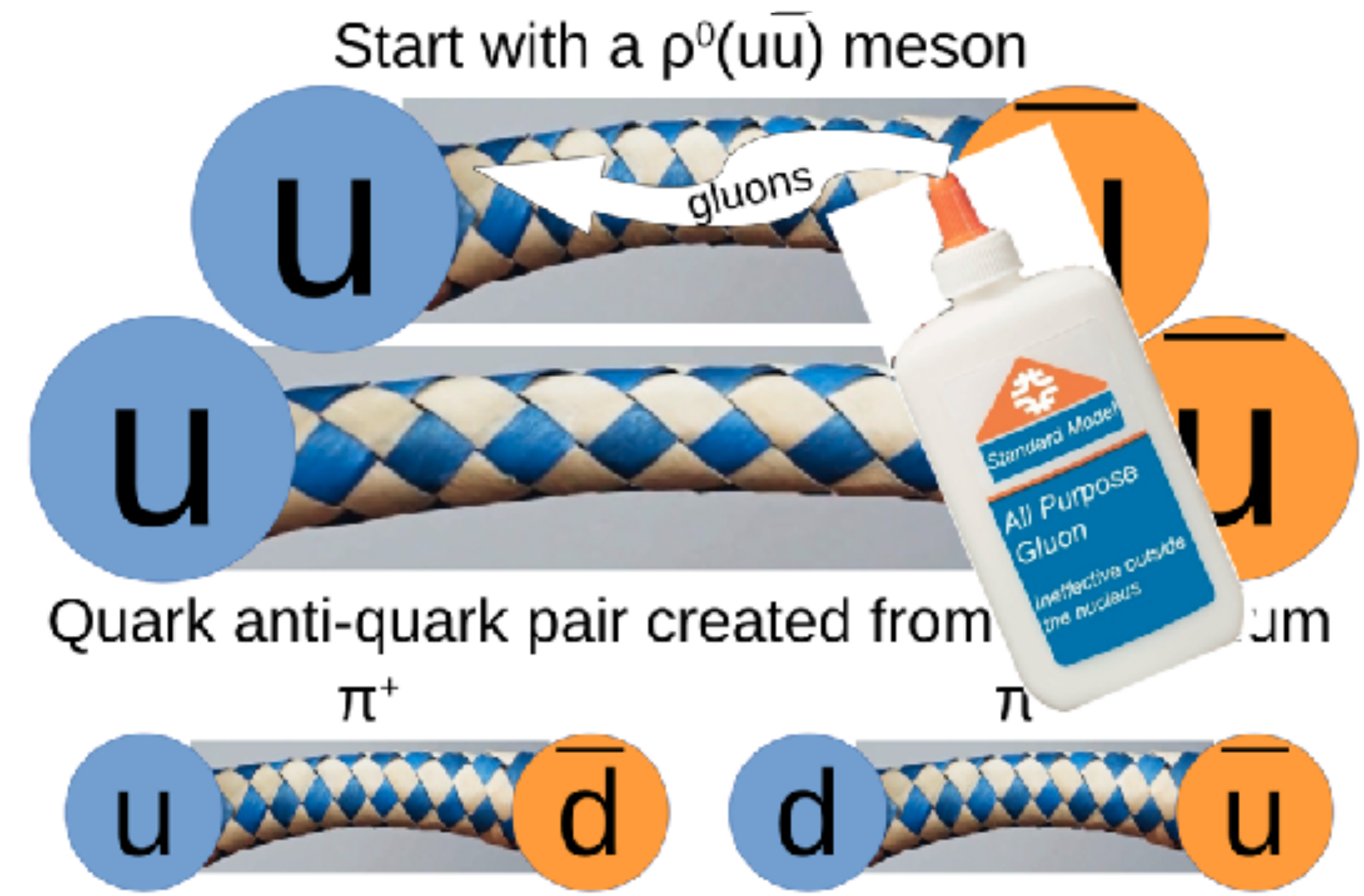
Theory of the strong force: Quantum Chromodynamics (QCD)

Quantum Chromodynamics (QCD)- David Politzer, Frank Wilczek and David Gross 1973 (2004 Nobel Prize for Physics)

Bound states=Hadrons

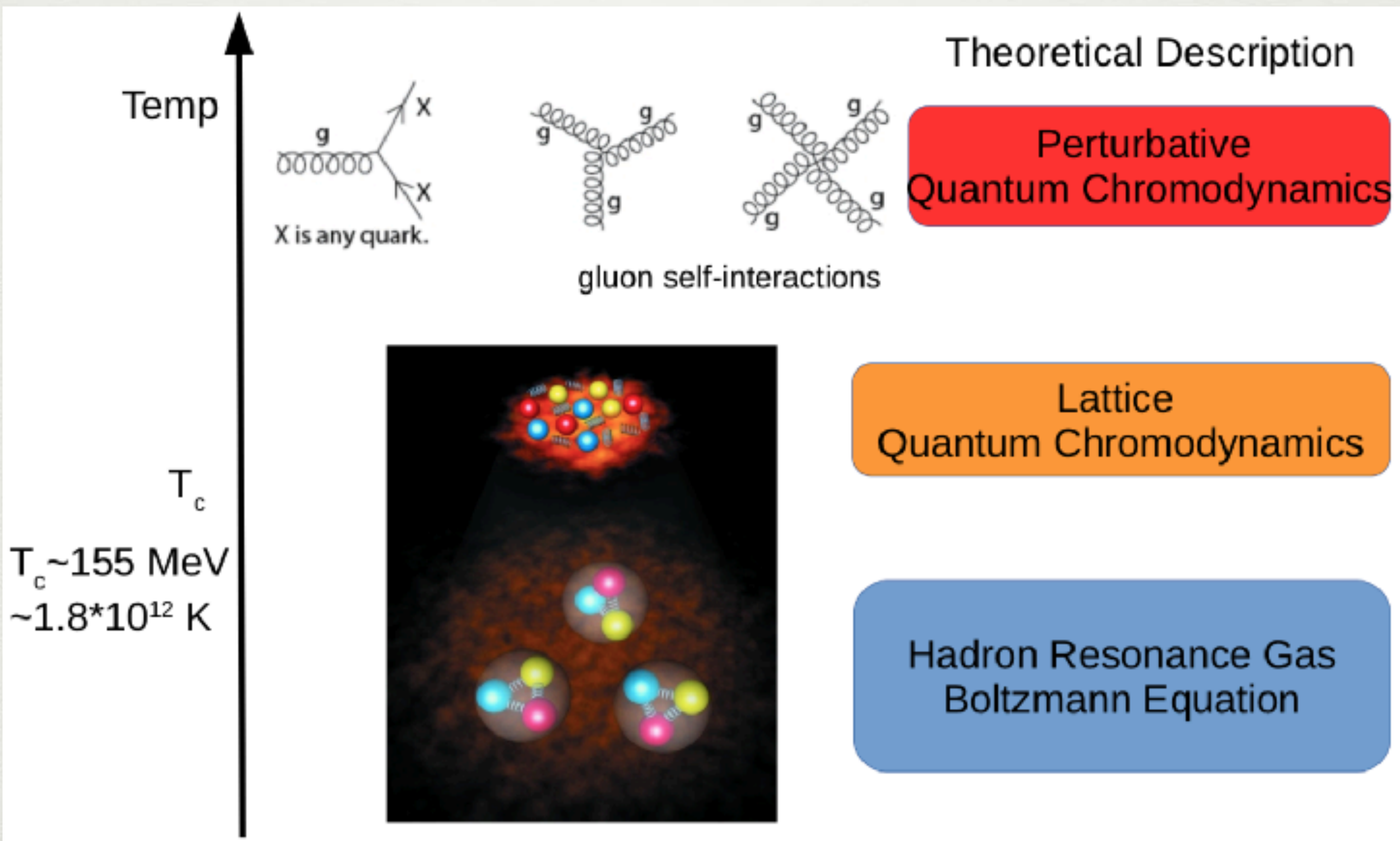


Confinement - no free quarks



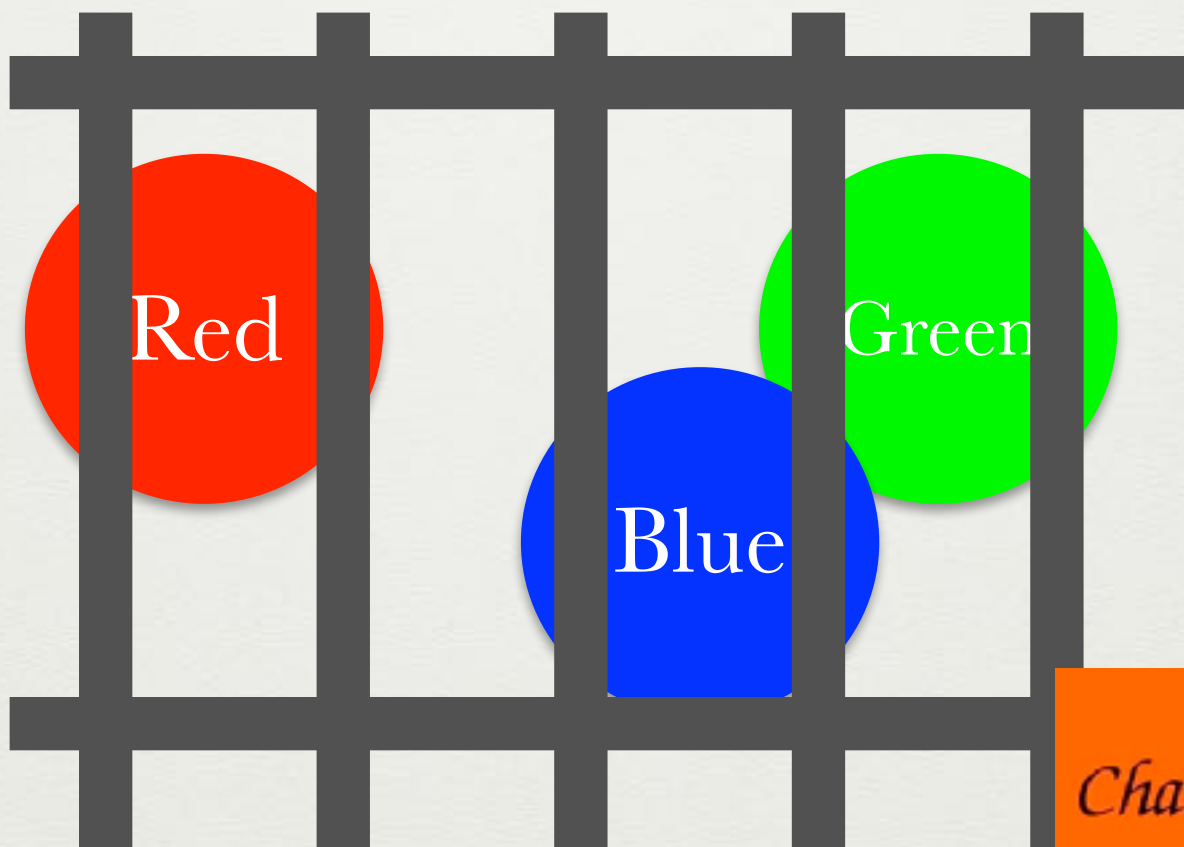
Solving Quantum Chromodynamics

$$L_{QCD} = \underbrace{-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}}_{\text{Gluon Interactions}} + \underbrace{\bar{\psi}^q\left(i\gamma_\mu D^\mu - m_q\right)\psi^q}_{\text{Quark Interactions}} \quad \text{where } D^\mu = \partial^\mu - ig\underbrace{A^\mu(x)}_{Gluons}$$



Deconfinement of QCD matter

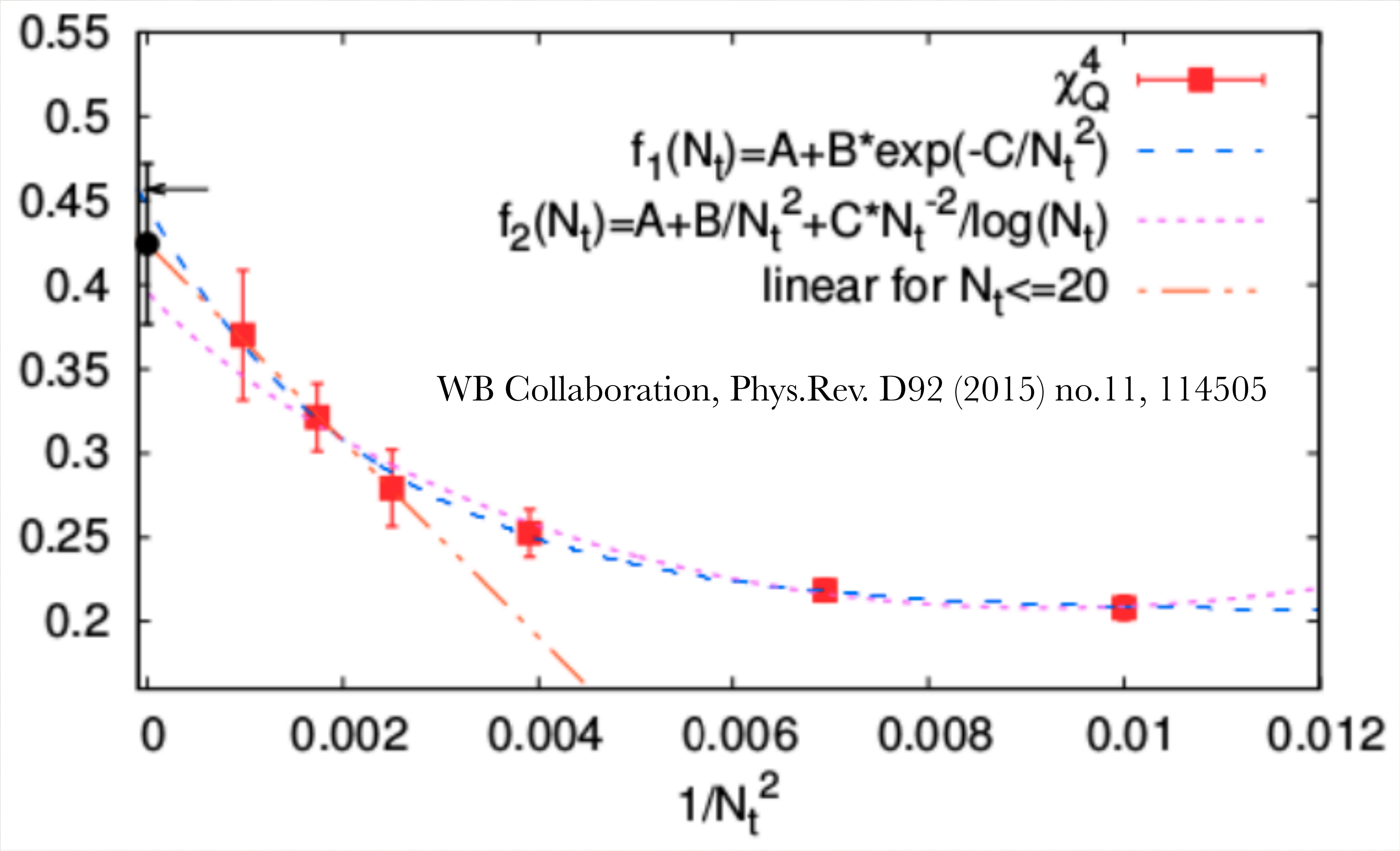
High enough temperatures/large enough energies can deconfine quarks and gluons



Finite Temperature QCD

- Analytical solutions are not possible, however, solving Quantum Chromodynamics on a lattice makes calculations possible.
- Limits of QCD $\mathscr{L}$
  - $m_q \rightarrow \infty$ pure gluon theory (quenched lattice)
  - $m_q \rightarrow 0$ chiral limit
  - $m_q \neq 0$ but u,d,s stronger interactions (non-perturbative)

Caution: Lattice Spacing and finite T observables



Too coarse of lattice spacing can lead to misleading results!

Lattice QCD basics

Temperature: $T = \dfrac{1}{N_T a}$

N_T - number of links in time direction
a - lattice spacing

Volume: $V = \left(N_S a\right)^3$

Continuum Limit:
$N_s \to \infty$
$N_t \to \infty$
$a \to 0$
While $T = const$ and $V = const$

Partition function and thermodynamics

- Sampling over gauge configurations gives the action and from there one calculate the partition function

$$Z = \int \Pi_{n,\mu} dU_\mu(n) e^{-S_g} \det(M[U])$$

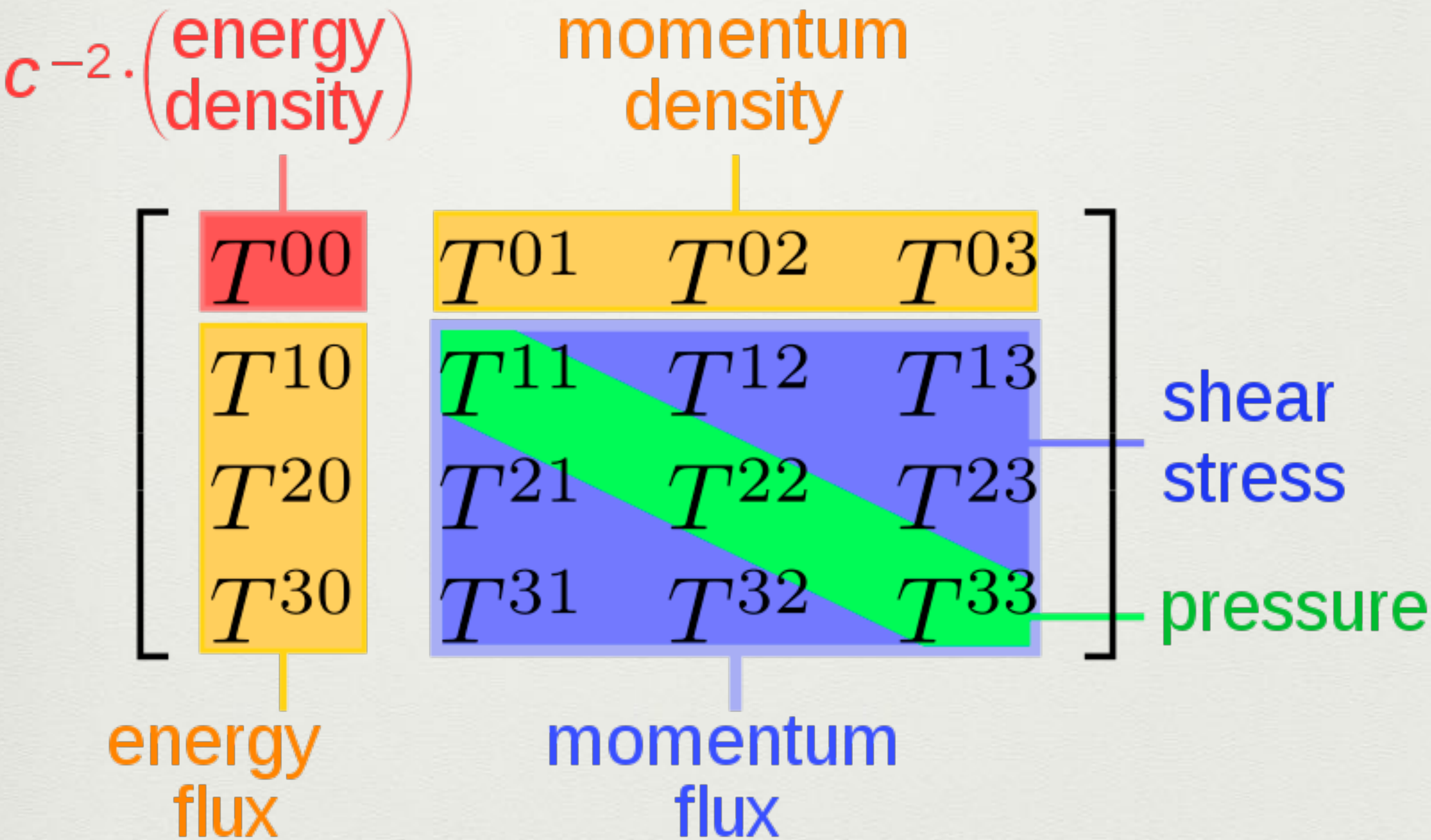
- From Z, one can calculate the pressure ($V \to \infty$ limit):

$$p = \frac{T}{V} \ln Z$$

or trace anomaly:

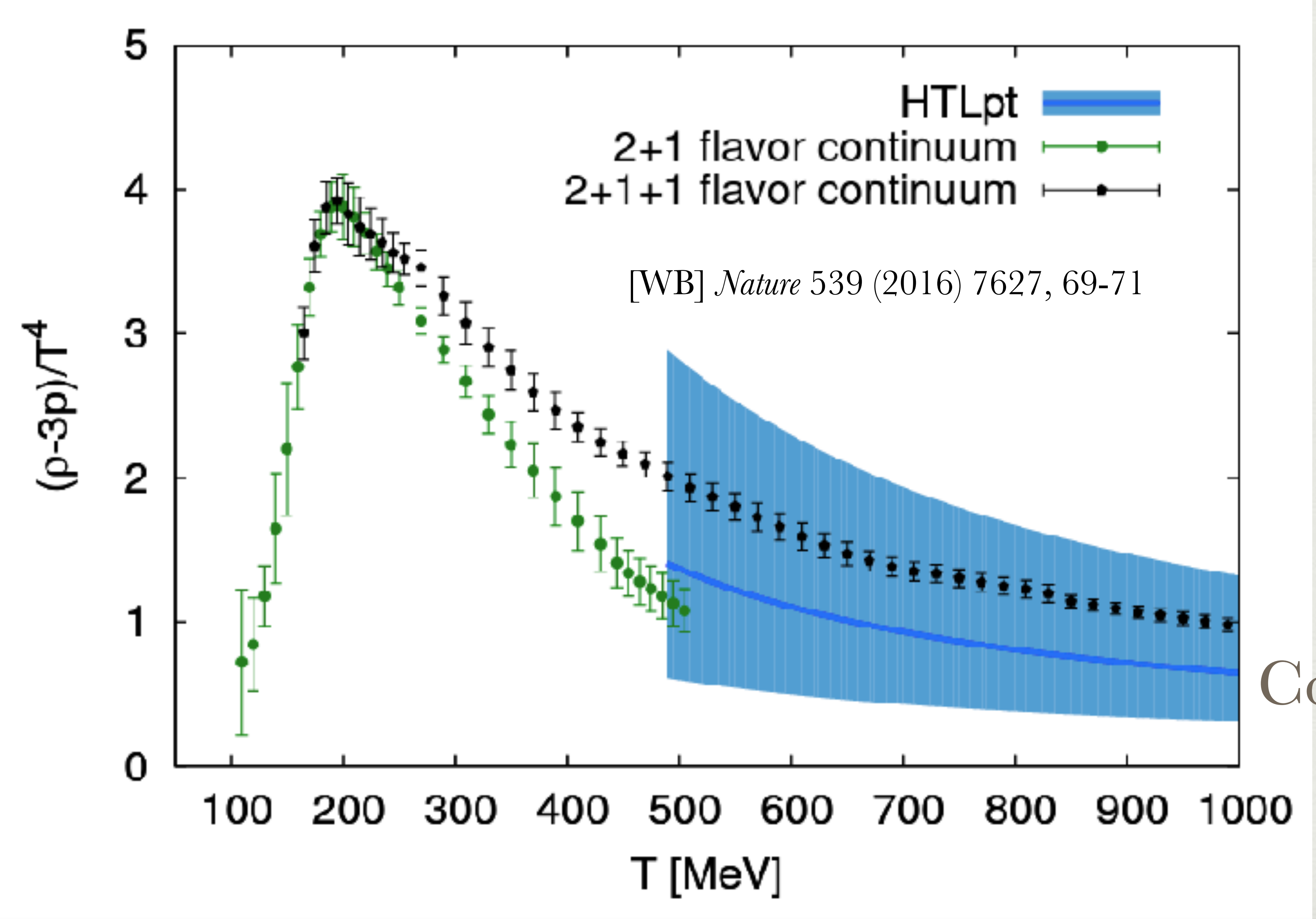
$$\varepsilon - 3p = -\frac{T}{V} \frac{d \ln Z}{d \ln a}$$

Trace anomaly $\varepsilon - 3p$ for $\{+, -, -, -\}$



Trace anomaly

Depends on the numbers of quarks

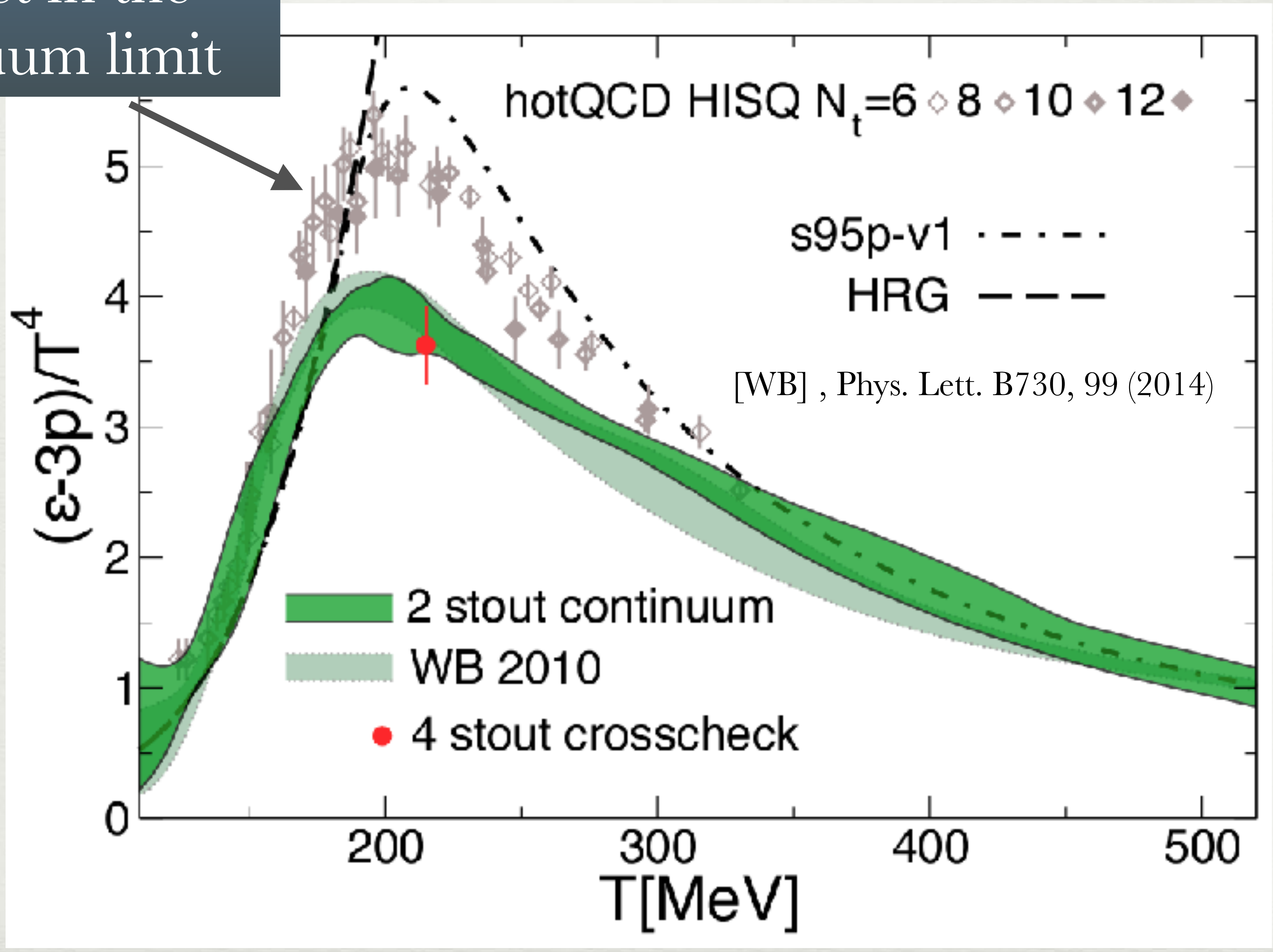


[WB] *Nature* 539 (2016) 7627, 69-71

Conformal
$\varepsilon = 3p$

Influence of lattice spacing

Not yet in the continuum limit



Thermodynamic relations at $\mu_B = 0$

From the trace anomaly $(\varepsilon - 3p)$, one can work out the rest

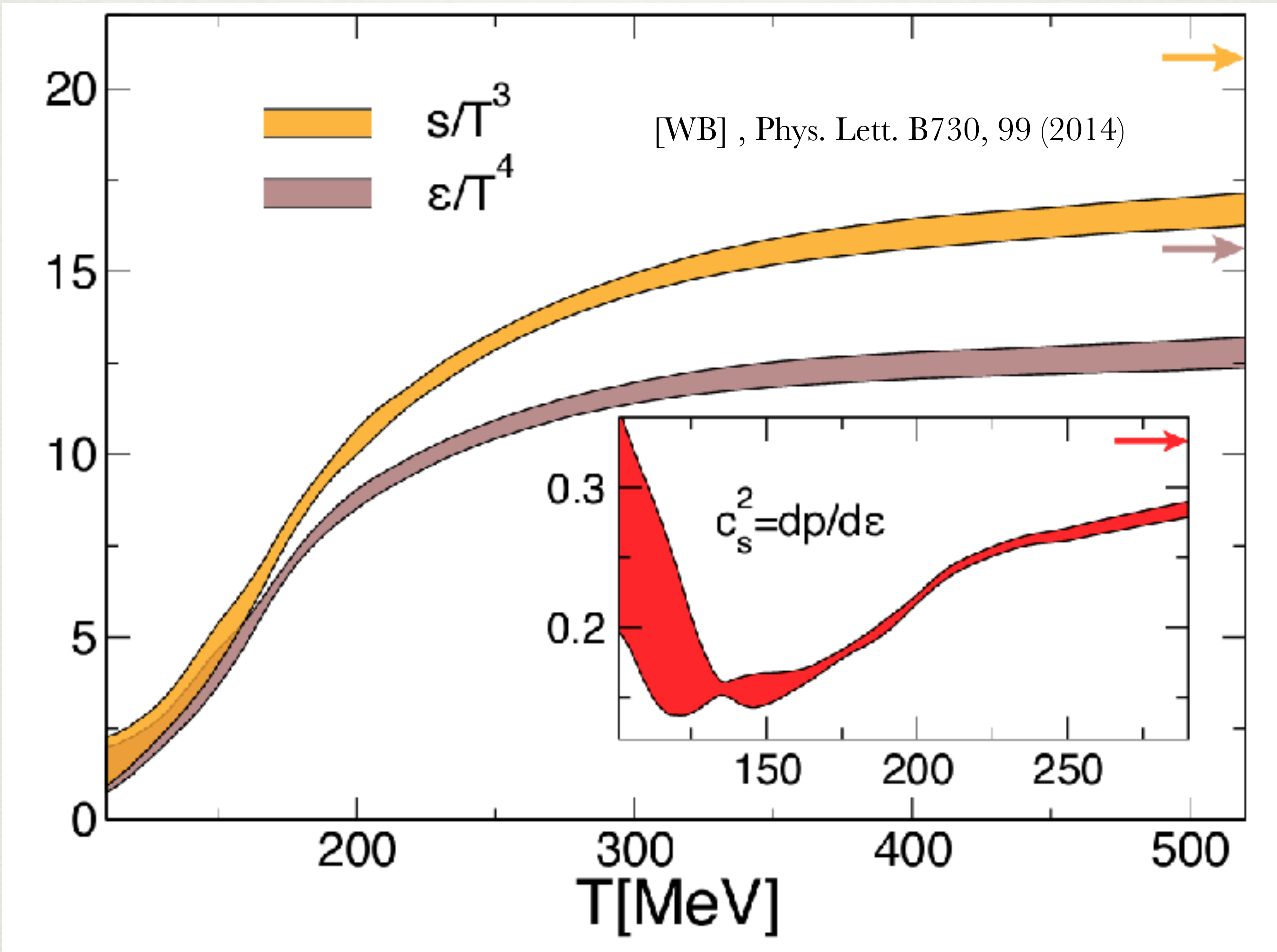
Pressure: $$\frac{p}{T^4} = \int_0^T dT \frac{1}{T}\left(\frac{\varepsilon - 3p}{T^4}\right)$$

Energy density: $$\frac{\varepsilon}{T^4} = \left(\frac{\varepsilon - 3p}{T^4}\right) + 3\frac{p}{T^4}$$

Entropy density: $$\frac{s}{T^3} = \left(\frac{\varepsilon - 3p}{T^4}\right) + 4\frac{p}{T^4}$$

Speed of sound: $$c_s^2 = \frac{dp}{d\varepsilon} = \frac{s}{T}\frac{dT}{ds}$$

Speed of sound

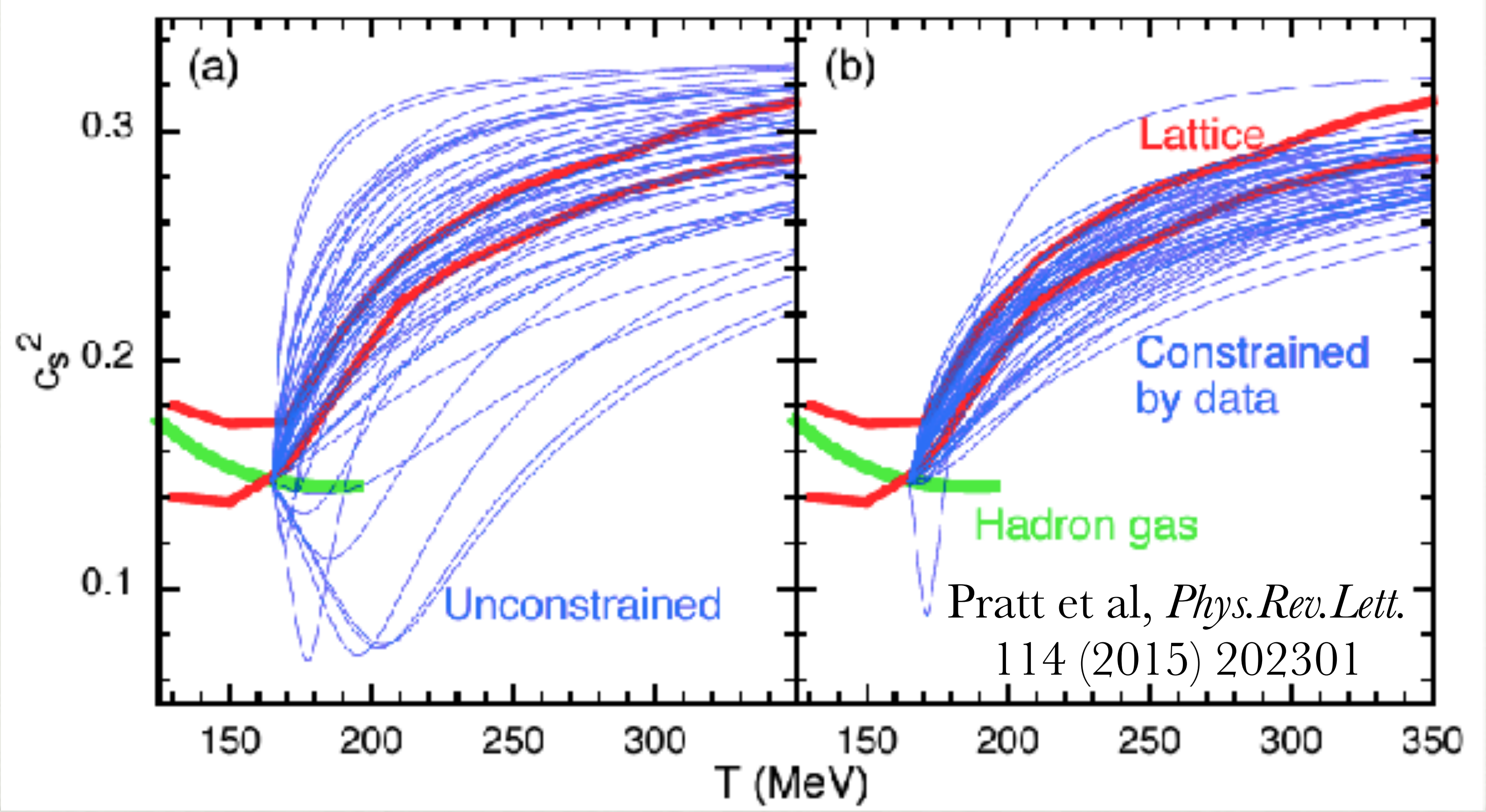


Equation of State

Equation of state: $p(\varepsilon)$ + derivatives

Unfortunately, we cannot directly probe $p(\varepsilon)$ experimentally

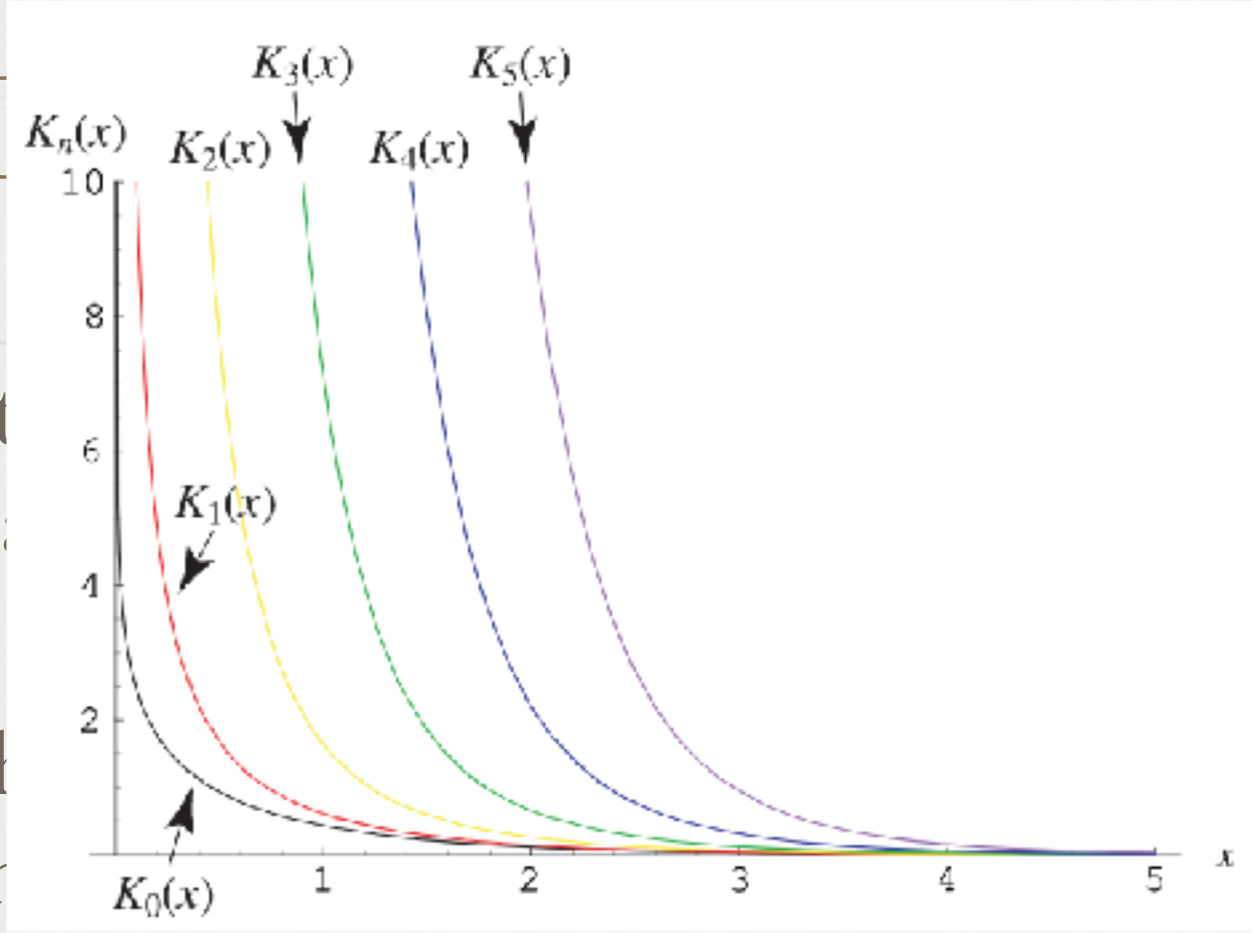
However, use $p(\varepsilon)$ as input for relativistic viscous hydrodynamic simulations, compare to heavy-ion data (tomorrow's lecture)



Pratt et al, *Phys.Rev.Lett.* 114 (2015) 202301

Low T gas

At "low" temperat ar matter to be
a g



We call this the l e ideal gas of
many, r nces

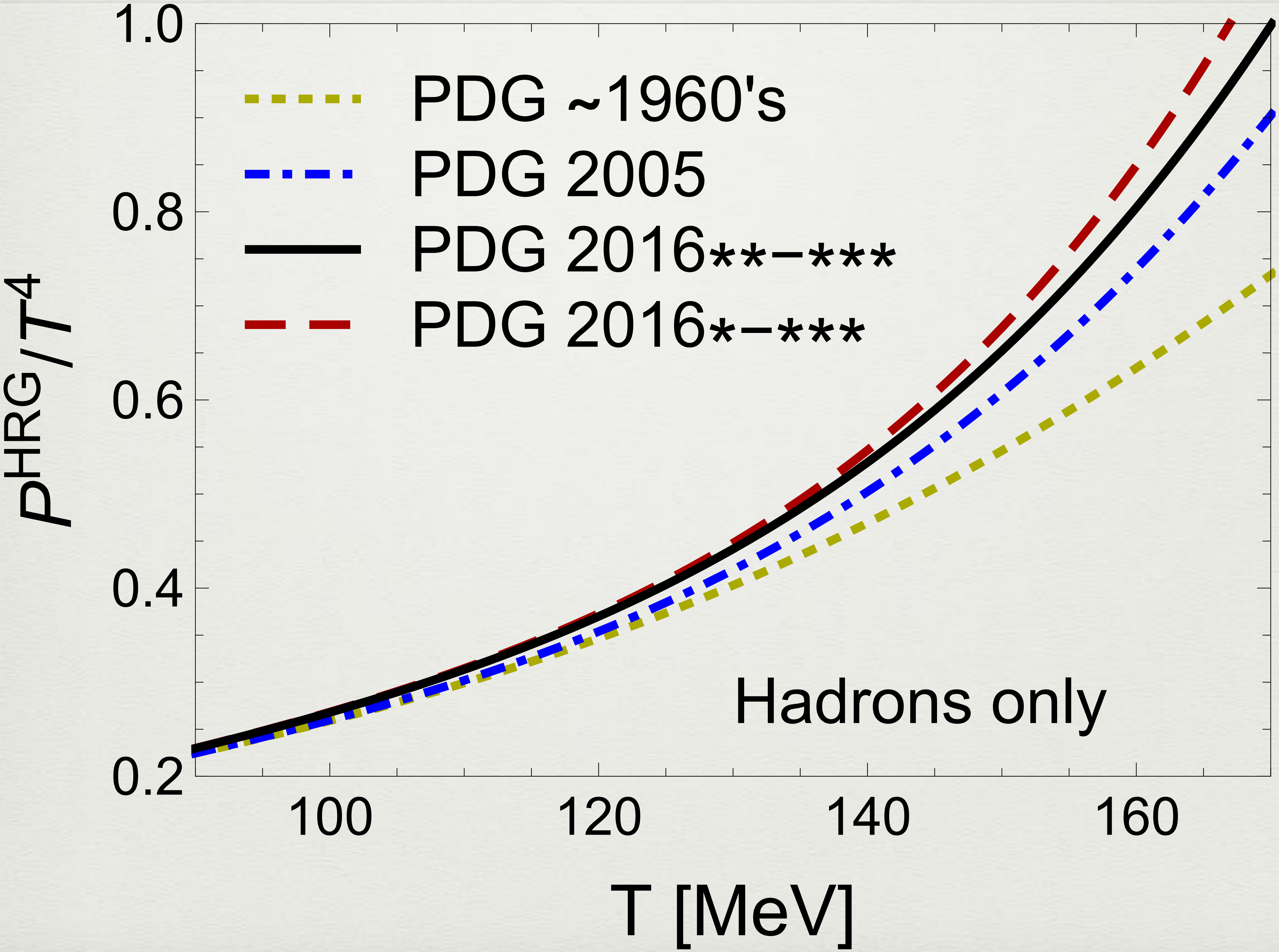
$$\ln Z_i(T,\vec{\mu}) \sim \frac{d_i}{2\pi}\left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} \frac{(-1)^{(|B_i|-1)(k+1)}}{k^2} K_2\left(\frac{k m_i}{T}\right) \cosh\left[k\vec{\mu}_i/T\right]$$

Where $\vec{\mu}_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$

In principle, one should describe their interactions. Instead, using all the resonance states does a reasonable job.

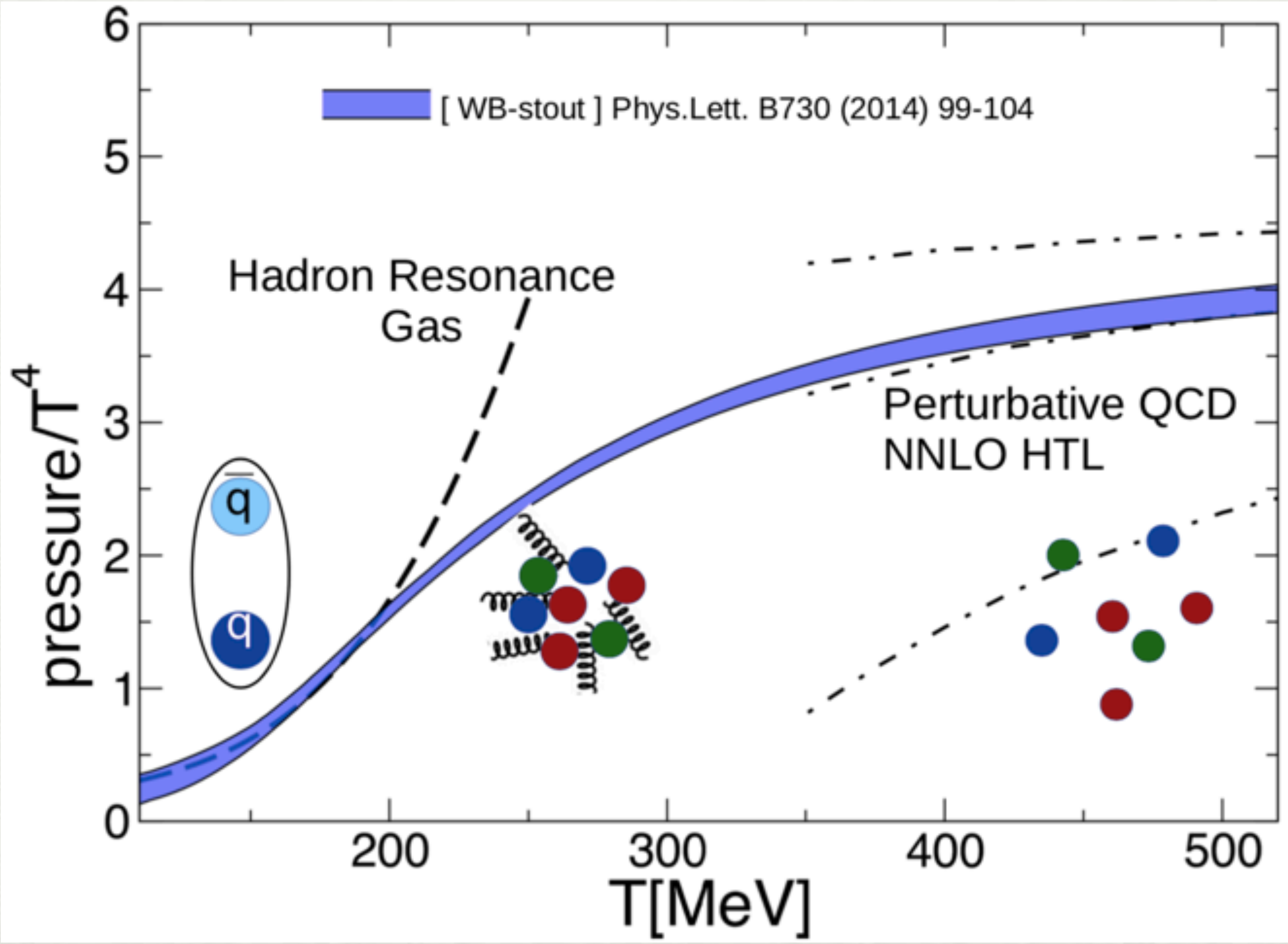
BUT at large μ_B, need interactions!

Influence of heavy resonances

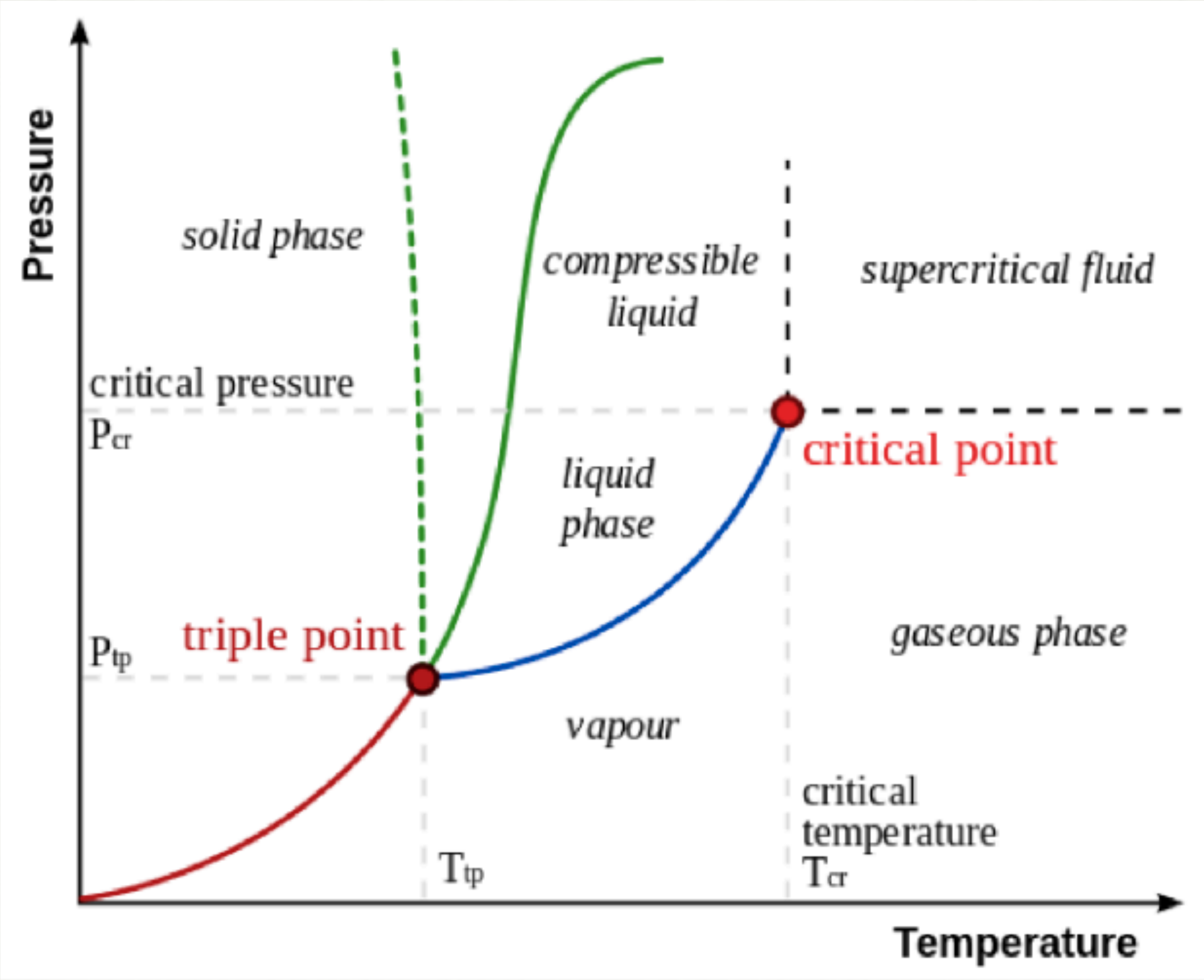


Lattice QCD: Phase Transition

Cross-over phase transition $T \sim 155\ MeV$



Water phase diagram



Water at atmospheric pressure has 2 first-order phase transitions

Finding a phase transition: order parameter

Order parameter - thermodynamic quantity the goes from 0 to 1 for different phases

Example: compressibility $K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

Solid or liquid - can't compress further with added pressure $K_T \sim 0$

Gas - inversely scales with pressure $K_T = p^{-1} \neq 0$

What is the order parameter in QCD?

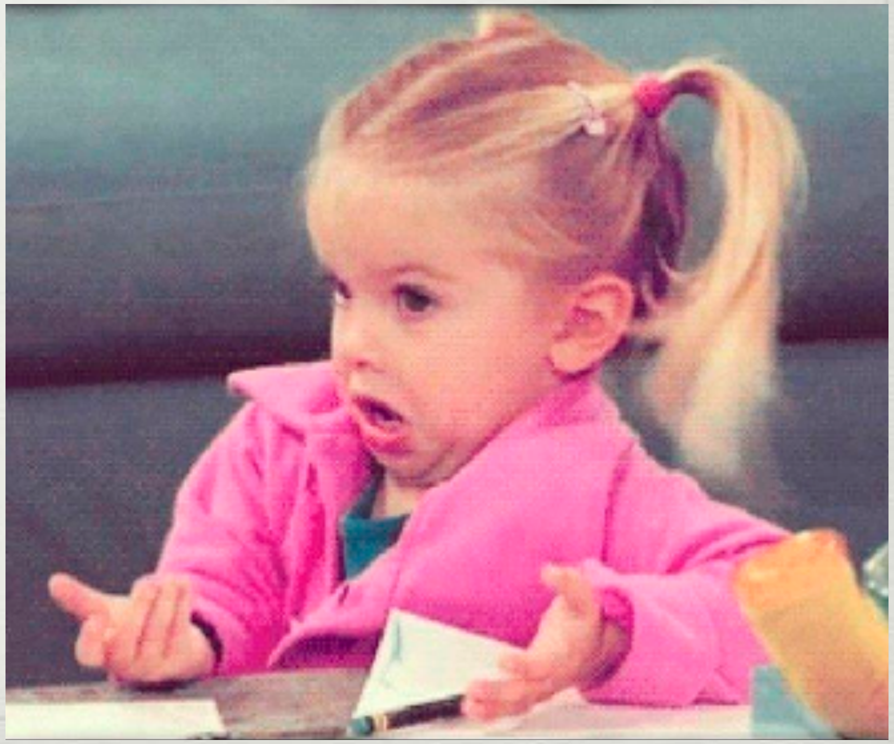
Baryon density n_B

Chiral condensate $\langle \psi \bar{\psi} \rangle = \frac{T}{V} \frac{\partial \ln Z}{\partial m_0}$

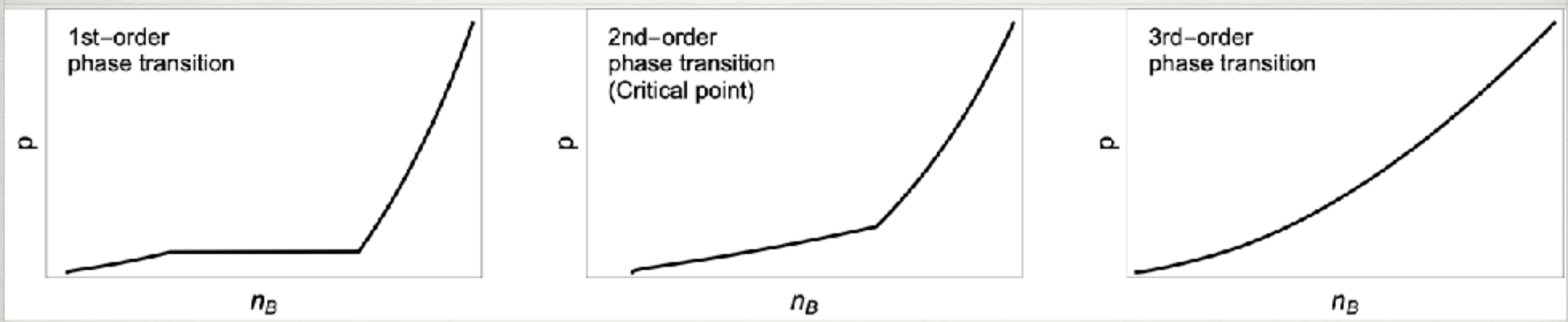
Proxy for deconfinement

Bazavov A et al. 2012 Phys. Rev. D85 054503

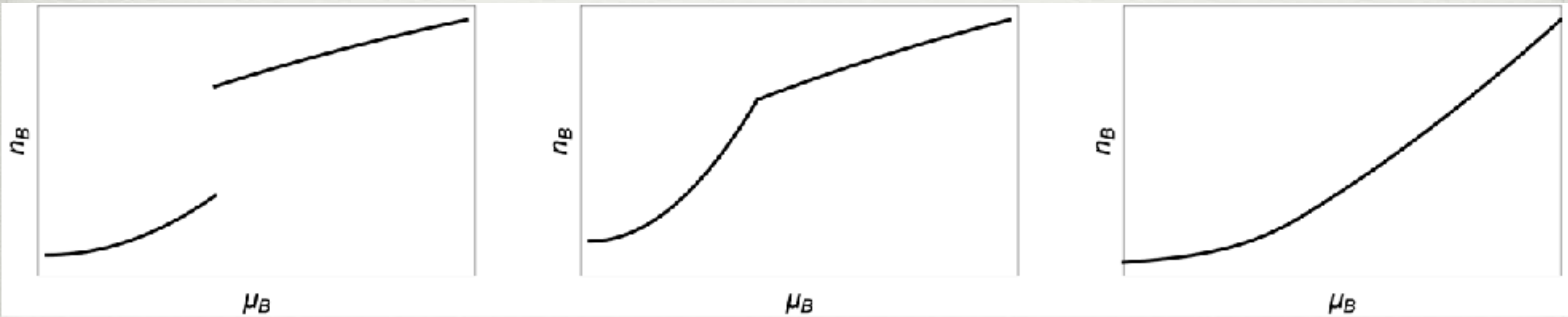
Other charge densities:
strangeness n_s or electric n_Q



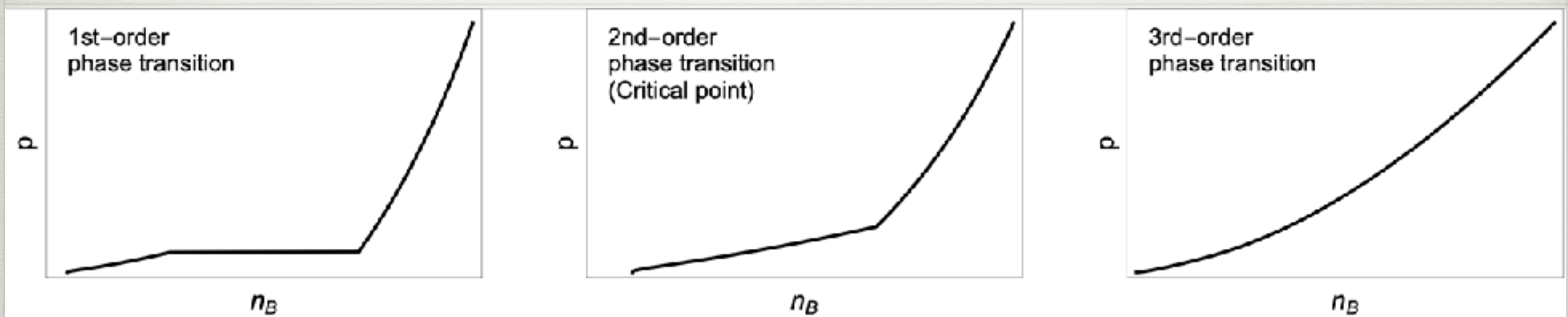
Order of the phase transition



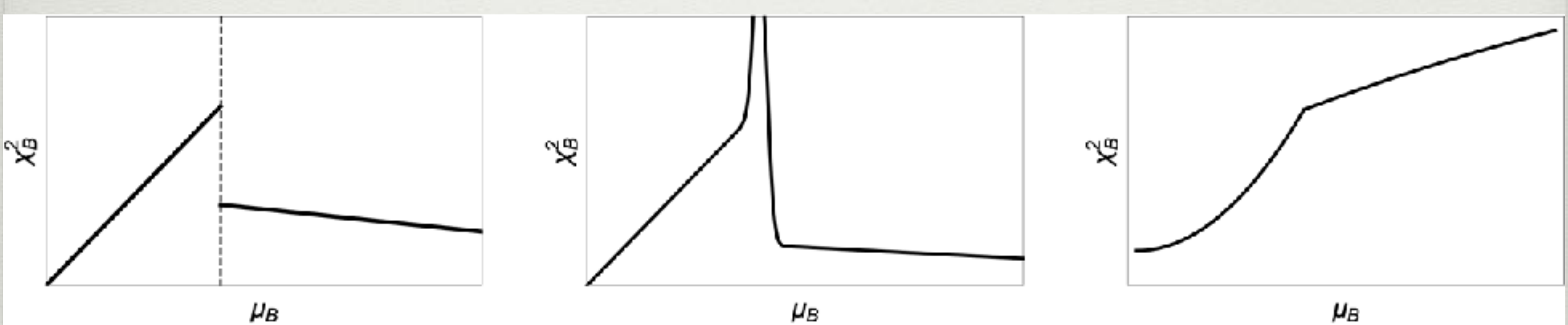
Baryon density $n_B = \frac{\partial p}{\partial \mu_B}$



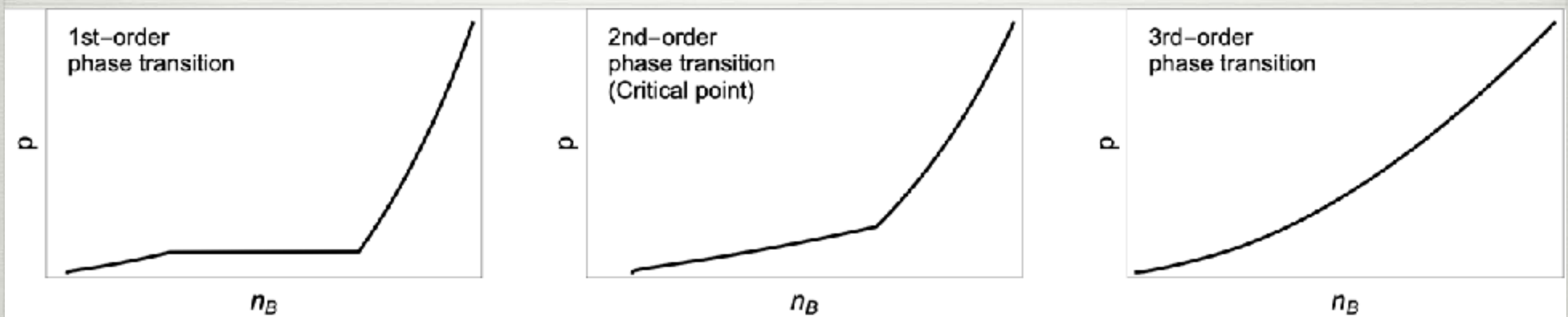
Order of the phase transition



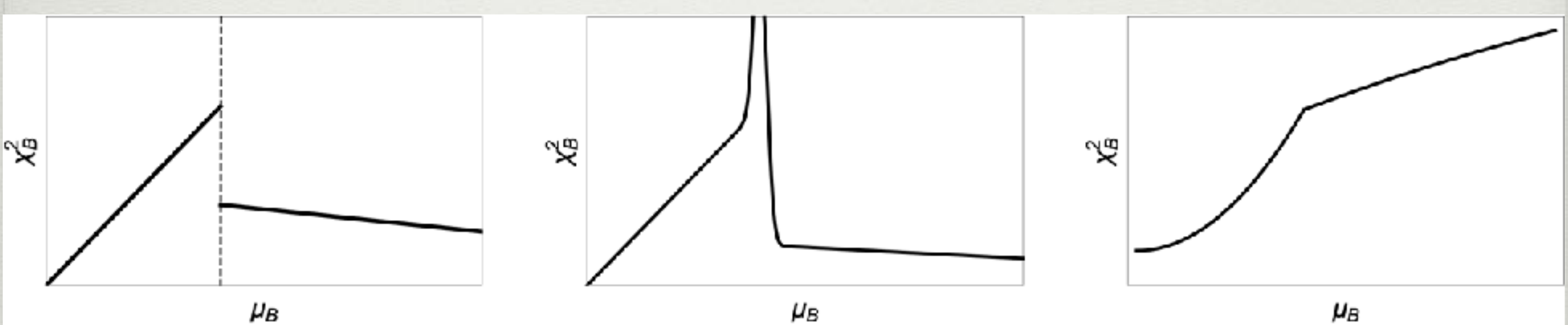
Susceptibilities $\chi_B^n = \dfrac{\partial^n \left(p/T^4\right)}{\partial \left(\mu_B/T\right)^n}$



Order of the phase transition



Susceptibilities $c_s^2 = \left(\frac{dp}{d\varepsilon}\right)_{s/n_B}$



What is the order of the QCD deconfinement transition?

Let’s review its history to understand…

1981 Pure Glue SU(2)

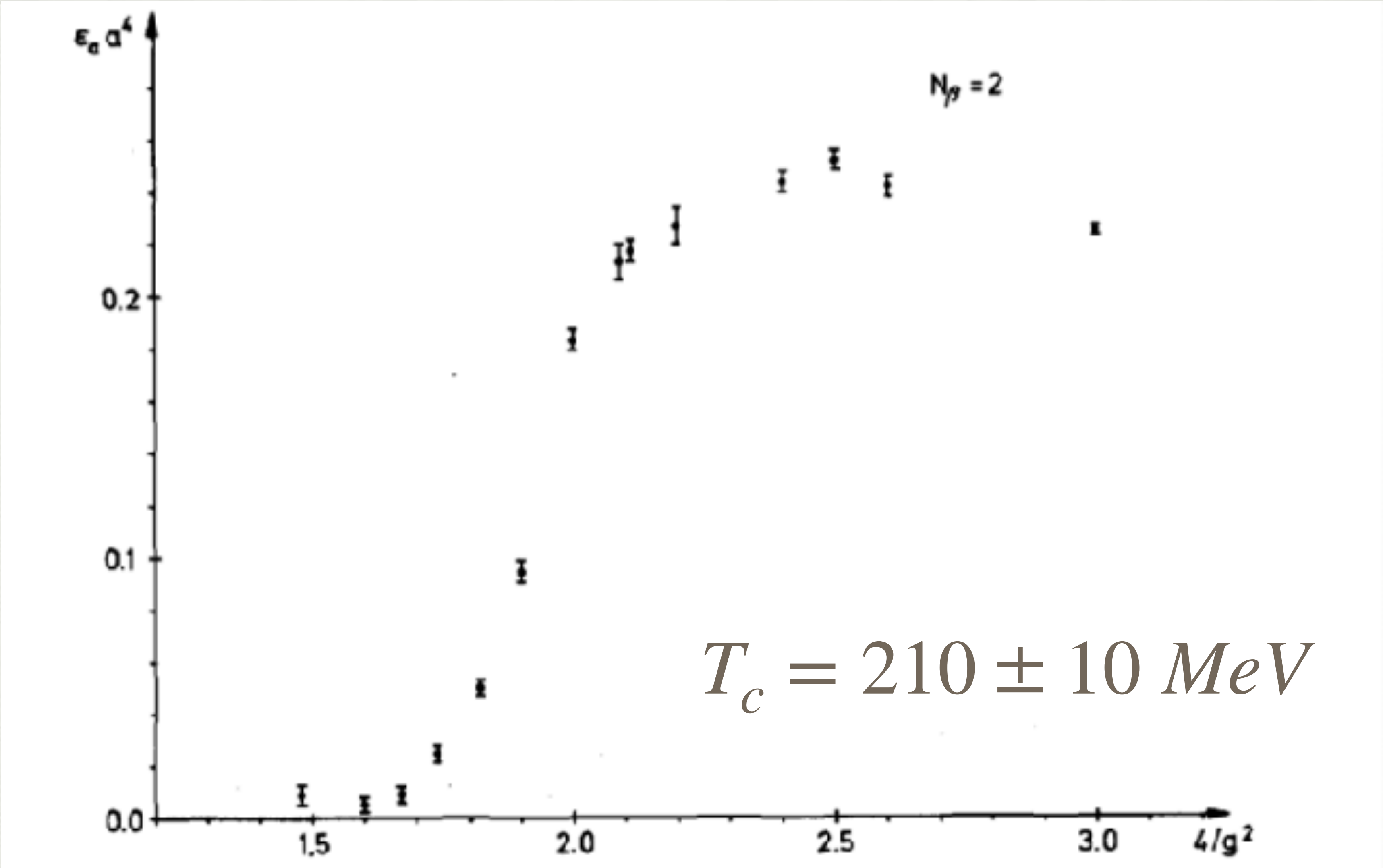
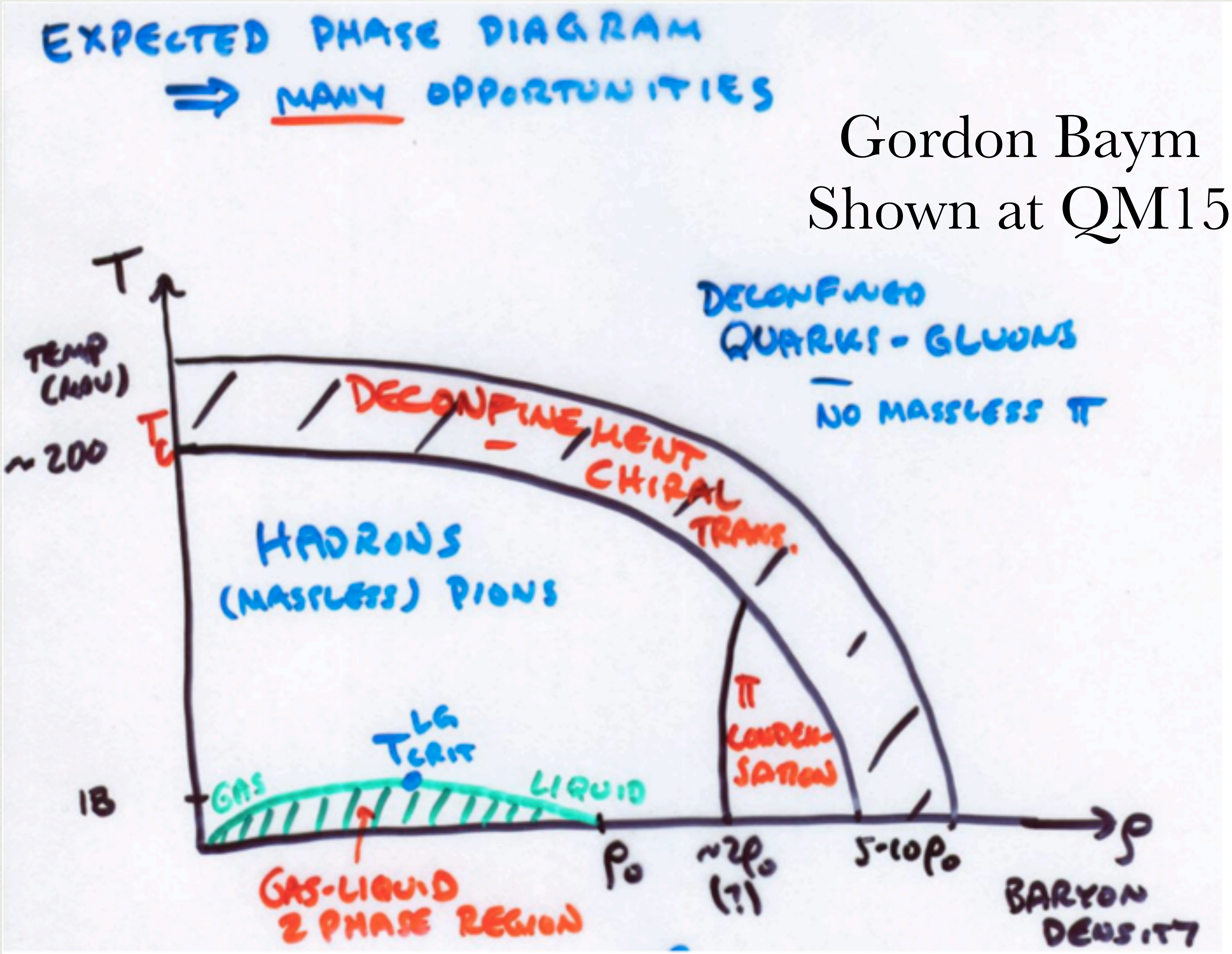


Fig. 3. Energy density of gluon matter versus $4/g^2$, at fixed lattice size $N_\beta = 2$, after about 500 iterations.

Phys.Lett. B101 (1981) 89

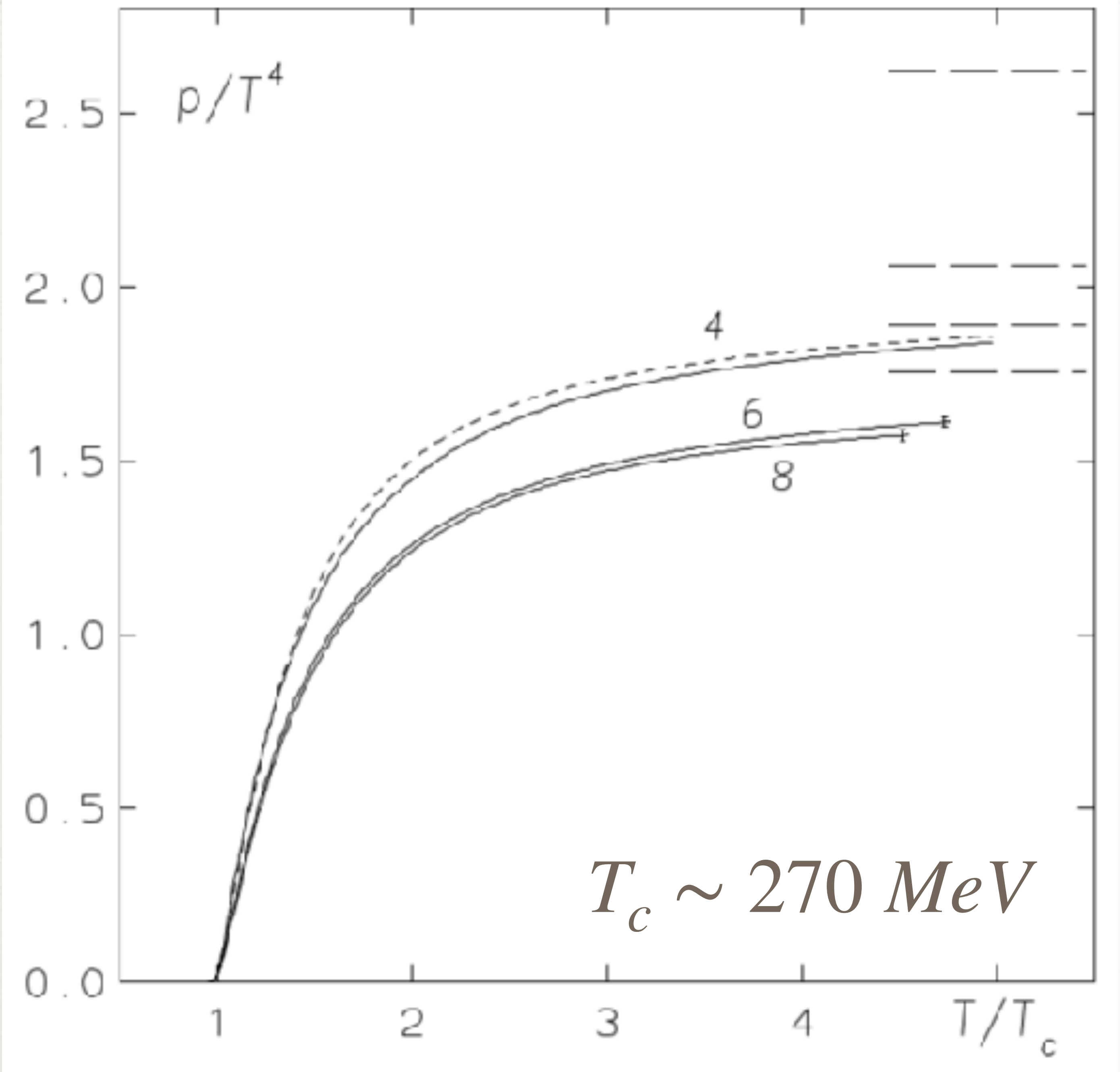
2nd order phase transition (Polyakov Loop McLerran and Svetitsky Phys.Rev. D24 (1981) 450)

~ 1980's QCD phase diagram



Gordon Baym
Shown at QM15

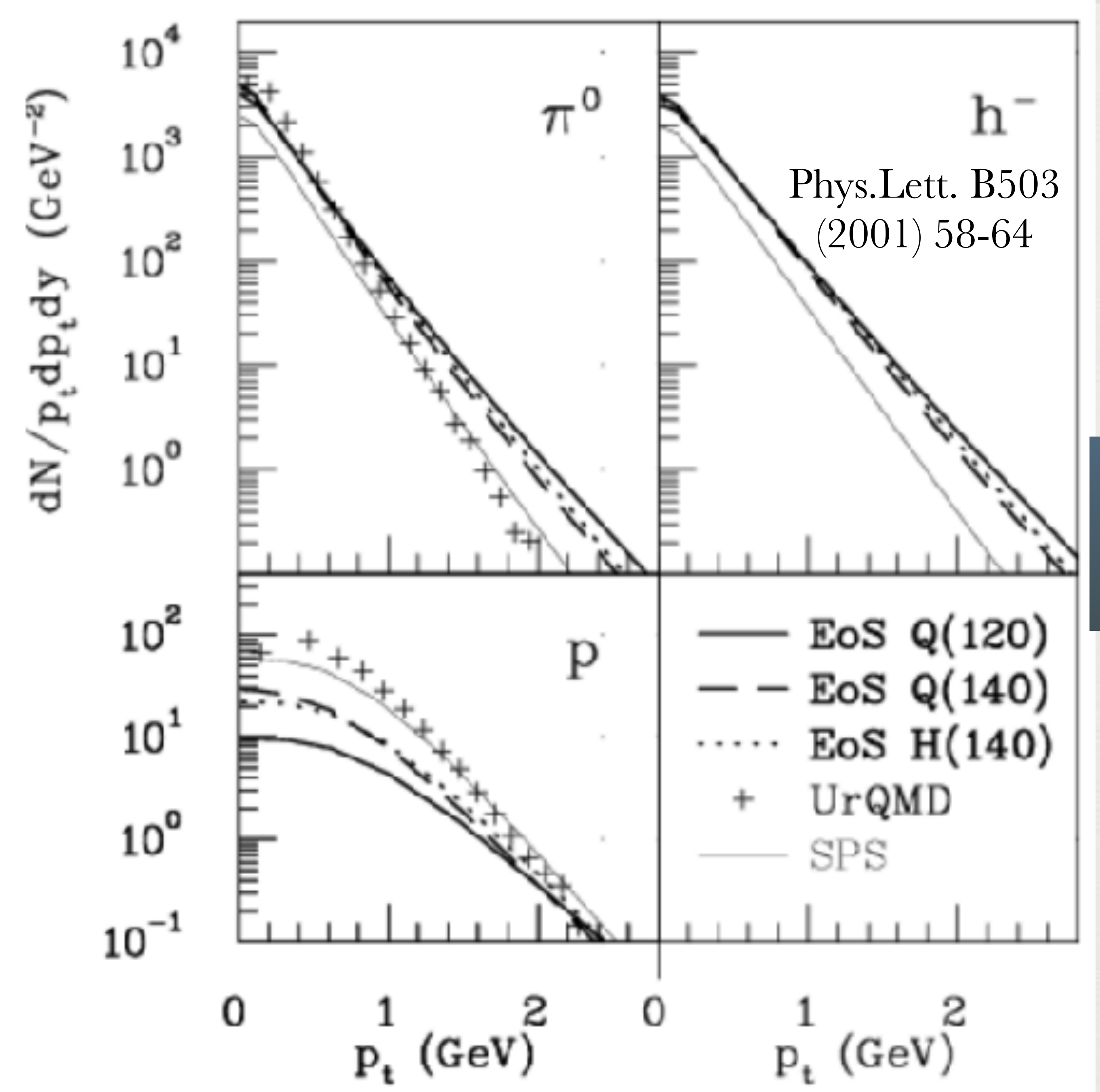
1996 Pure Glue SU(3)



Nucl.Phys. B469 (1996) 419-444

1st order phase transition

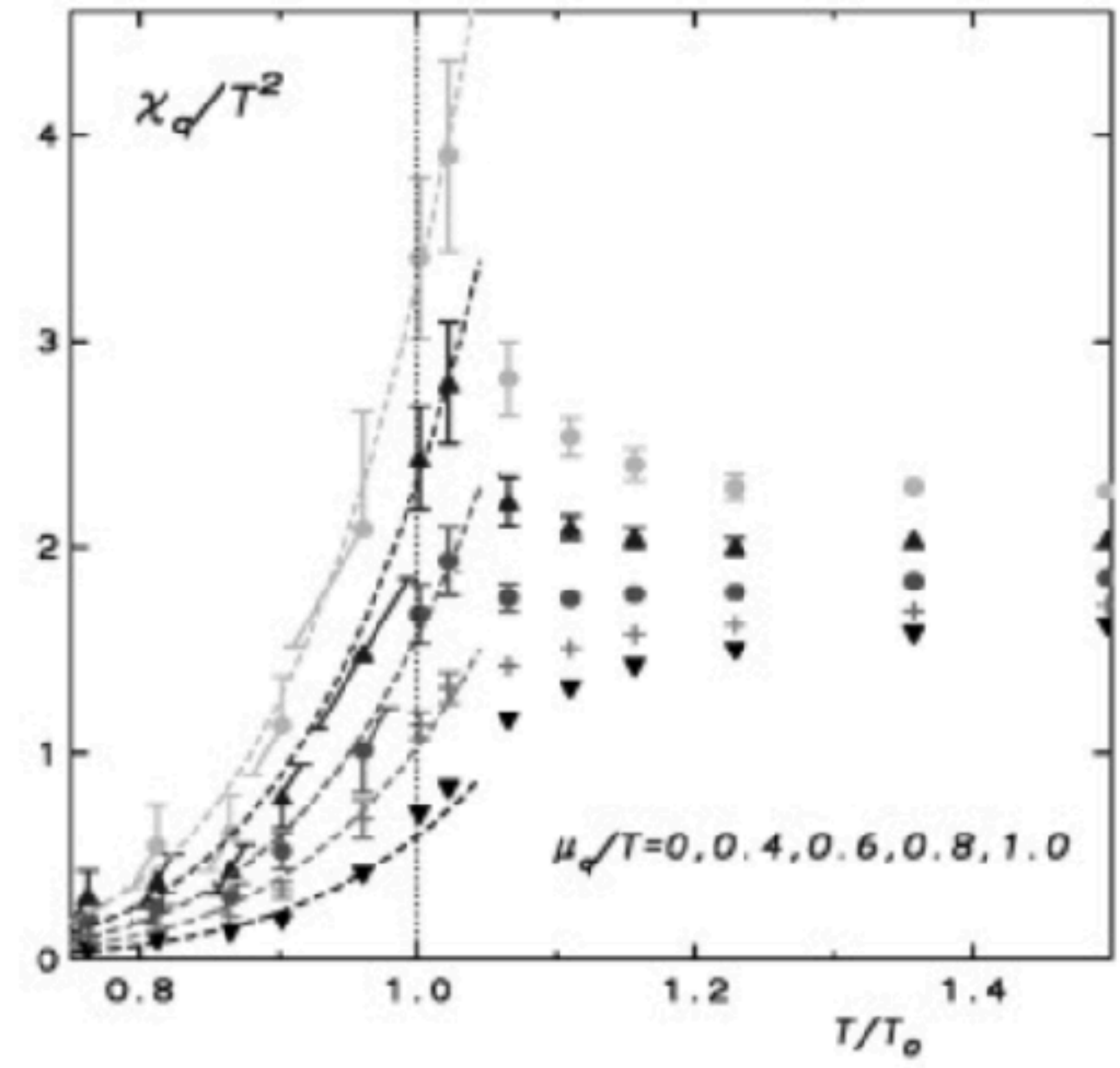
2001 Comparisons to heavy-ion data



See tomorrow's talk

First-order phase transition equation of state compared to data

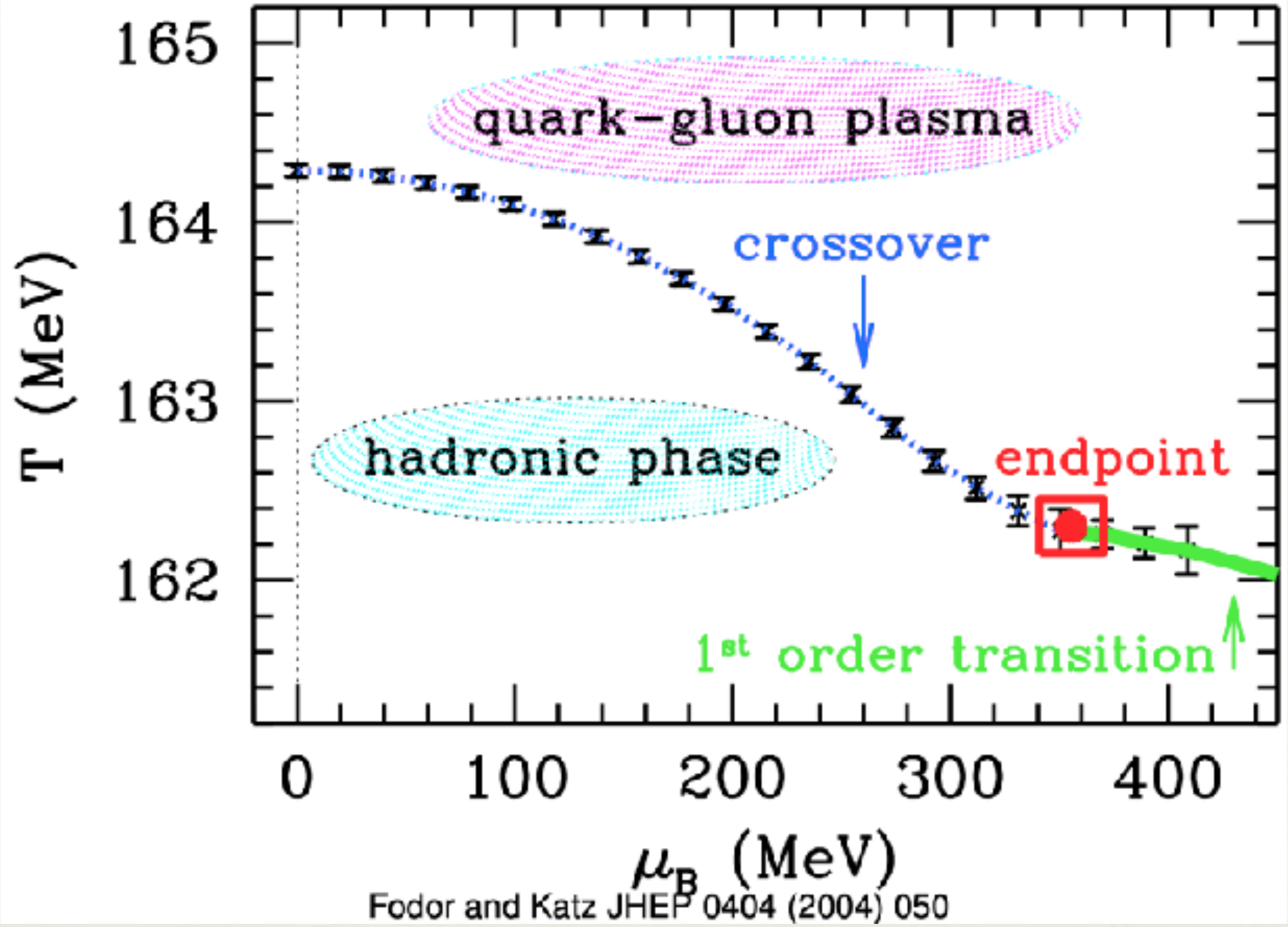
2003 Matching lattice QCD to hadron resonance gas



Karsch, Redlich, Tawfik Phys.Lett. B571 (2003) 67-74

Hadron masses readjusted to smoothly match lattice QCD results

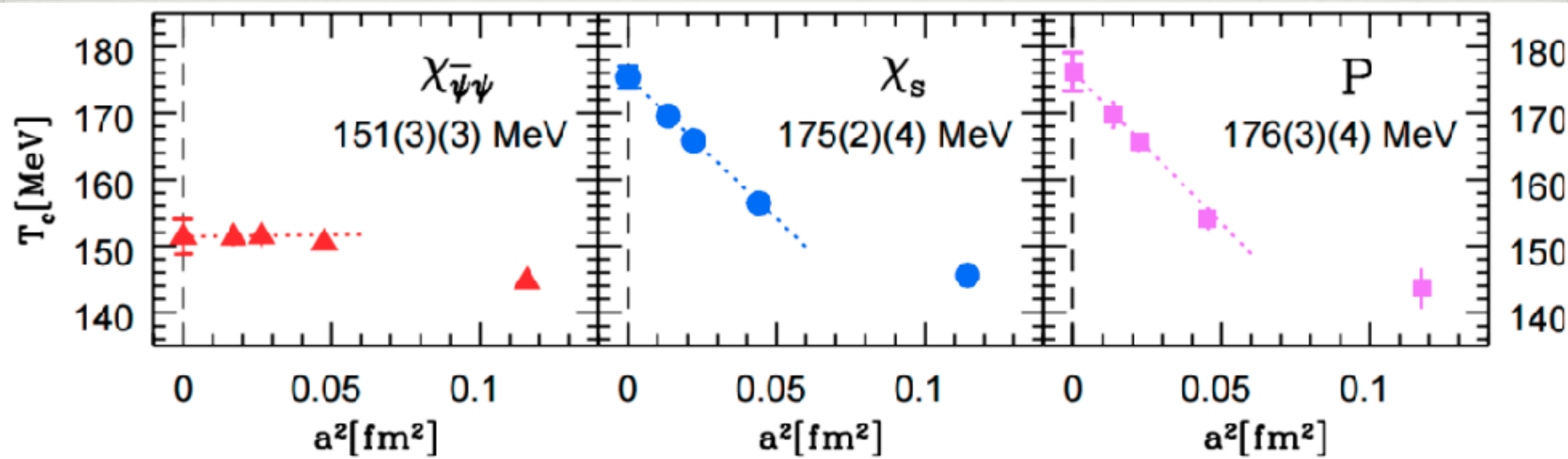
2004 Cross-over at $\mu_B = 0$ and critical point



Reweighting scheme with a coarse lattice

Modern results: *[WB] Phys.Rev.D* 105 (2022) 5, L051506

2006 Cross-over Phase Transition



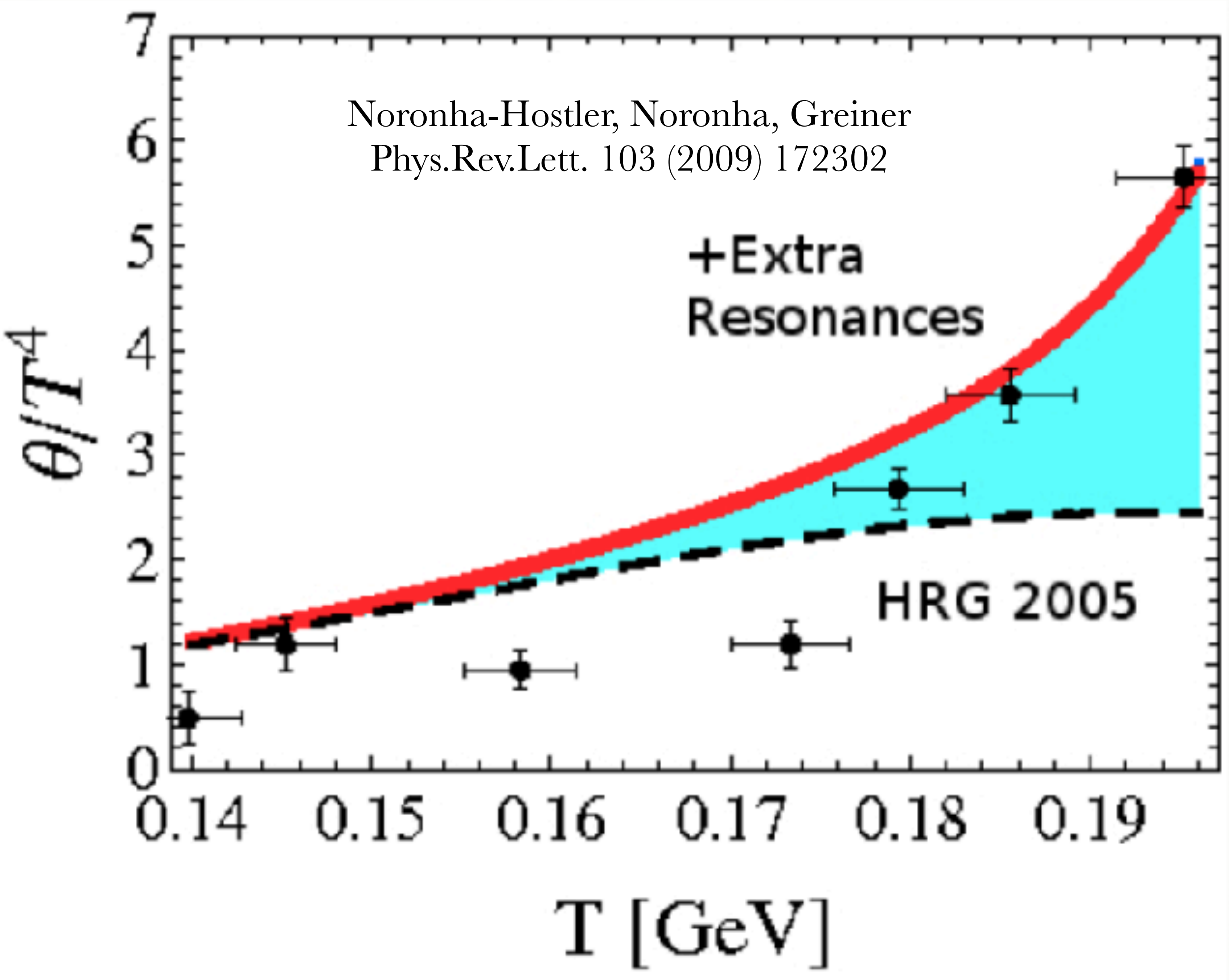
[WB] Phys.Lett. B643 (2006) 46-54; Nature 443 (2006) 675-678

Figure 4: Continuum limit of the transition temperatures obtained from the renormalized chiral susceptibility ($m^2\Delta\chi_{\bar\psi\psi}/T^4$), strange quark number susceptibility (χ_s/T^2) and renormalized Polyakov-loop (P_R).

Checked 3 different "order parameters", all lead to different T

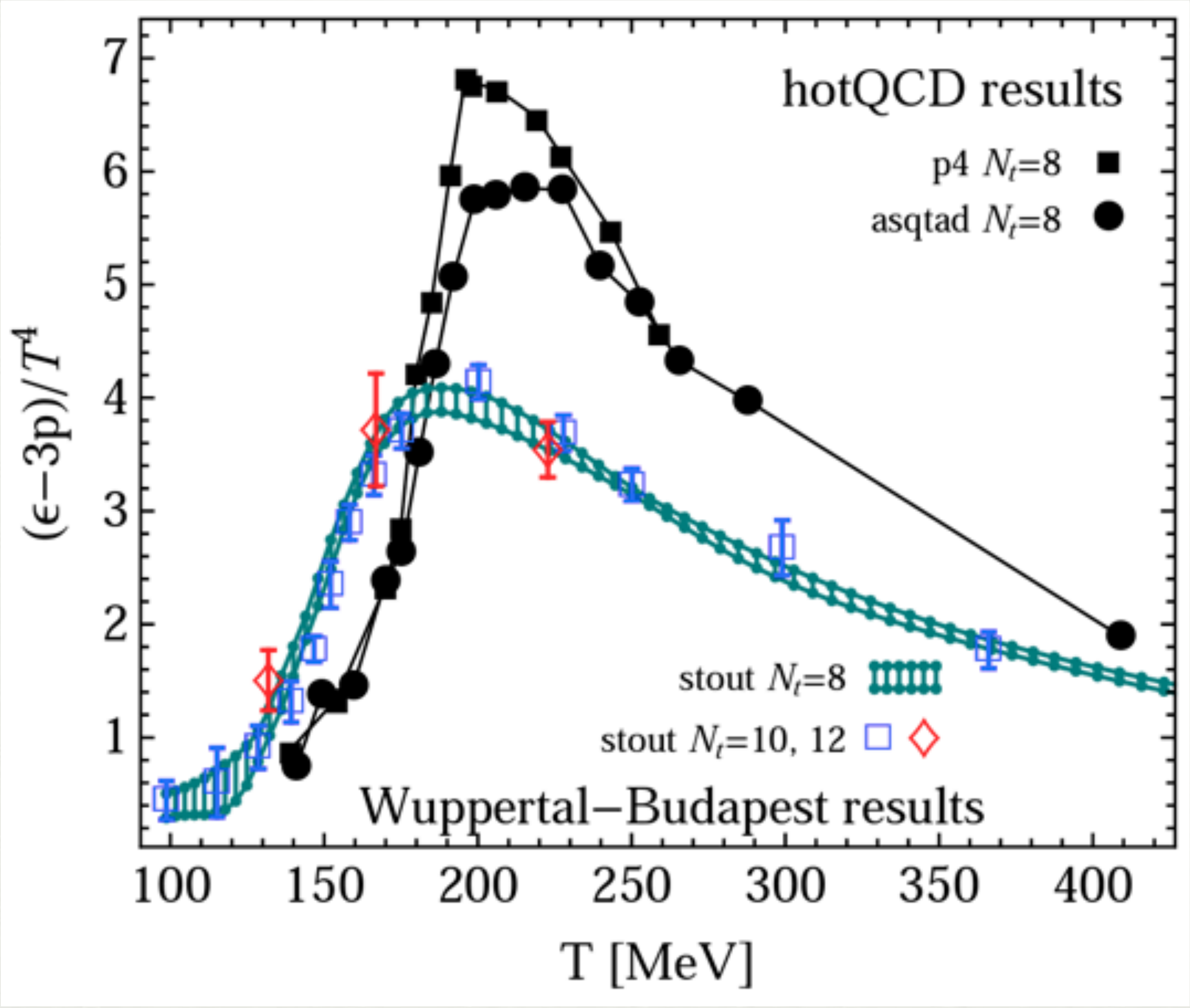
Smooth cross-over phase transition from hadrons to quarks & gluons!

2009 Extra Resonance needed for Cross-over



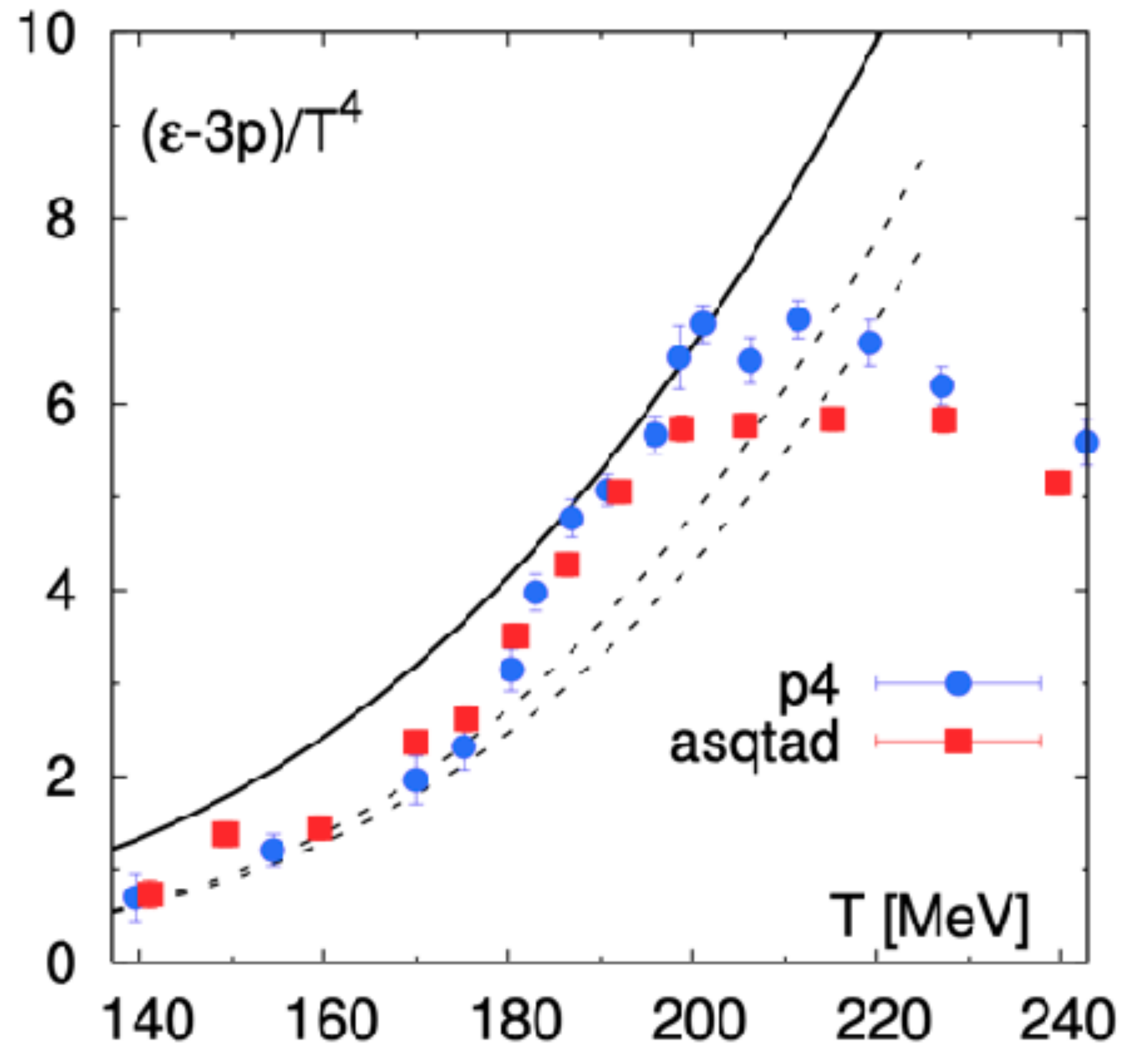
Extend mass spectrum with e^{m/T_H}

~2010 Lattice QCD did not agree on T_c



hotQCD T_c ~ 196 MeV
WB T_c ~ 176 MeV

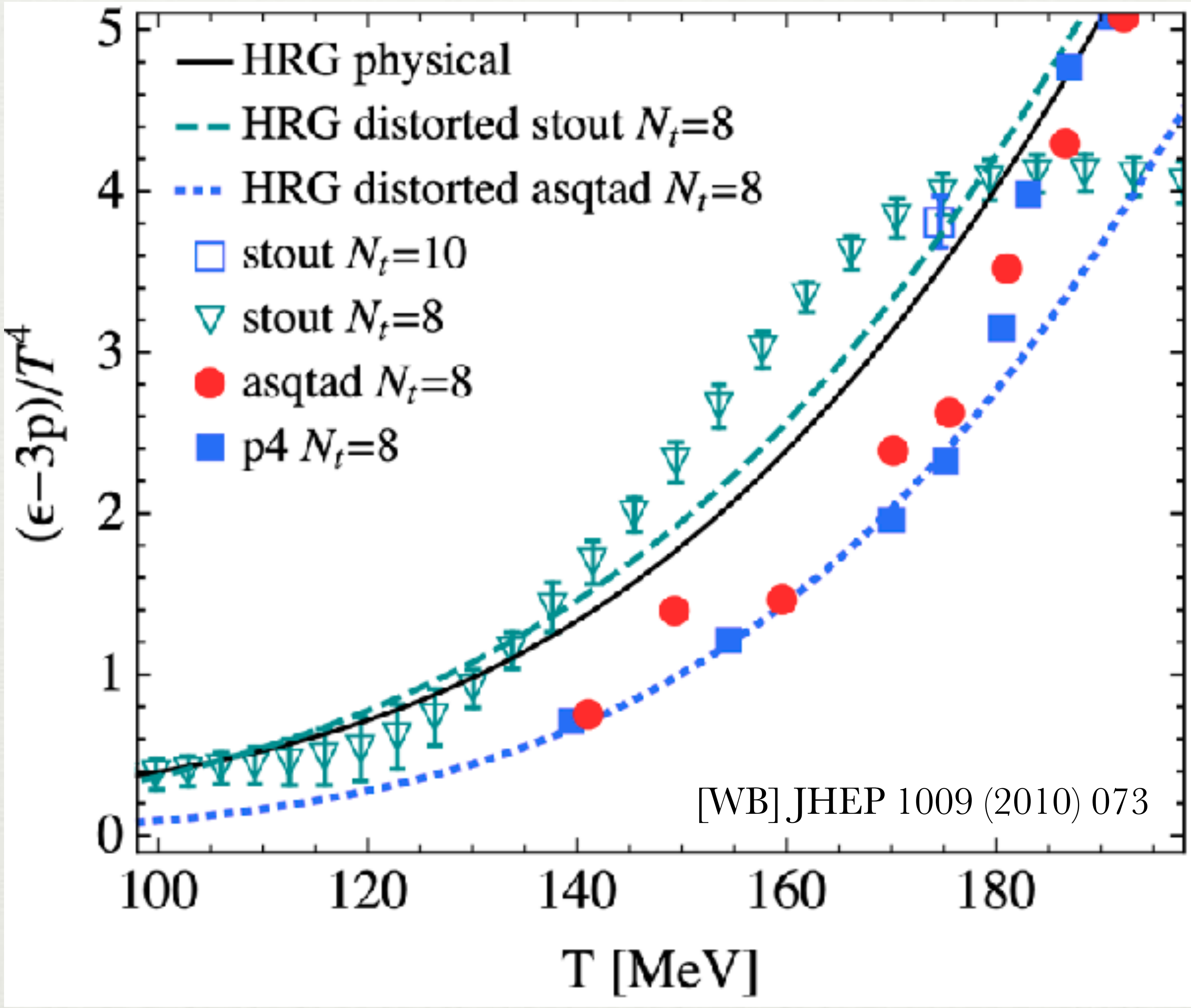
~2010 rescaling hadron masses



Huovinen and Petreczky Nucl.Phys. A837 (2010) 26-53

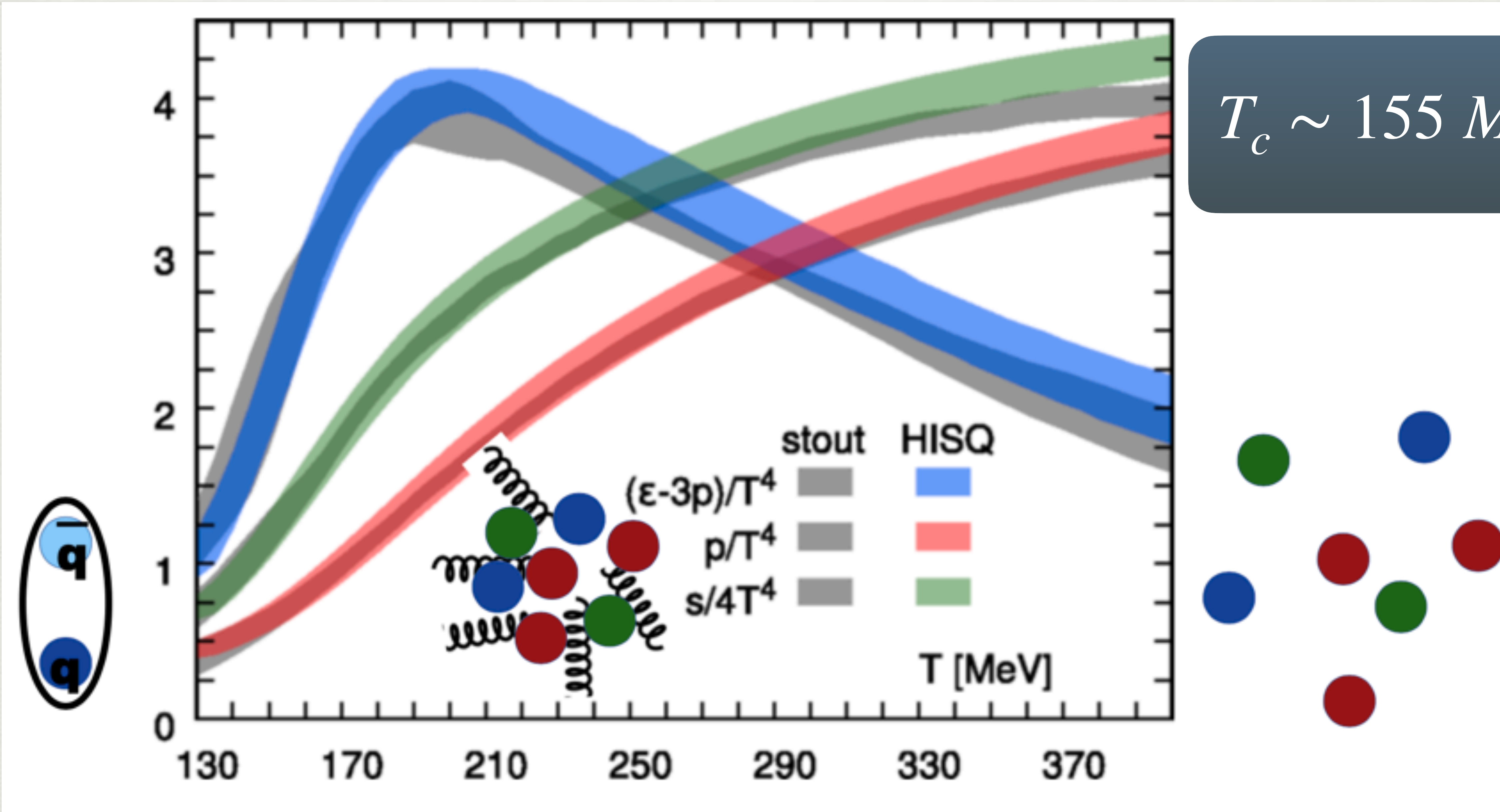
Before reaching the continuum limit (physical m_π), was difficult for HRG to match lattice. Not all hadrons included

2010 Hadron Resonance Gas almost matches Lattice QCD



Continuum limit significantly improved match with HRG

2014 Finally, the actions agree

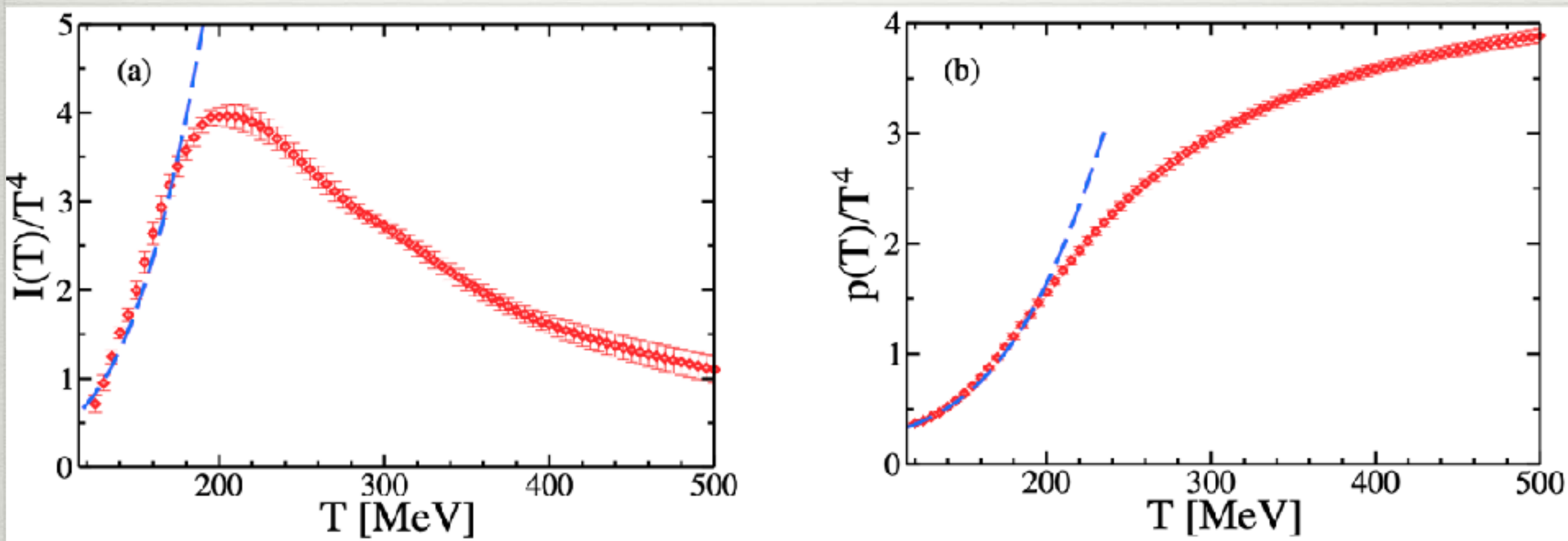


$T_c \sim 155\ MeV$

[WB] Phys.Lett. B730 (2014) 99-104 ; [HotQCD] Phys.Rev. D90 (2014) 094503

Agreement between different collaborations/actions (WB-stout and HotQCD-HISQ)

2014 Hadron resonance gas and lattice QCD finally fit?



Nucl.Phys. A929 (2014) 157-168

Once the continuum limit was reached, wealth of new observables were possible.

Does the HRG actually fit?

What's next?

- Tomorrow: Connecting the equation of state directly to data (out-of-equilibrium effects)
- Weds: Today we focused on $\mu_B = 0$ results, what happens at large baryon densities?
- What can't we get from lattice QCD?
- Tuesday & Wednesday: Open Questions

Summary
Lattice QCD at finite T, $\mu_B = 0$

- We now know precisely the equation of state for the QCD deconfinement phase transition
- This equation of state matches smoothly to a hadron resonance gas with a large number of particles
- The QCD phase transition is a smooth cross-over
- Light and strange particles appear to hadronize/freeze-out at different temperatures