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Lecture 2 on Hot QCD Matter: Heavy-Ion Collisions

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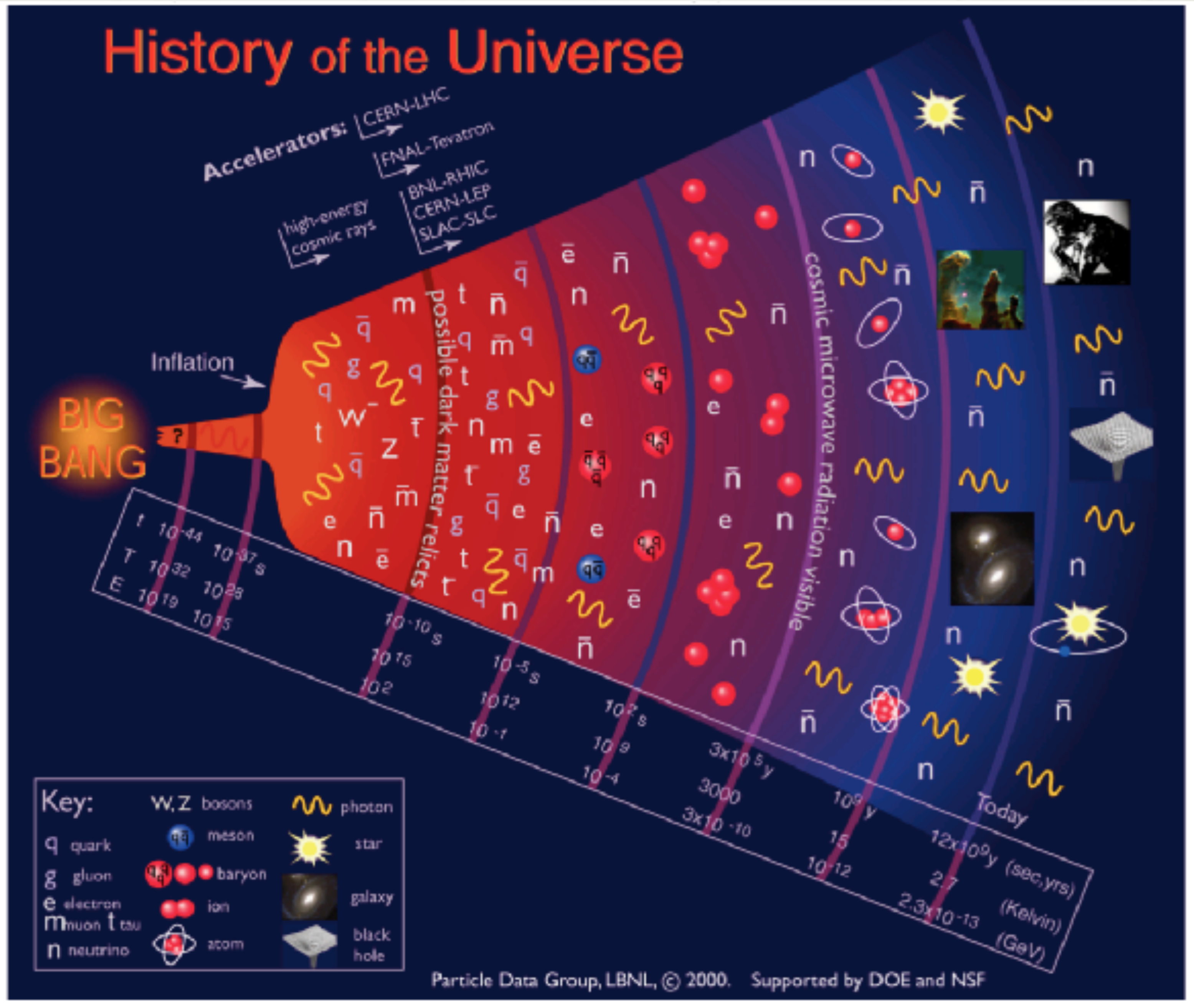
National Nuclear Physics Summer School
MIT 2022

Deconfined Quarks and Gluons in the Early Universe

$\sim 10^{-6}\ s$ after the Big Bang → Quark-Gluon Plasma

(1975 Collins and Perry)

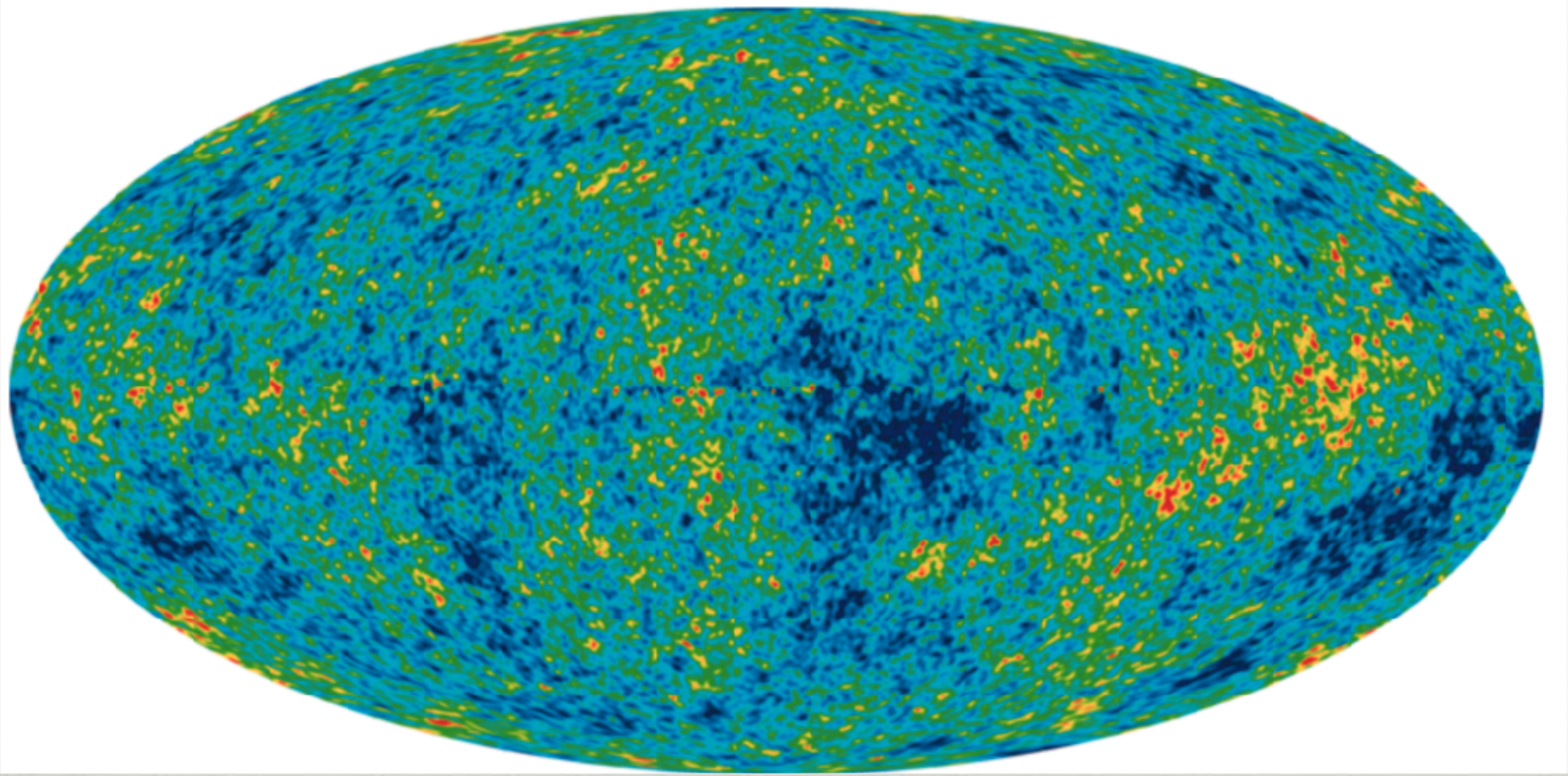


Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

How far back in time can we see?

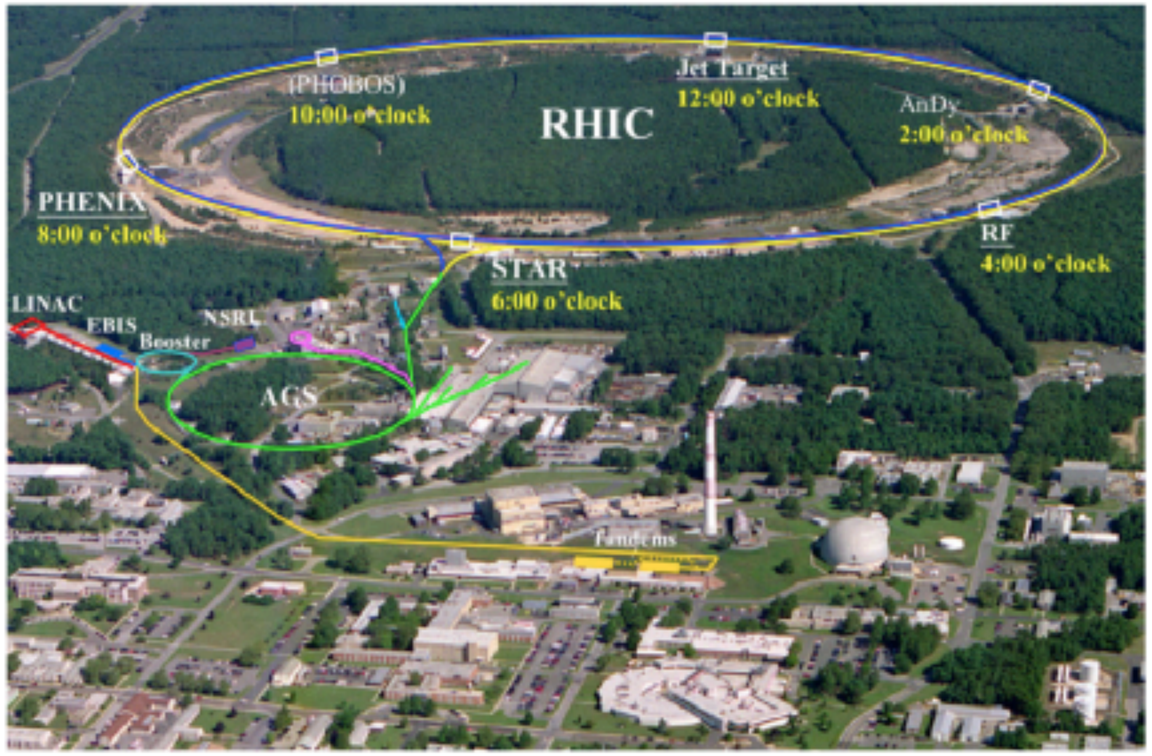
Cosmic Microwave Background ~ 10^5 **years** after Big Bang

Quark-Gluon Plasma existed ~ 10^{-6} **seconds**



Little Bangs in the Lab

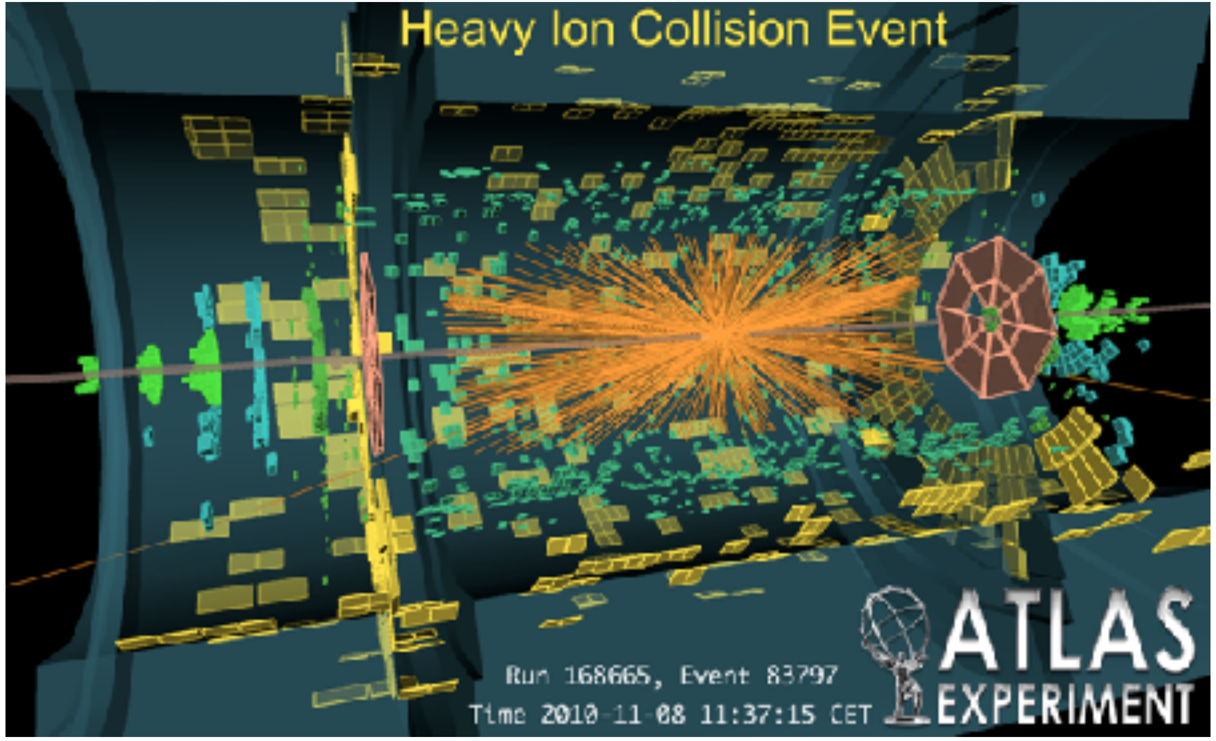
The Large Hadron Collider and RHIC create "little bangs”: deconfined quarks and gluons in the lab



T=4 trillion degrees Celsius

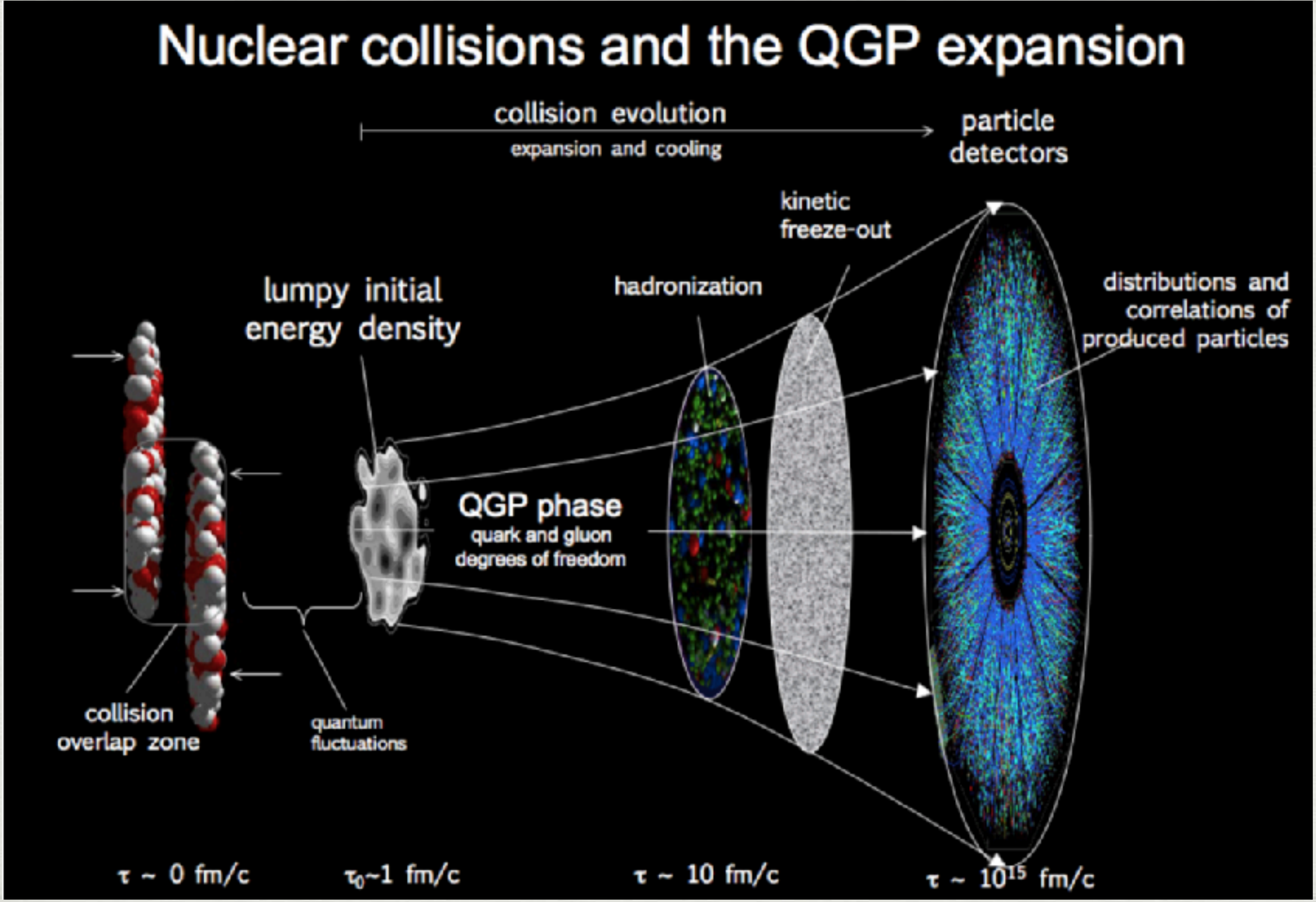
GUINNESS WORLD RECORDS

10^8 times hotter than the sun!

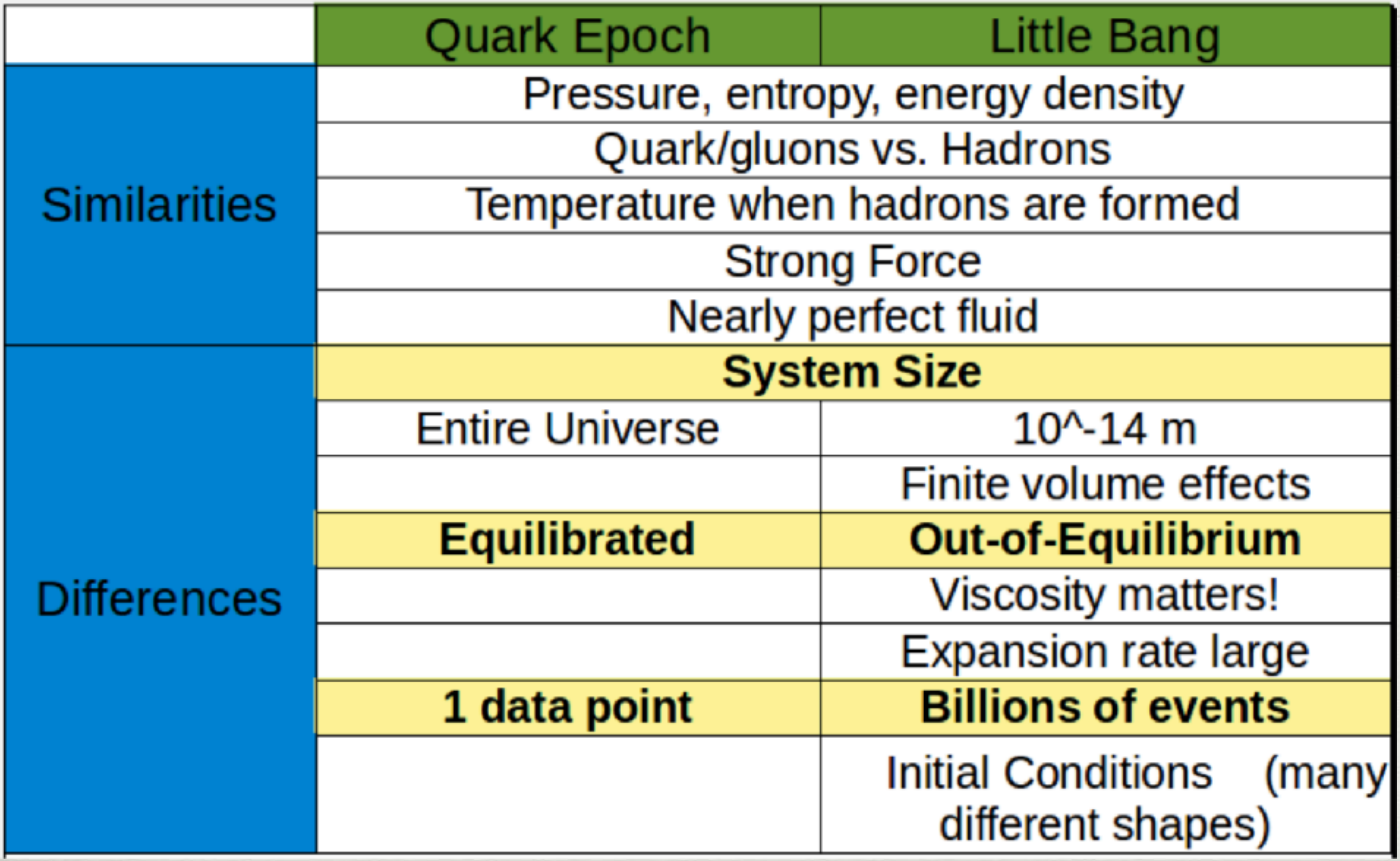


Evolution of a heavy-ion collision

Smashing two gold ions at the speed of light



Big Bang vs. Heavy-Ion Collisions



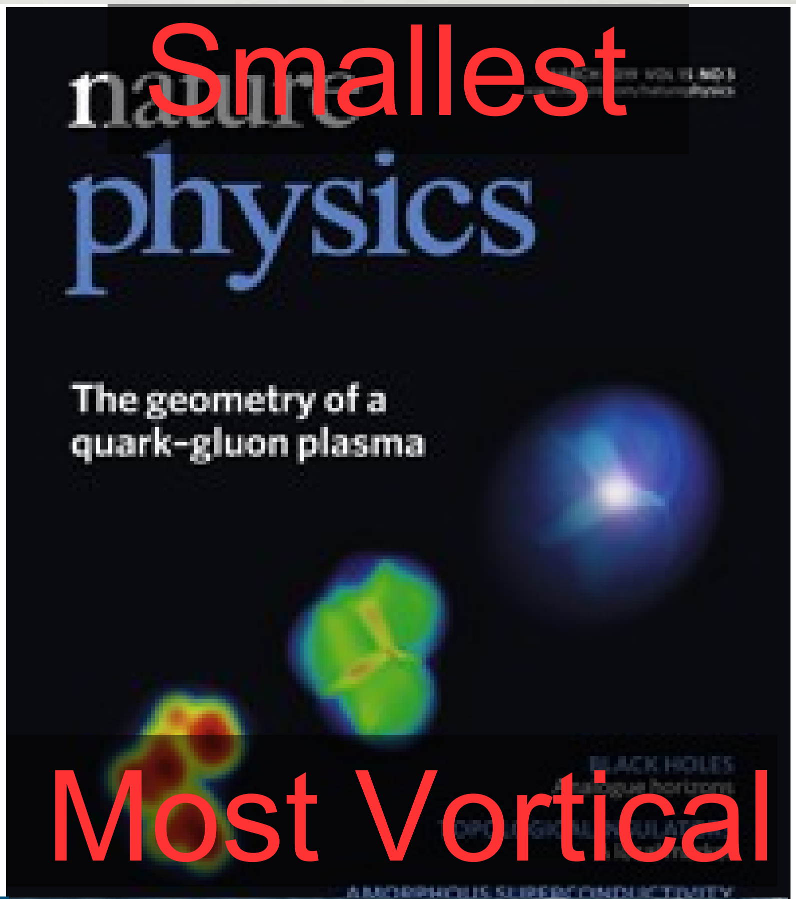
<table>
<tr><th></th><th>Quark Epoch</th><th>Little Bang</th></tr>
<tr><td rowspan="5">Similarities</td><td colspan="2">Pressure, entropy, energy density</td></tr>
<tr><td colspan="2">Quark/gluons vs. Hadrons</td></tr>
<tr><td colspan="2">Temperature when hadrons are formed</td></tr>
<tr><td colspan="2">Strong Force</td></tr>
<tr><td colspan="2">Nearly perfect fluid</td></tr>
<tr><td rowspan="9">Differences</td><td colspan="2">System Size</td></tr>
<tr><td>Entire Universe</td><td>10^-14 m</td></tr>
<tr><td></td><td>Finite volume effects</td></tr>
<tr><td>Equilibrated</td><td>Out-of-Equilibrium</td></tr>
<tr><td></td><td>Viscosity matters!</td></tr>
<tr><td></td><td>Expansion rate large</td></tr>
<tr><td>1 data point</td><td>Billions of events</td></tr>
<tr><td></td><td>Initial Conditions (many different shapes)</td></tr>
</table>

Collisions of fluids vs. solids

The Quark Gluon Plasma is the ...

Hottest

Smallest



physics

The geometry of a quark-gluon plasma

BLACK HOLES

Analogue horizons

AMORPHOUS SUPERCONDUCTIVITY

Most Perfect

ANTICANCER BLOCKBUSTER? • RISE AND FALL OF THE SLIDE RULE

SCIENTIFIC AMERICAN

Bringing DNA Computers to Life

MAY 2006
WWW.SCIAM.COM

Quark Soup

PHYSICISTS RE-CREATE THE LIQUID STUFF OF **THE EARLIEST UNIVERSE**

Stopping **Alzheimer's**

Birth of **the Amazon**

Future **Giant Telescopes**

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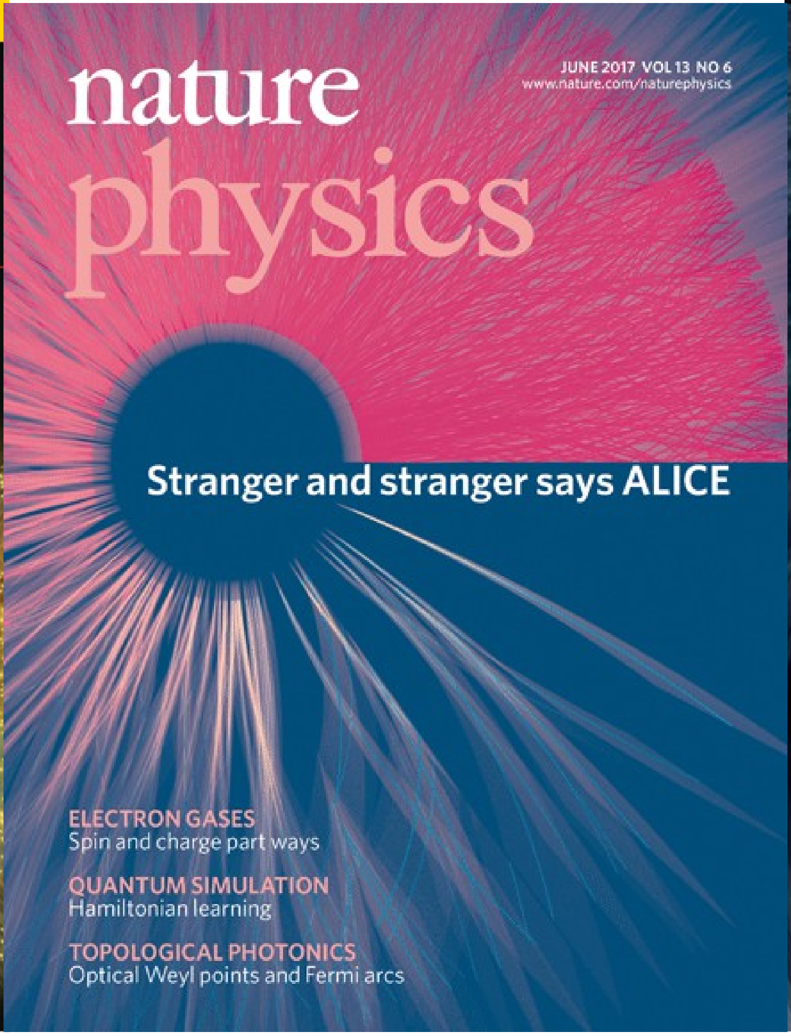
Strange



nature physics

JUNE 2017 VOL 13 NO 6
www.nature.com/naturephysics

Stranger and stranger says ALICE

ELECTRON GASES
Spin and charge part ways

QUANTUM SIMULATION
Hamiltonian learning

TOPOLOGICAL PHOTONICS
Optical Weyl points and Fermi arcs

Most Vortical

nature

THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

First observation of fluid vortices formed by heavy-ion collisions
PAGES 34 & 62

SUBATOMIC SWIRLS

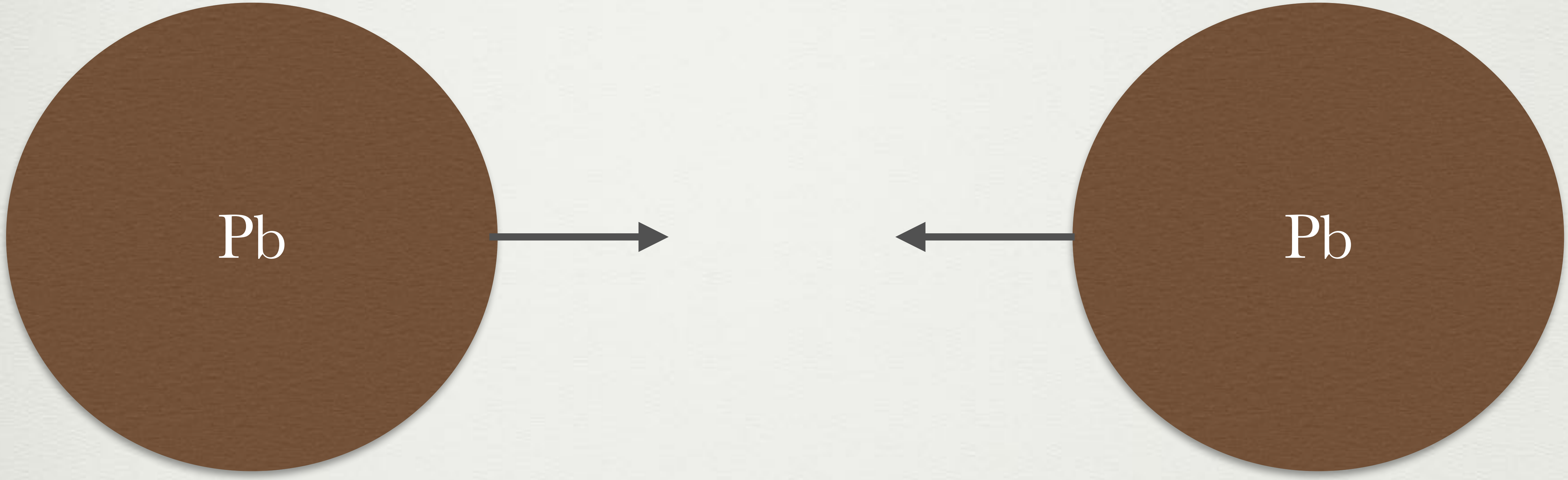
CLIMATE CHANGE
PARIS AGREEMENT
Time for nations to match words with deeds
PAGE 25

BOOKS
SUMMER SELECTION
Recommended reading for the holiday season
PAGE 28

Fluid

Kinematics of Heavy-Ion collisions

How do heavy-ions collide?



Pb nucleus $A = 208$

Center of mass energy *per nucleon* $\sqrt{s_{NN}} = 5.02$ TeV

Collider mode: $s = (E + E) = 4E^2$

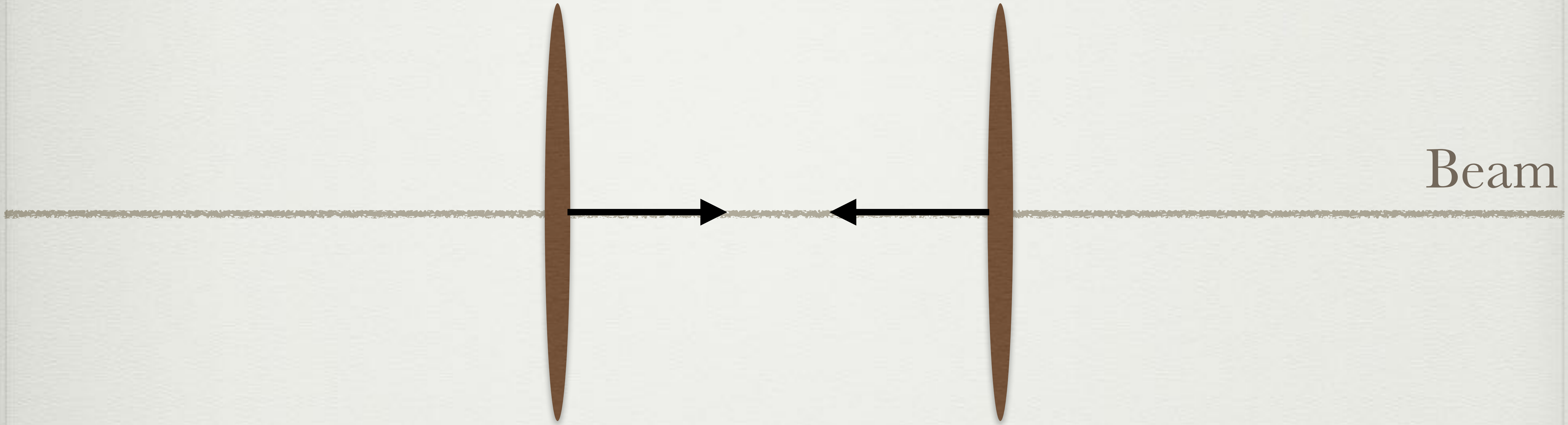
Radius along beam

$$E = \gamma m_N c^2$$

$$L = L_0/\gamma$$

$$R_{beam} = \frac{2R_0 m_N}{\sqrt{s}}$$

How do heavy-ions collide?

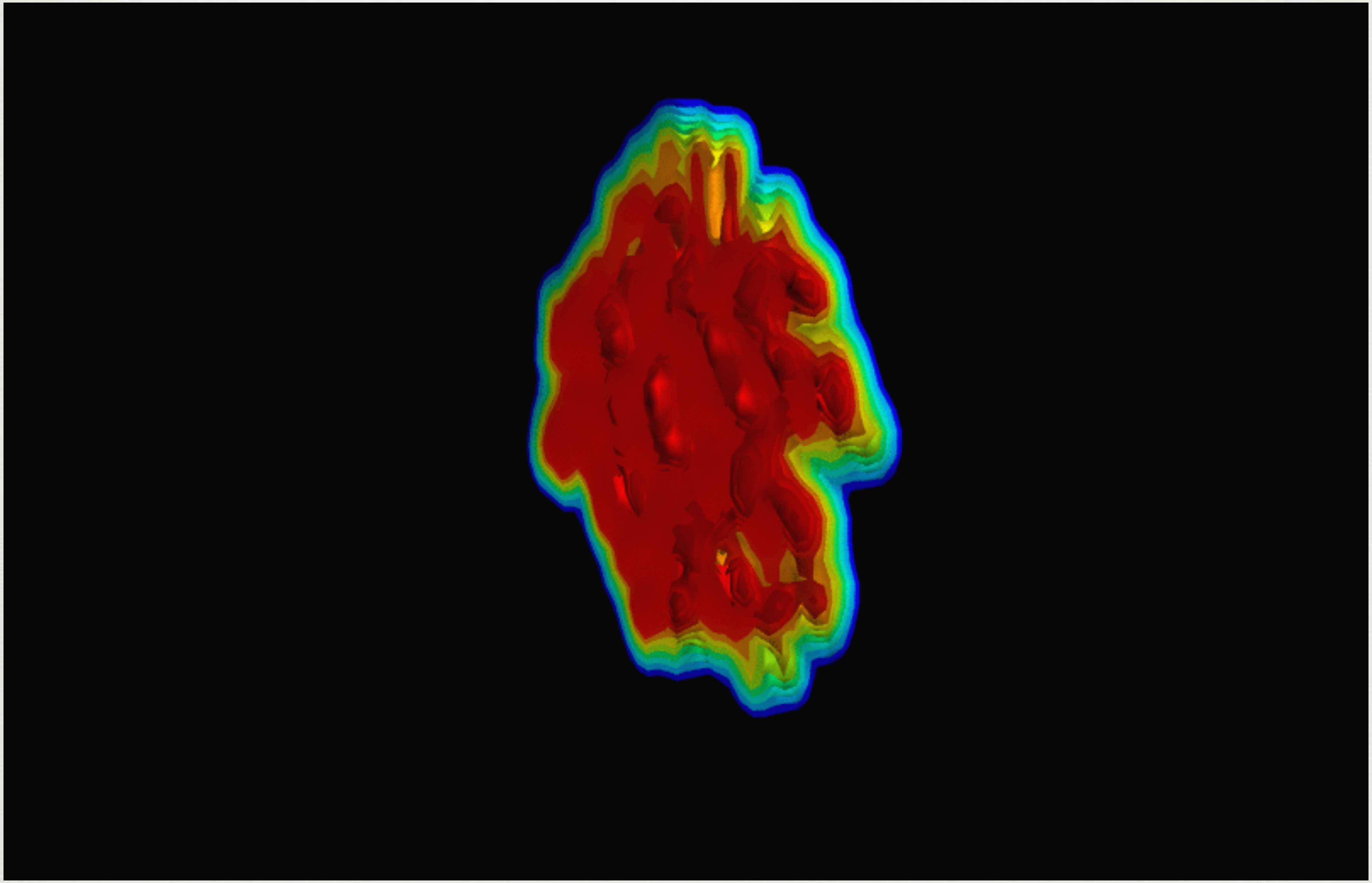


Radius along beam $R_{beam} = \dfrac{2R_0 m_N}{\sqrt{s}}$

$R_{Pb} \sim 6$ fm $= 1183.8$ MeV

$R_{beam} \sim 0.002$ fm

Heavy-ions: Beam direction



Schenke IP-Glasma+MUSIC

Boost-invariance: 3+1 vs 2+1D

Heavy-ion collisions are **boost-invariant-** identical slices along longitudinal direction (z)

$$v_z = \frac{z}{t}$$

Thus, we talk about the proper time τ and space-time rapidity y

$$t = \tau \cosh y,$$

$$z = \tau \sinh y,$$

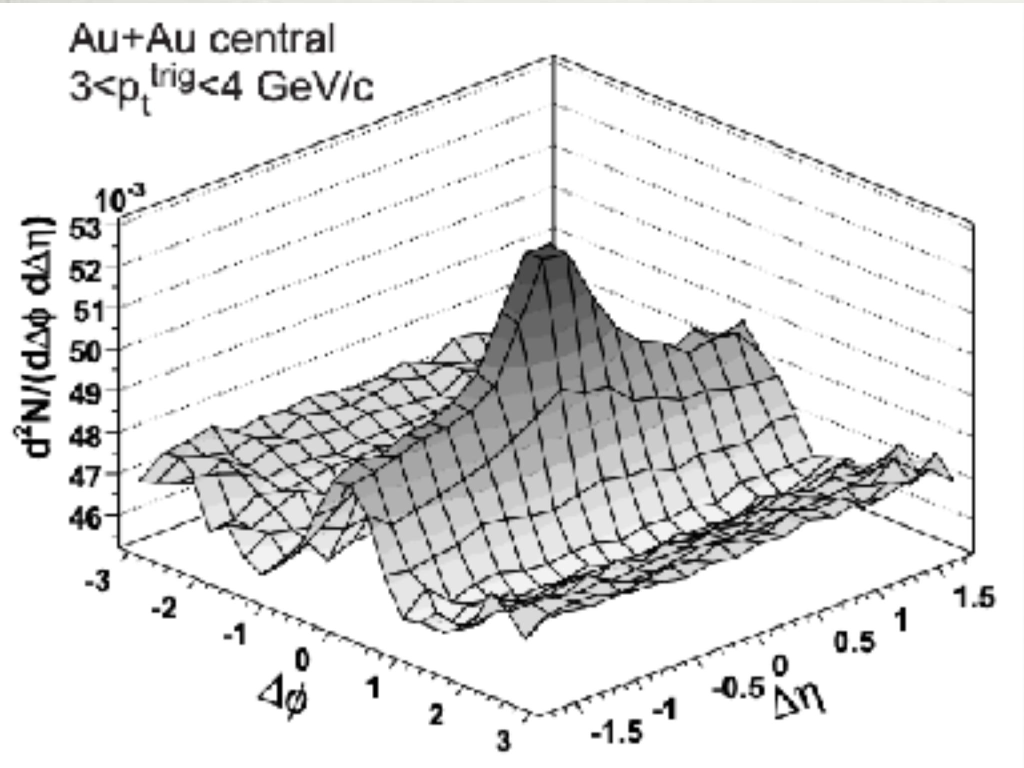
$$\tanh y = \frac{z}{t}$$

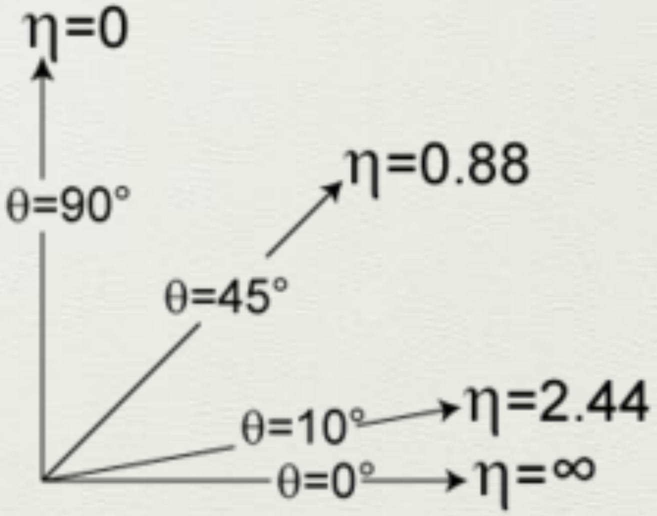
Pseudo-rapidity *η*

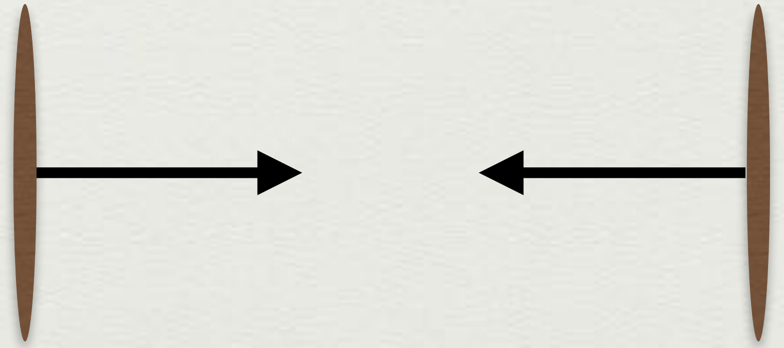
When a particle is traveling at the speed of light (massless) $y \sim \eta$

For massive particles, use **pseudo-rapidity** *η*

$$y = \ln\left(\frac{\sqrt{m^2 + p_{\mathrm{T}}^2 \cosh^2\eta} + p_{\mathrm{T}} \sinh\eta}{\sqrt{m^2 + p_{\mathrm{T}}^2}}\right),$$

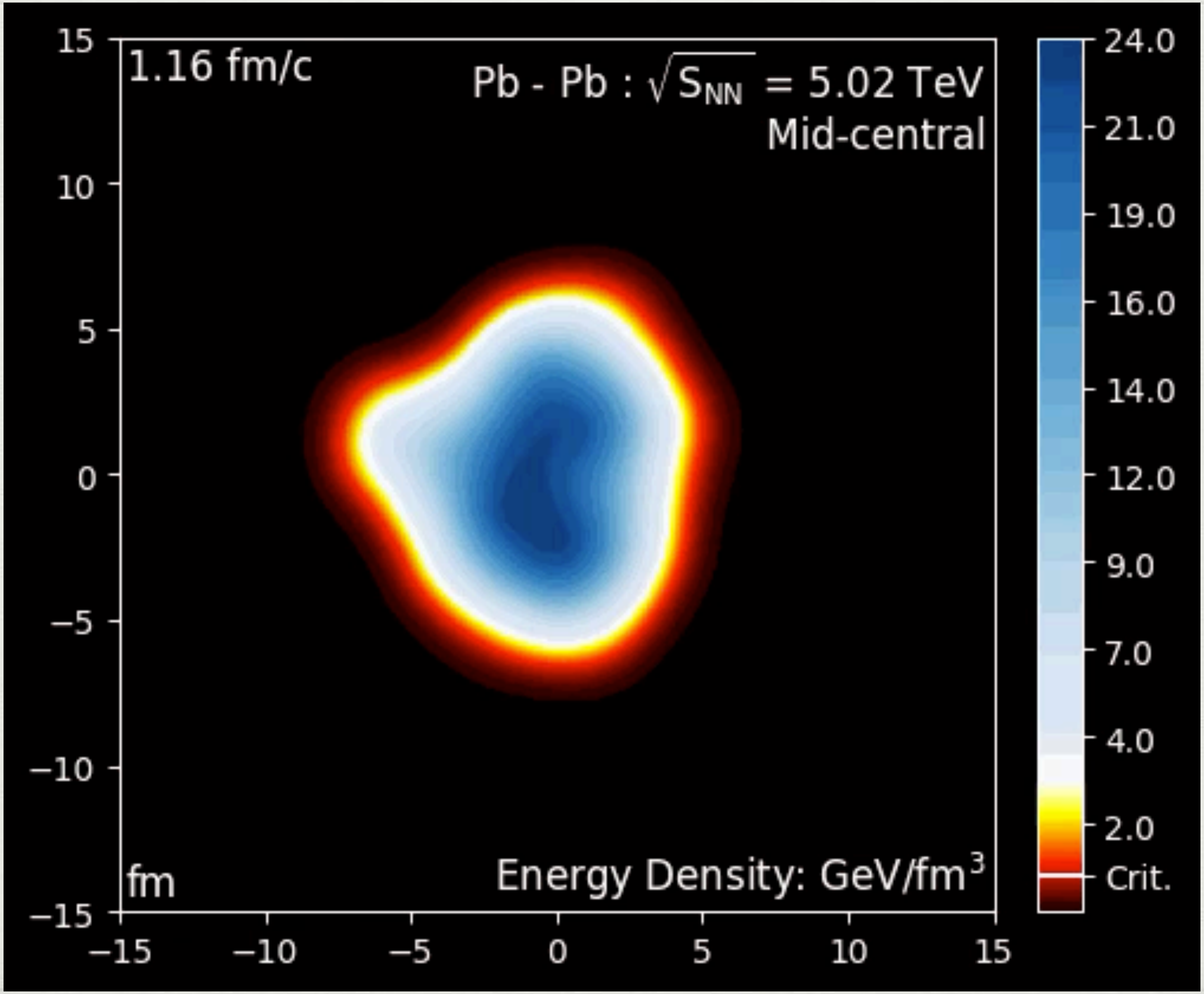




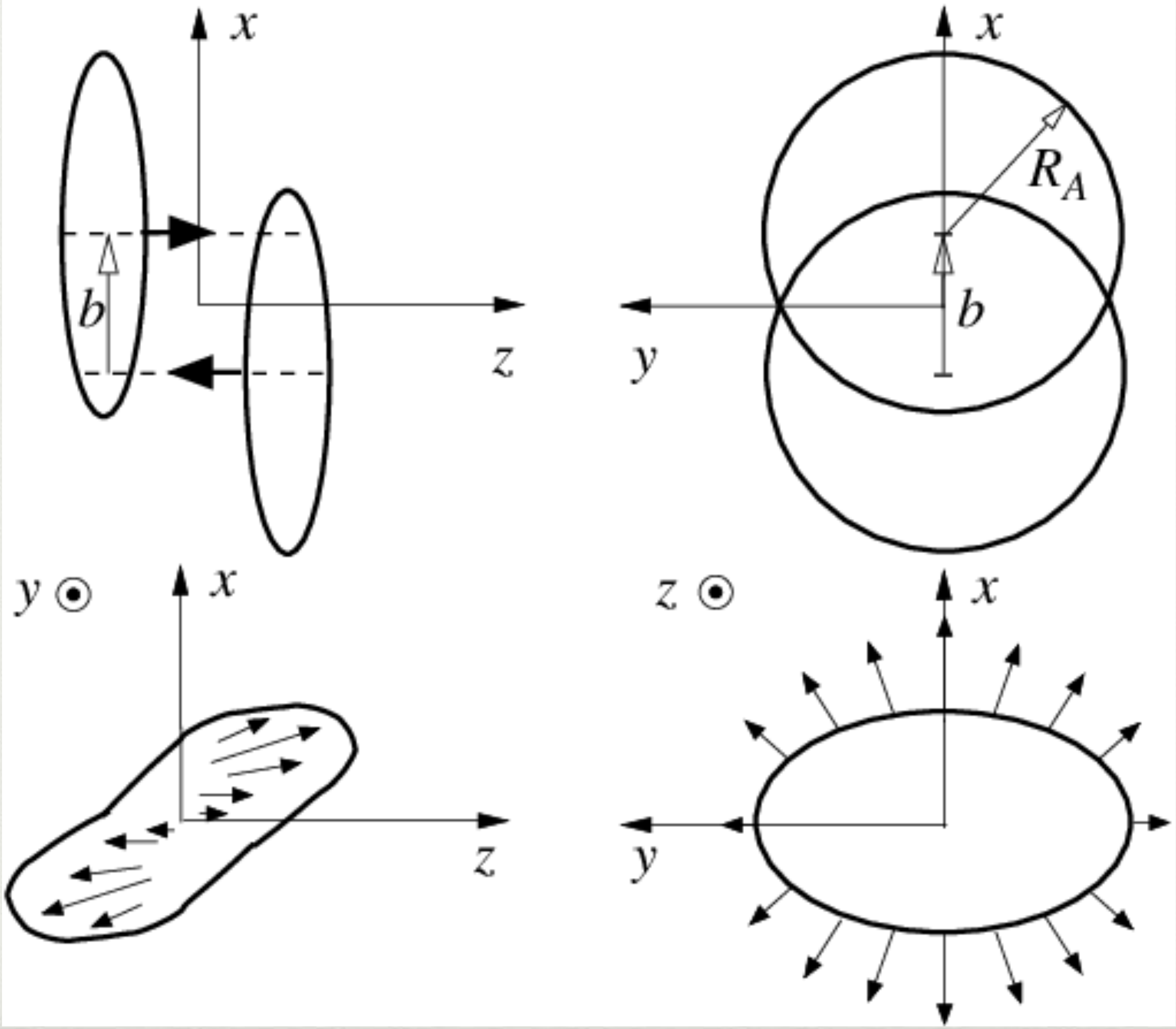


Beam

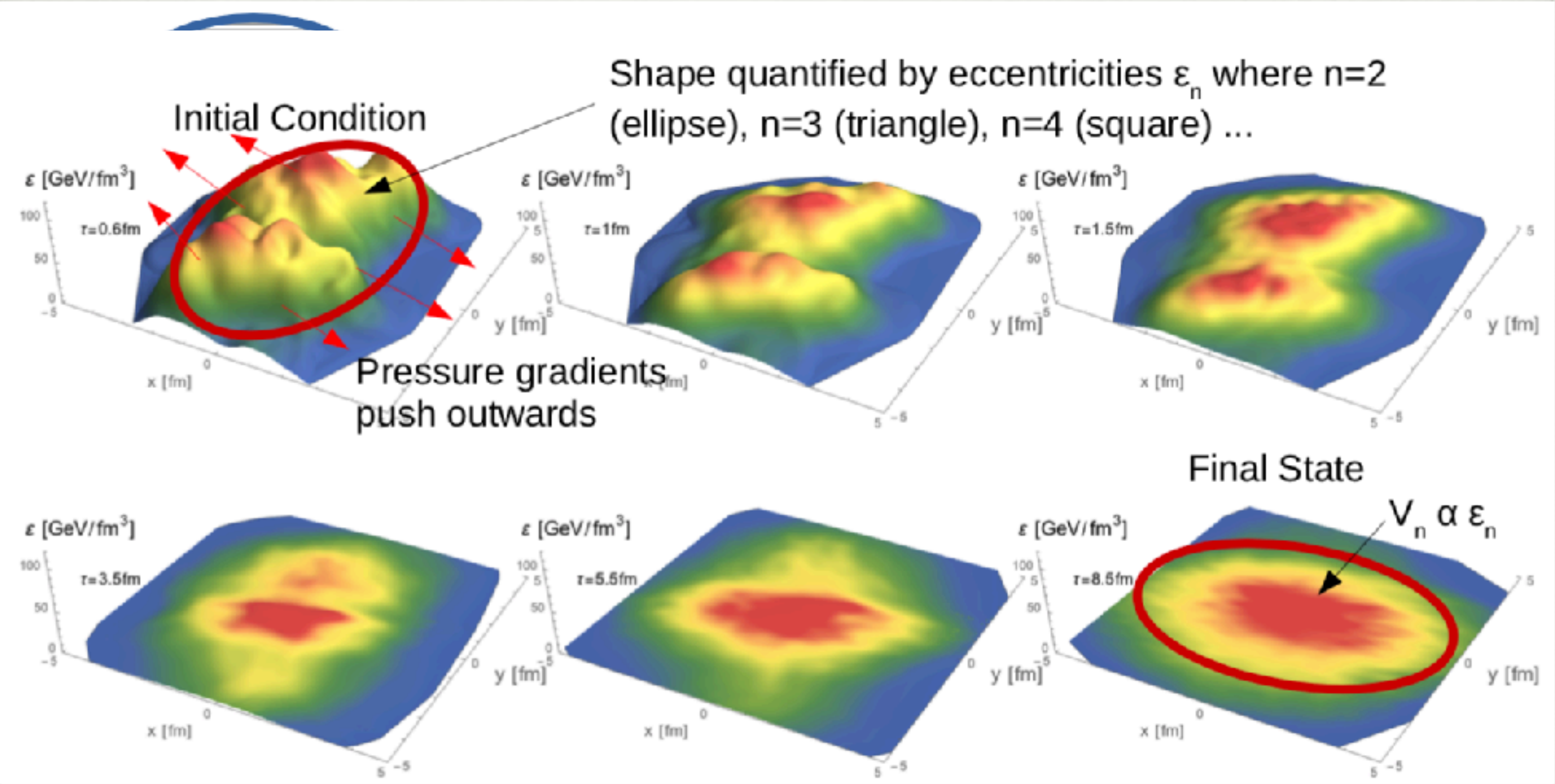
Heavy-ions: Transverse plane



Transverse plan vs. longitudinal



Initial conditions

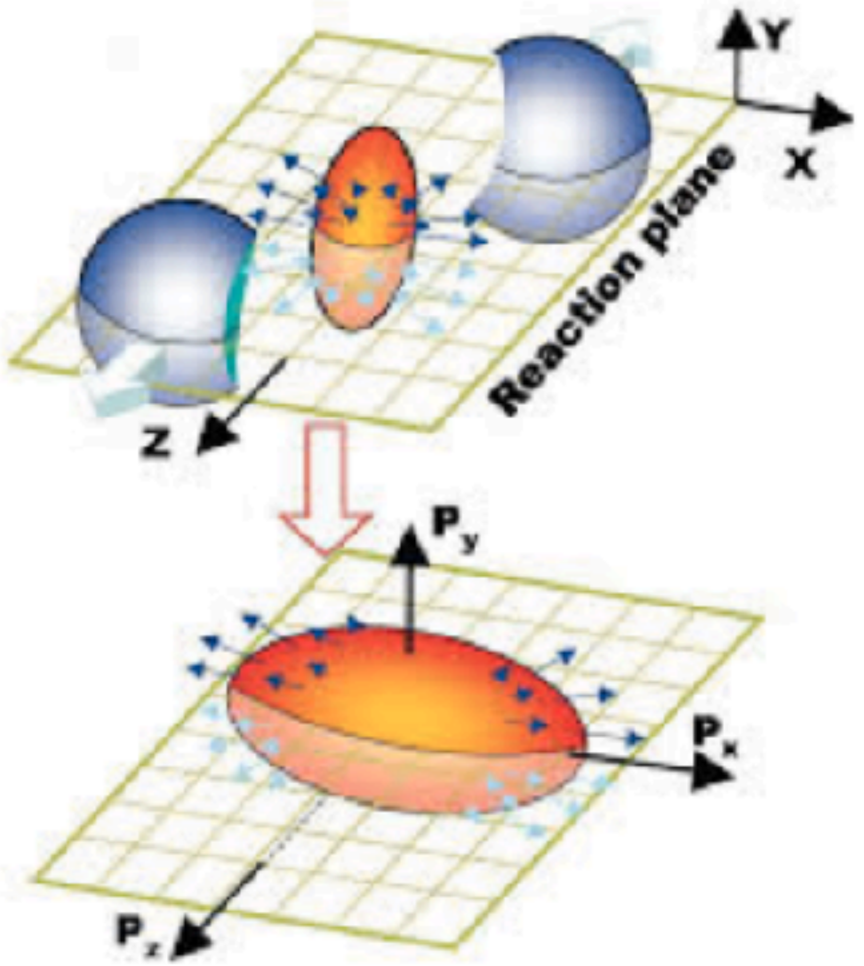


Shape quantified by eccentricities ε_n where n=2 (ellipse), n=3 (triangle), n=4 (square) ...

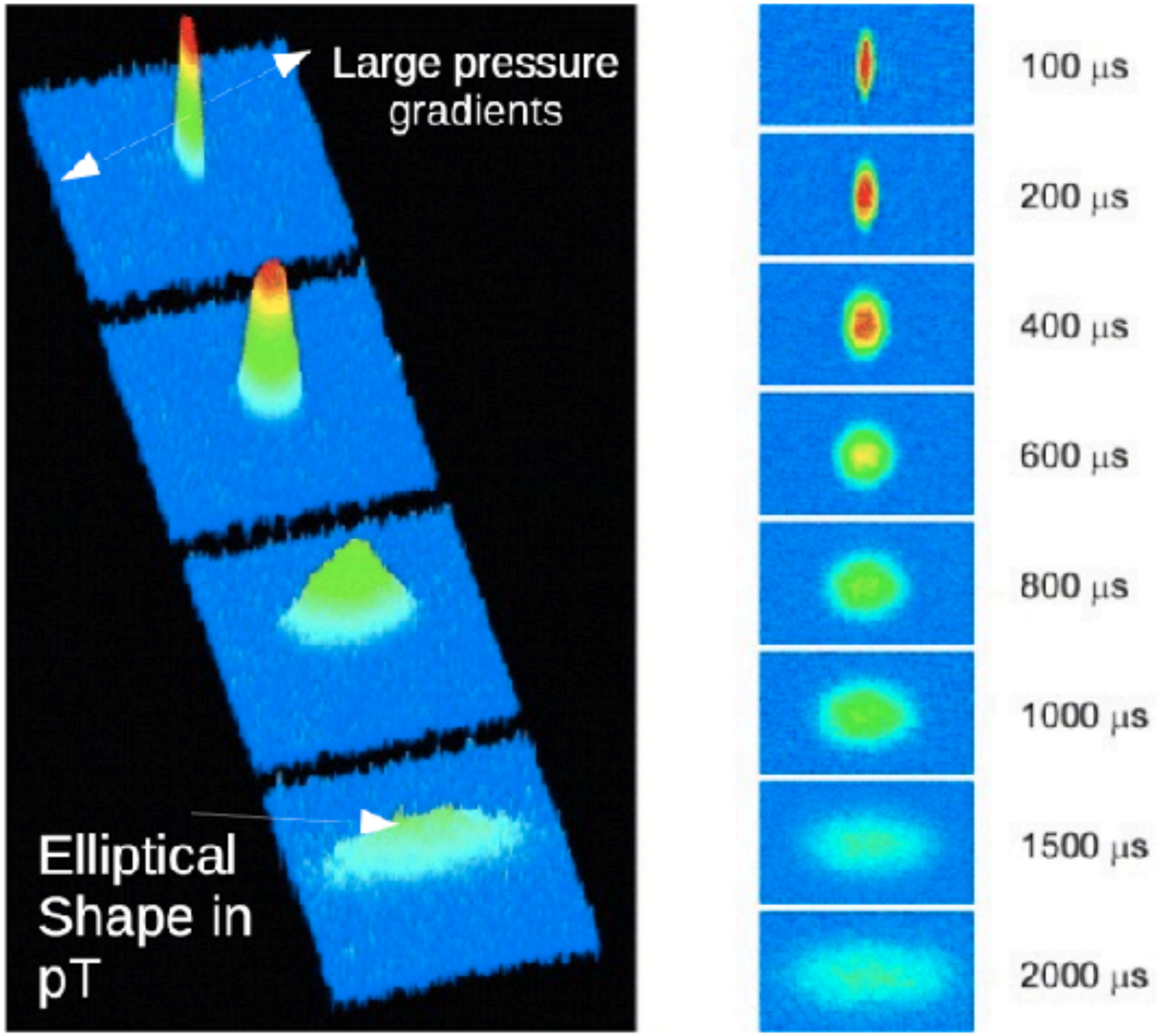
Eccentricities ε_2's are directly related to the final measured flow observables v_n's

How 2 heavy-ions collide

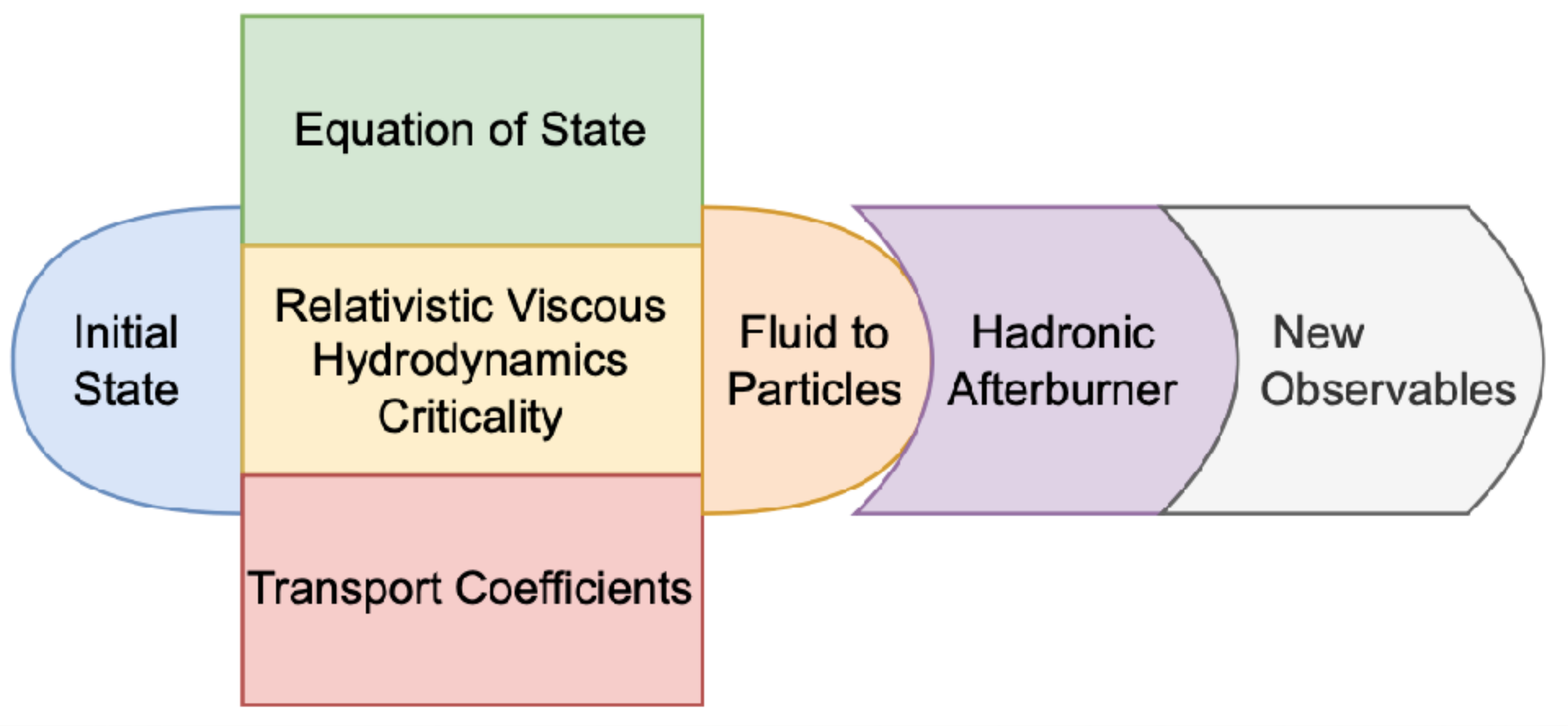
Assuming Gold ions are spheres...



Cold Atoms



Standard Model of HIC



Equation of State

Initial State

Relativistic Viscous Hydrodynamics Criticality

Fluid to Particles

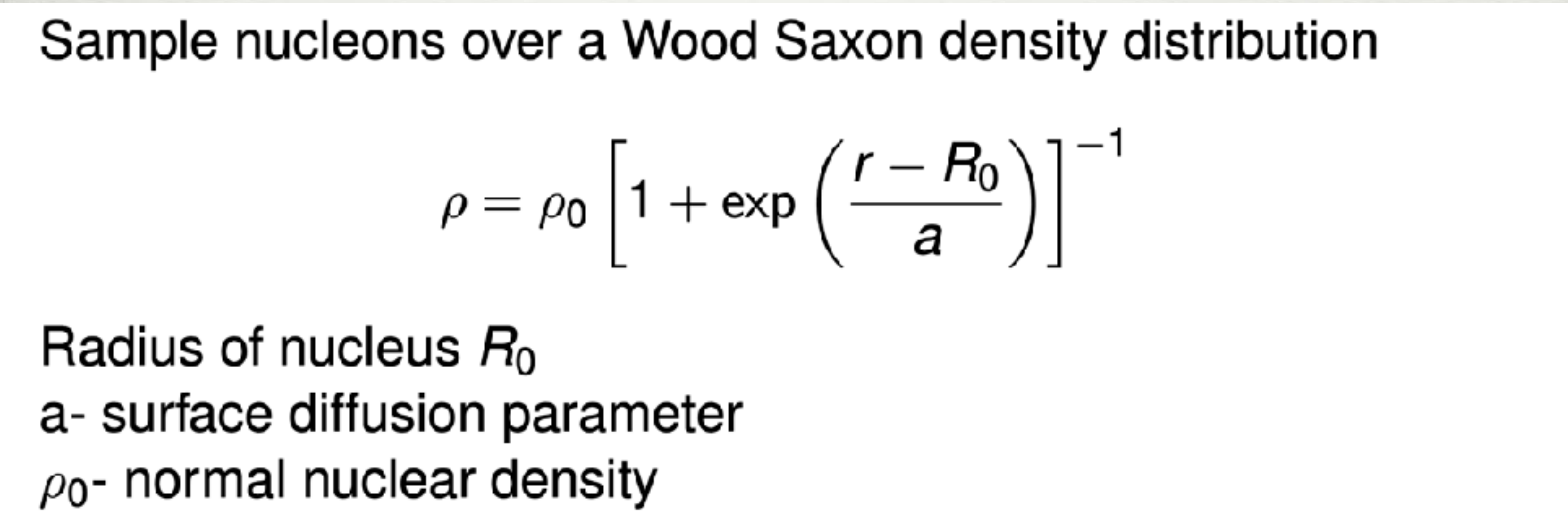
Hadronic Afterburner

New Observables

Transport Coefficients

Initial Conditions

Wood Saxon



Sample nucleons over a Wood Saxon density distribution

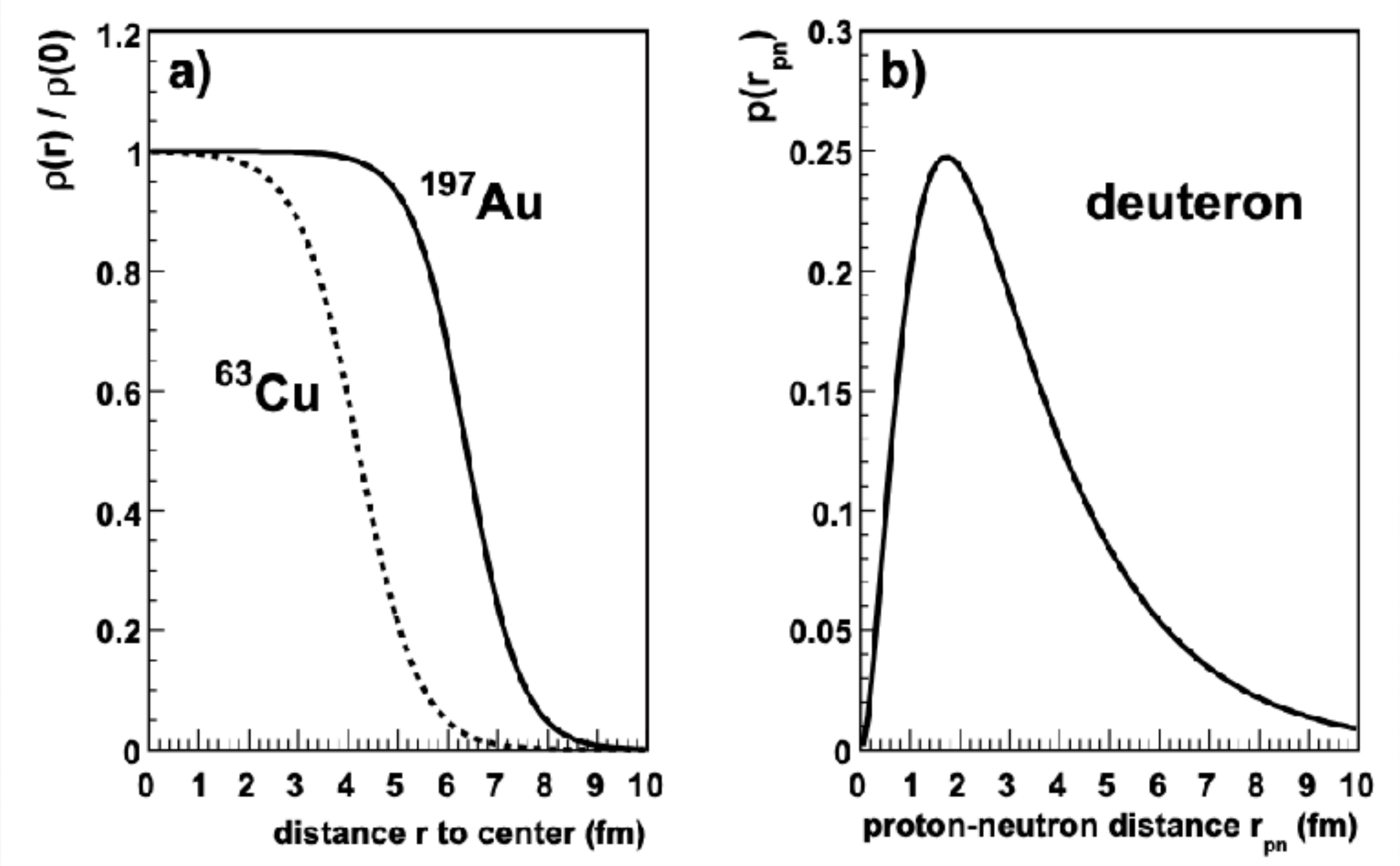
$$\rho = \rho_0 \left[1 + \exp\left(\frac{r - R_0}{a} \right) \right]^{-1}$$

Radius of nucleus R_0
a- surface diffusion parameter
ρ_0- normal nuclear density

We also consider deformed nuclei where $R_0 \neq const$

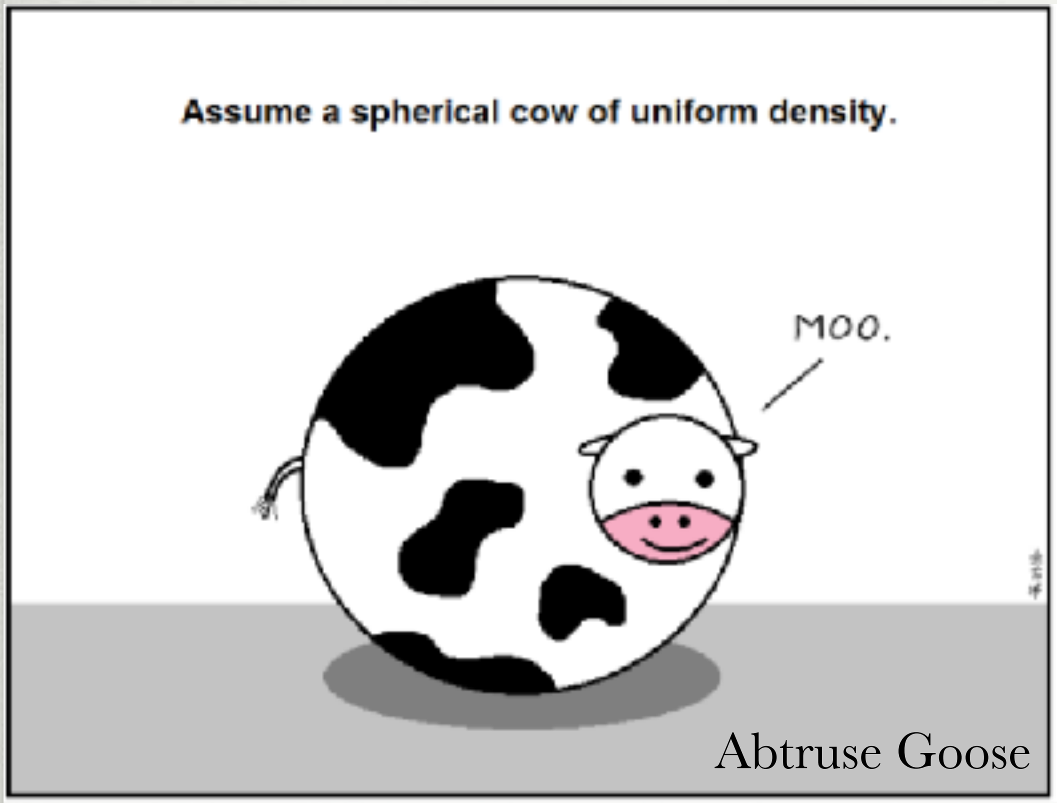
Alternative: nucleon configurations from low-energy nuclear structure calculations

Wood-Saxon continued

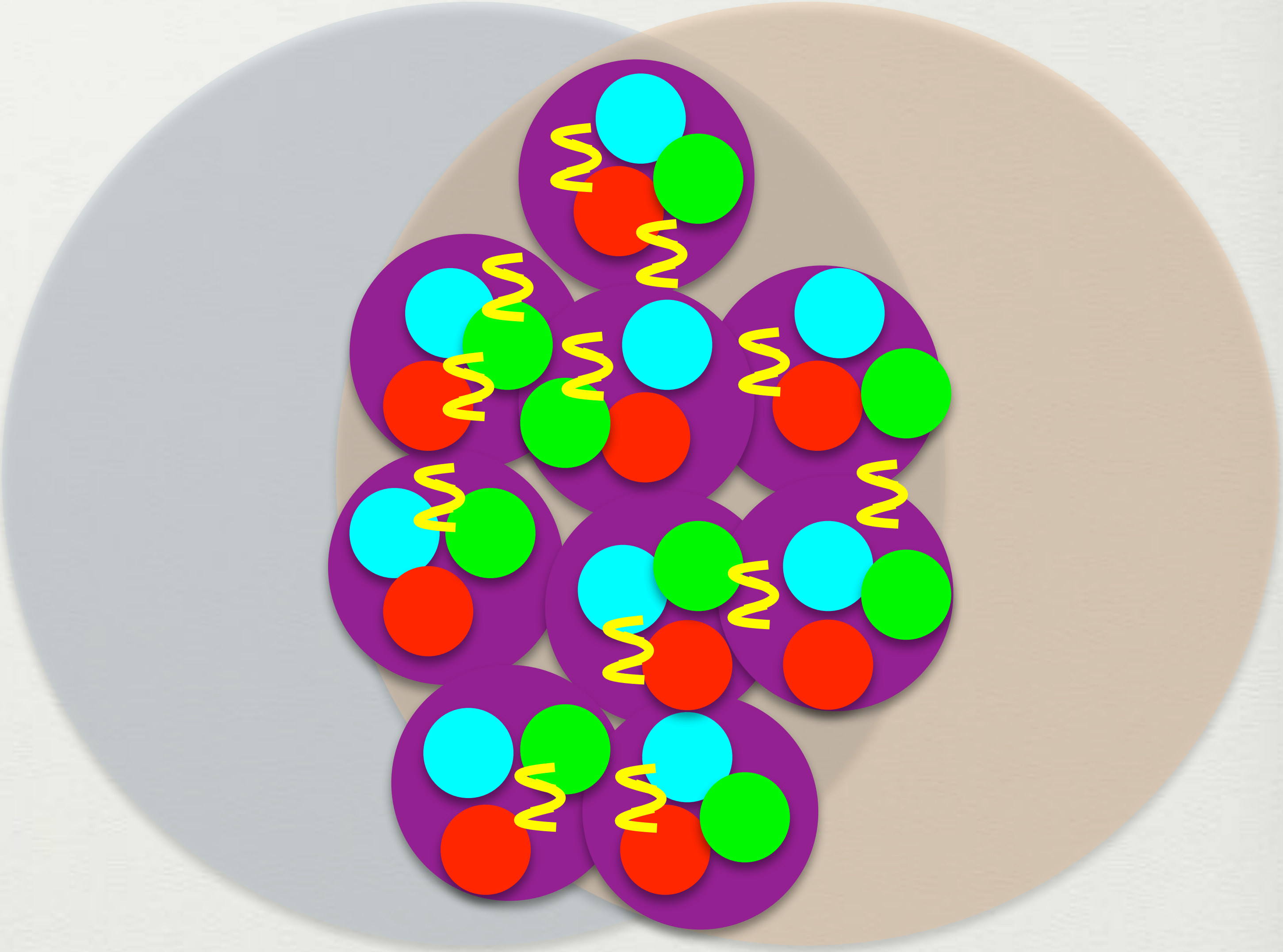


Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243

The spherical cow has bumps:
From optical Glauber to event-by-event fluctuations

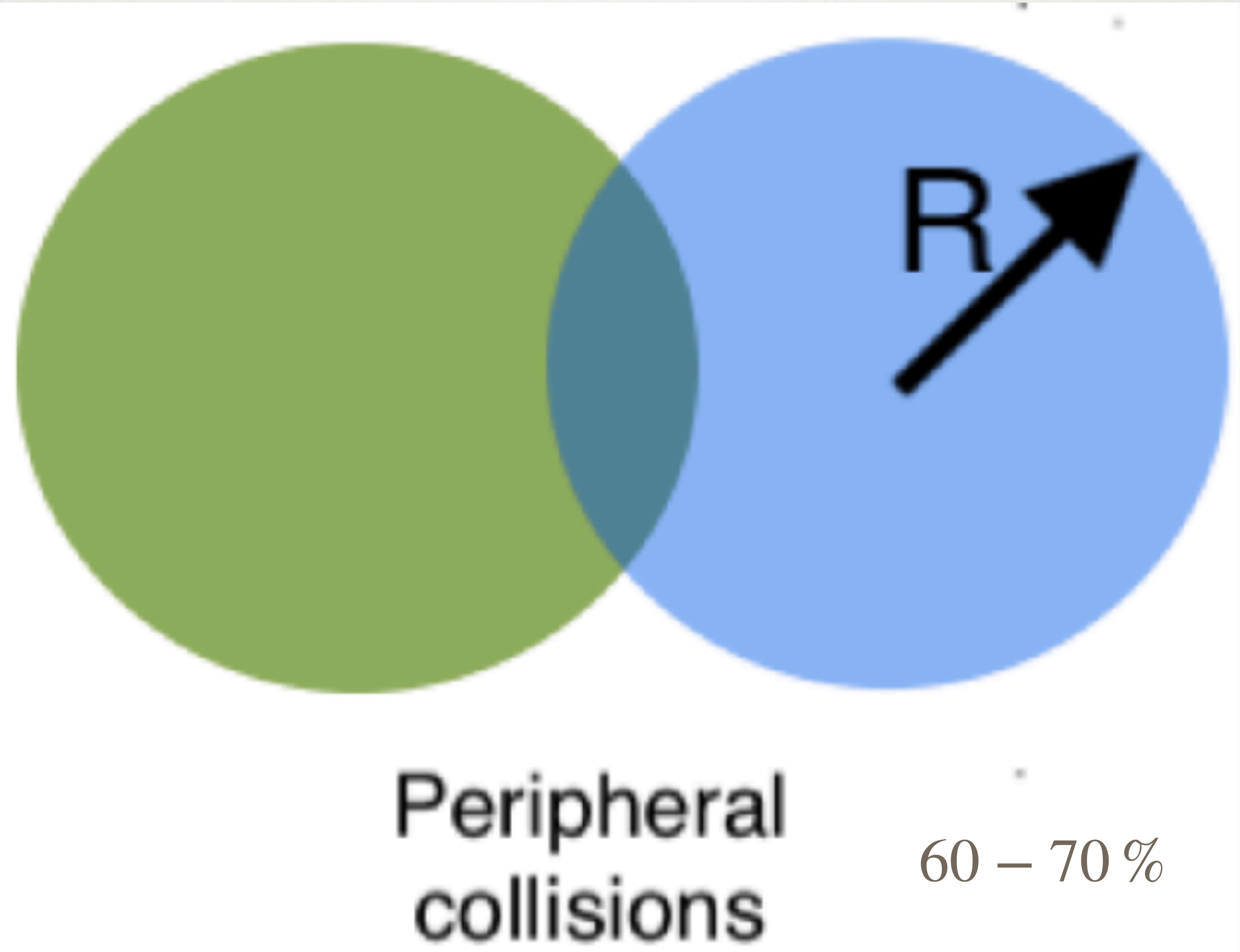


Abtruse Goose



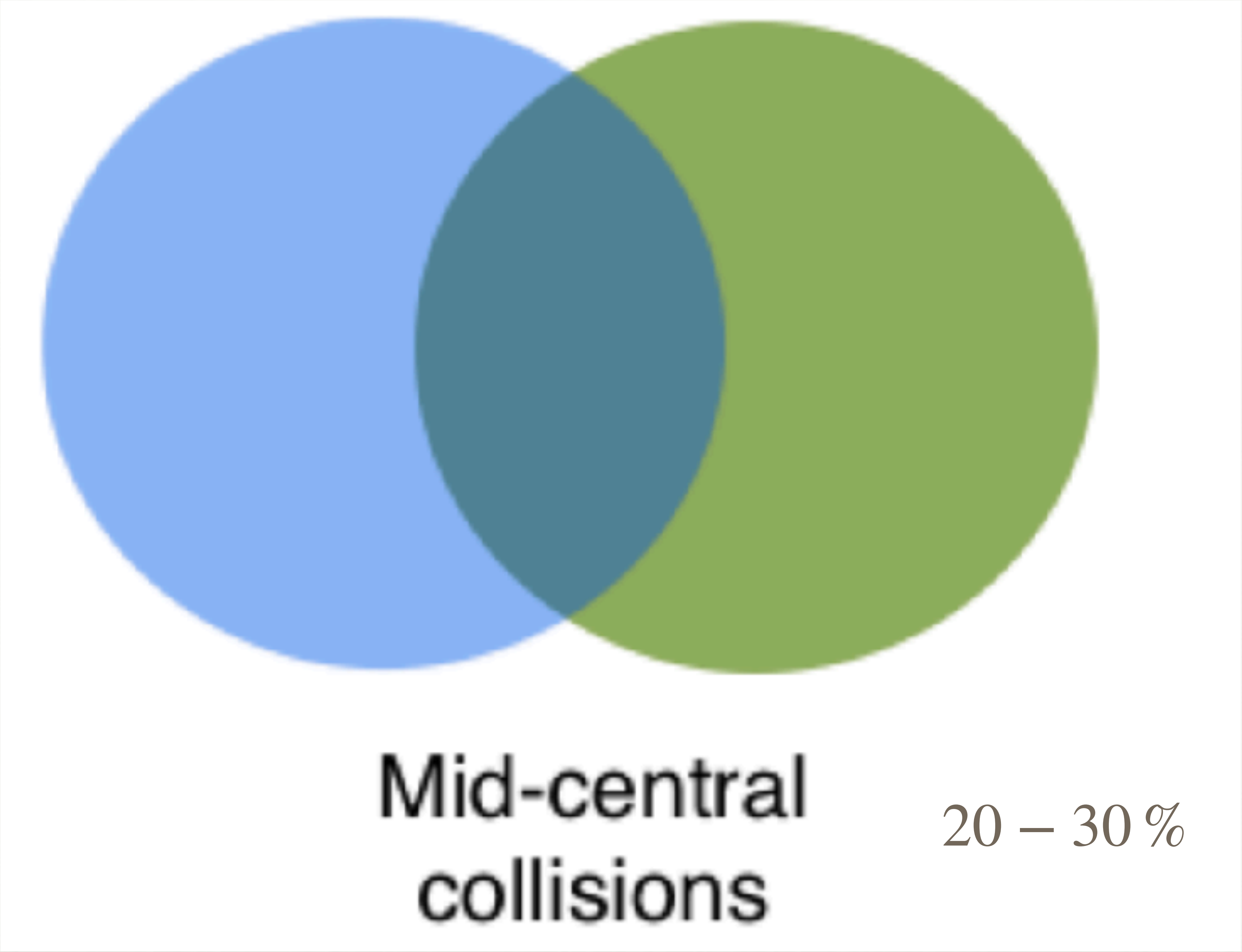
Sub-nucleonic Fluctuations

Centrality circa 2000’s



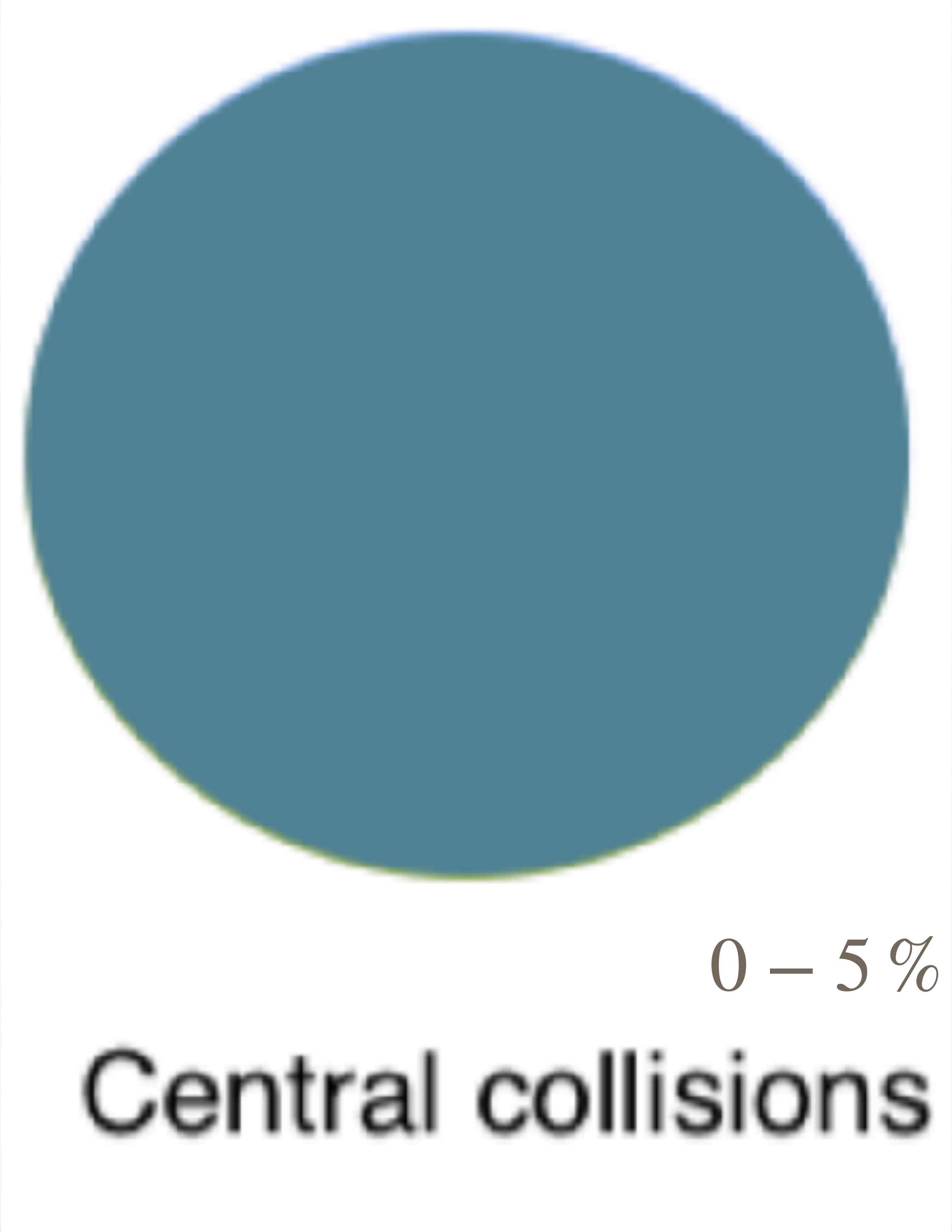
60 – 70 %

Centrality circa 2000's

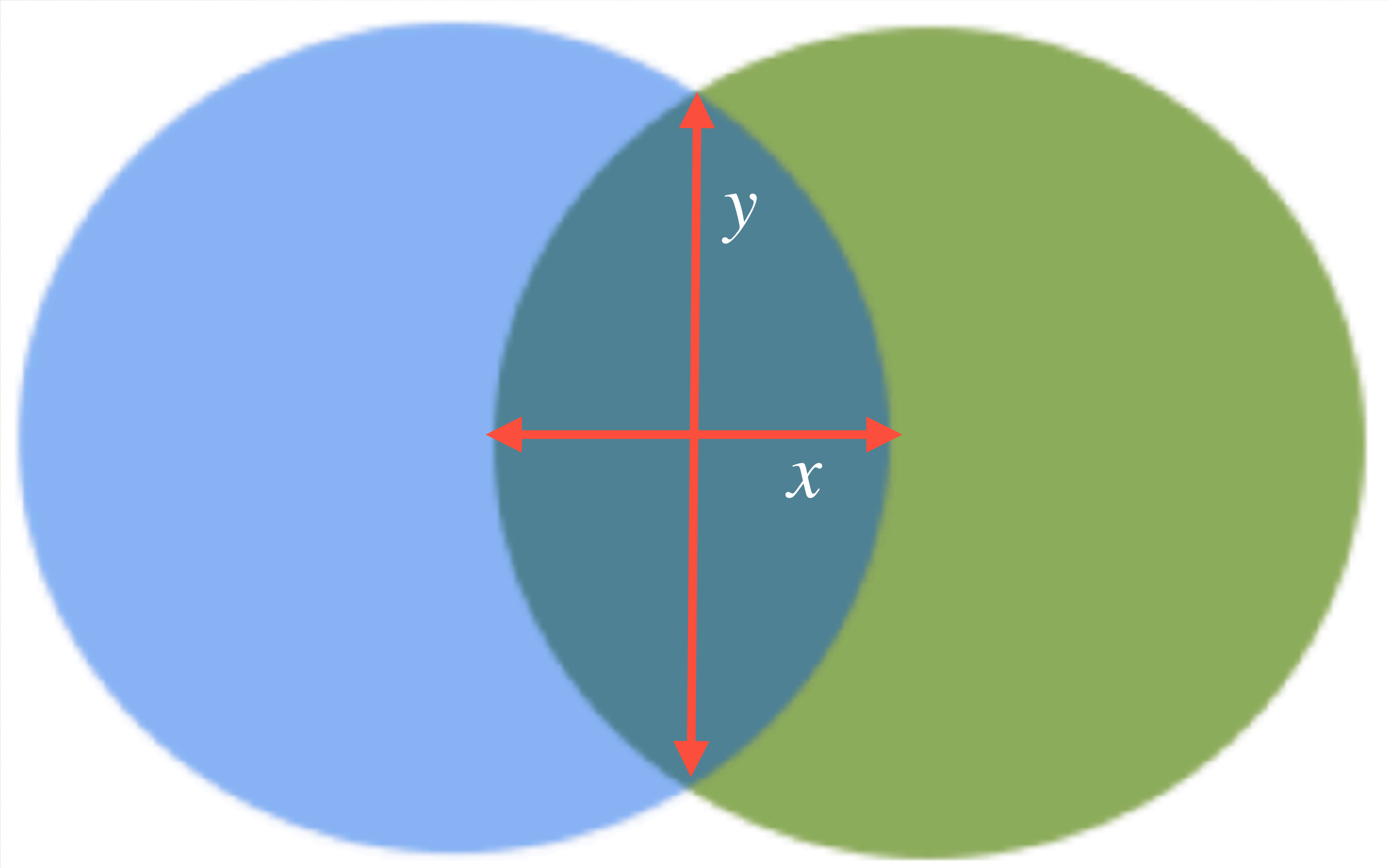


20 – 30 %

Centrality circa 2000’s



Eccentricity early 2000's

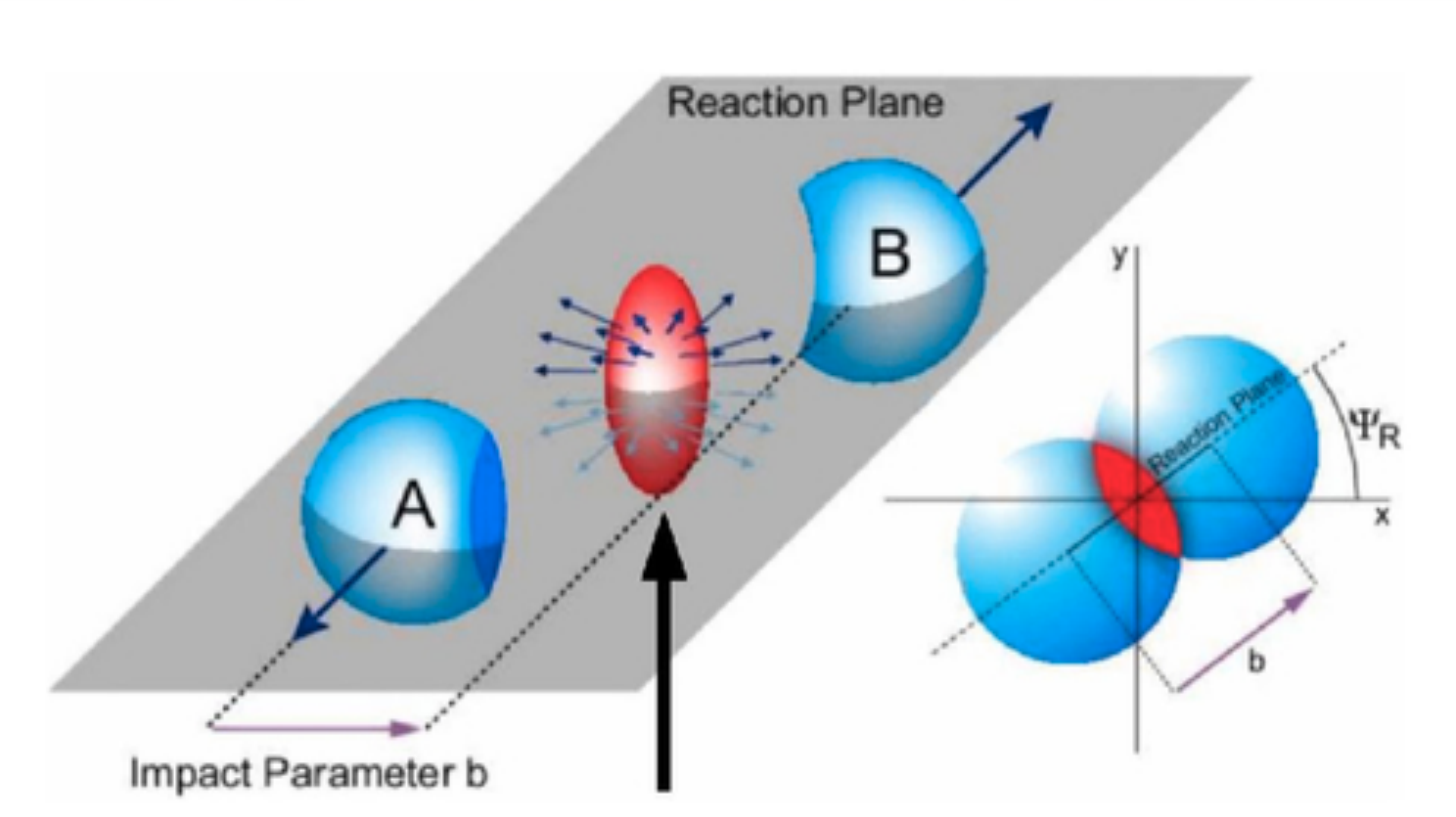


$$\varepsilon_n = \frac{\{y^2 - x^2\}}{\{y^2 + x^2\}}$$

$$\{\ldots\} \equiv \frac{\int d^3x e(\overrightarrow{x})\ldots}{\int d^3x e(\overrightarrow{x})}$$

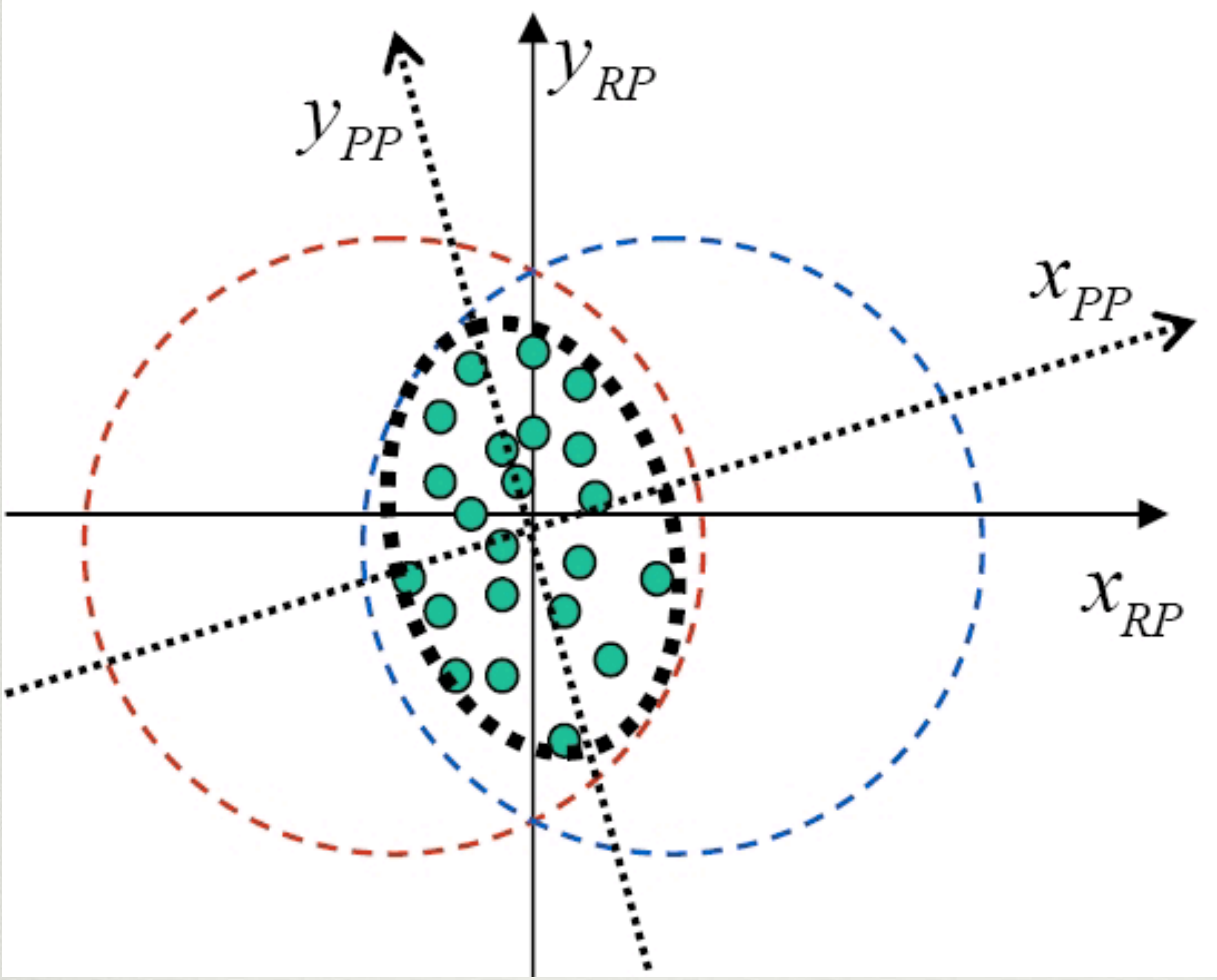
Defined with impact parameter

Collision: Lorentz contracted



Collisions impact parameter *b*, and reaction plane angle ψ_R

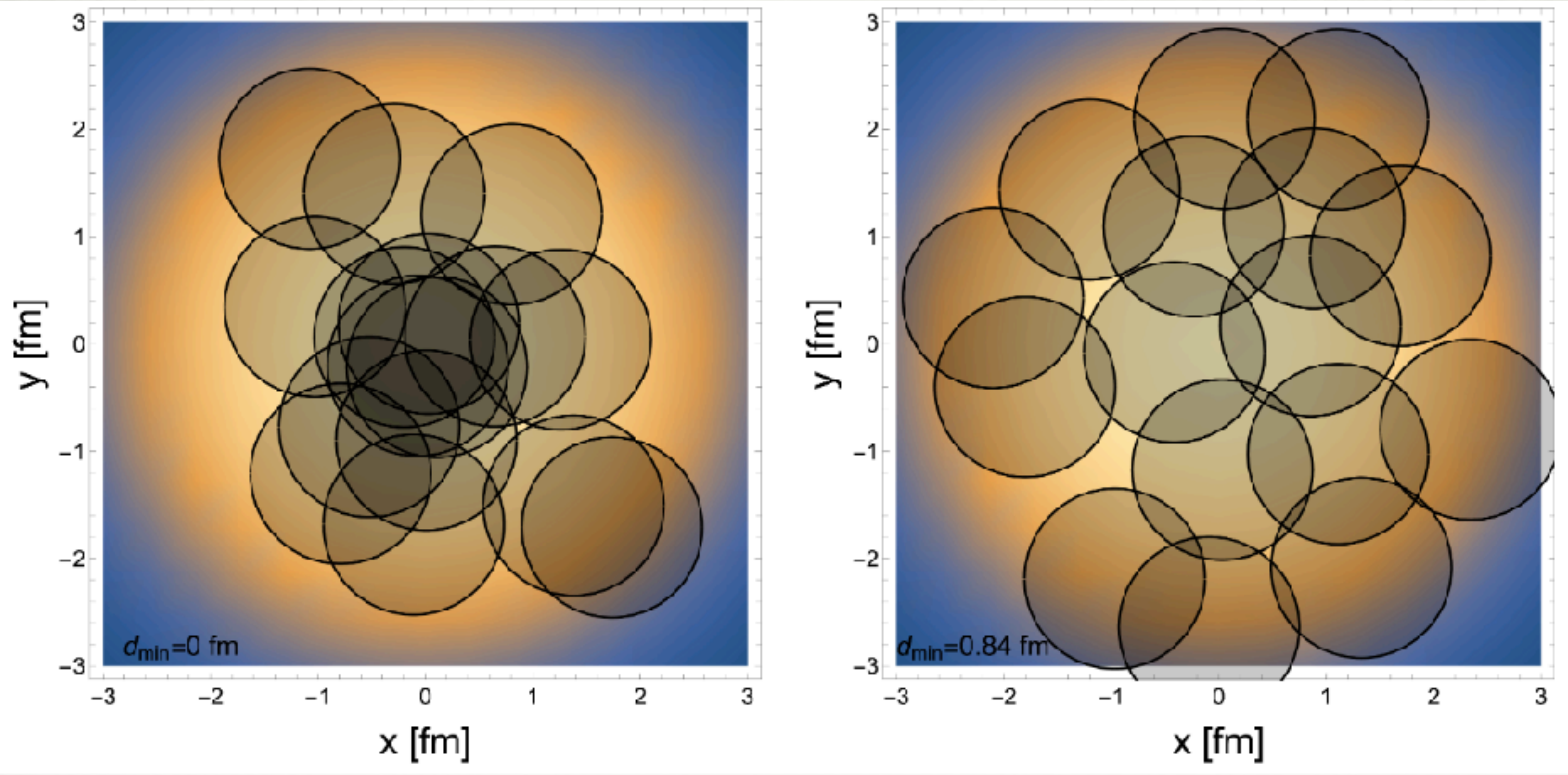
Participant Plane vs. Reaction Plane



Event plane angle ψ_{EP}

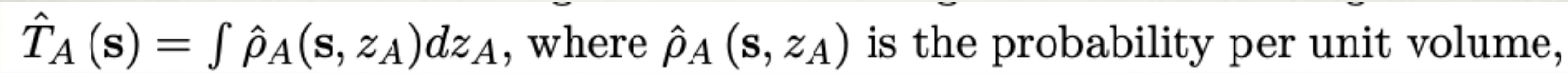
Now Eccentricity had a magnitude AND angle!

Monte Carlo sampling of ^{16}O



d_{min} Minimum distance apart

Thickness function



$\hat{T}_A(\mathbf{s}) = \int \hat{\rho}_A(\mathbf{s}, z_A)dz_A$, where $\hat{\rho}_A(\mathbf{s}, z_A)$ is the probability per unit volume,

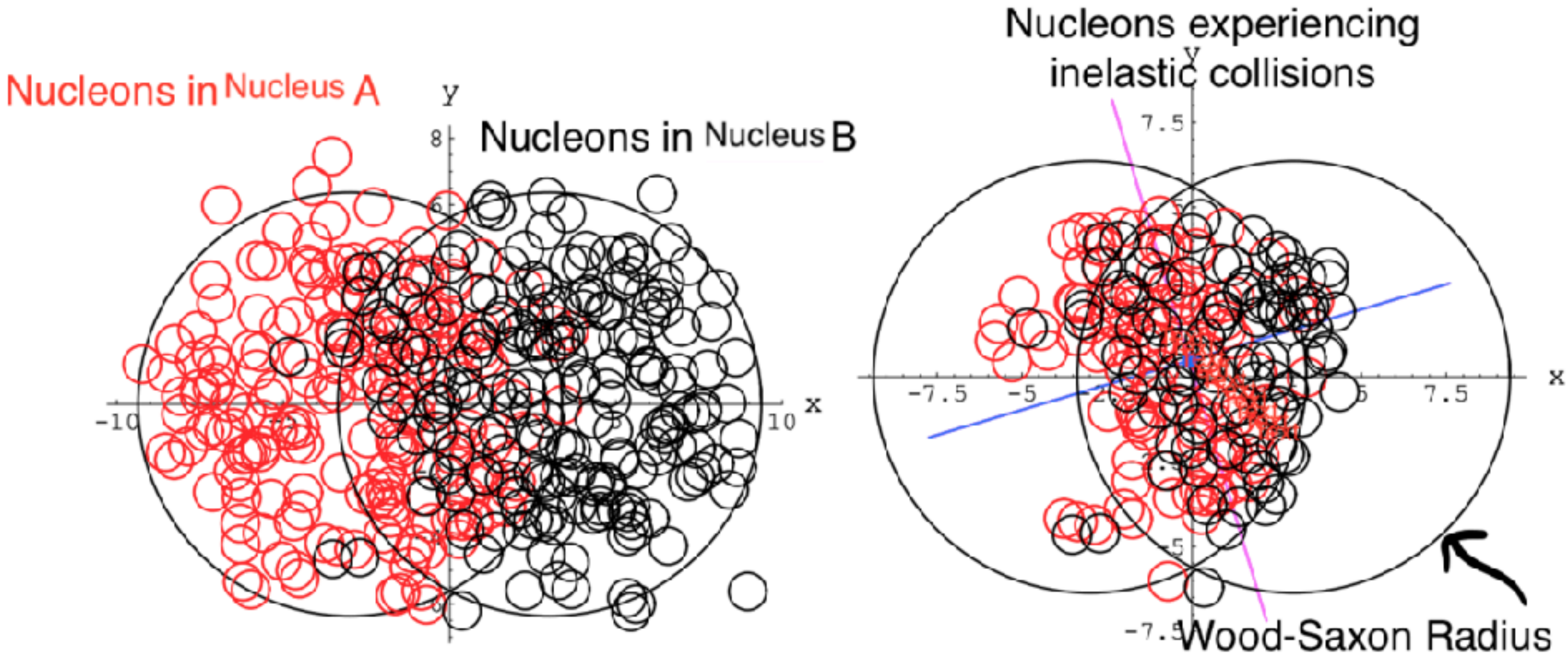
Joint probability per unit area of nucleons being located in the respective overlapping target and projectile flux tubes

Reduced thickness function $\hat{T}_{AB}(\vec{s}, \overrightarrow{b}) \equiv f\left[\hat{T}_A(\vec{s}), \hat{T}_B(\vec{s} - \overrightarrow{b})\right]$

Probability of a collision $p(\vec{s}, \overrightarrow{b}) = \sigma_{NN}^{inel}\hat{T}_{AB}(\vec{s}, \overrightarrow{b})$

$$\left(\frac{dS}{dy}\right)_{\tau=\tau_0} \propto f\left(T_A, T_B\right)$$

Monte Carlo sampling of collisions

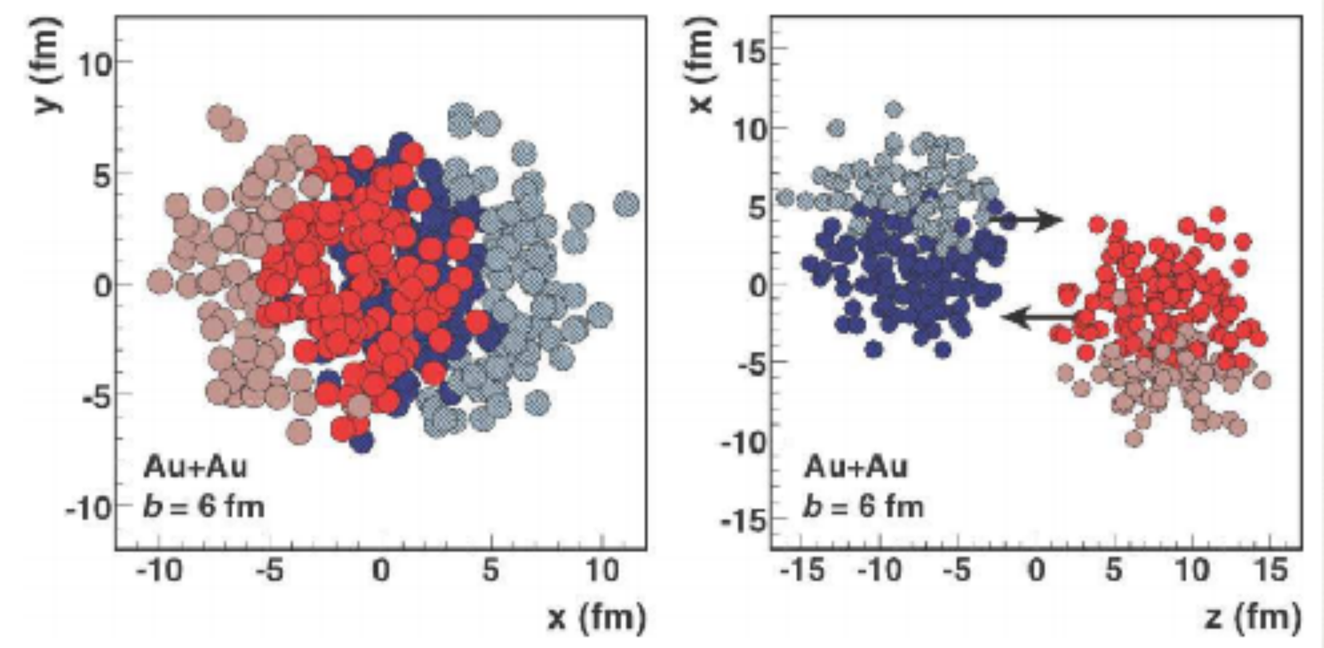


Keep the participating nucleons that collide in the overlap region

Collisions and Participants

$$N_{\text{coll}}\left(b\right)=\sum_{n=1}^{AB} nP\left(n,b\right)=AB\hat{T}_{AB}\left(b\right)\sigma_{\text{inel}}^{\text{NN}}$$

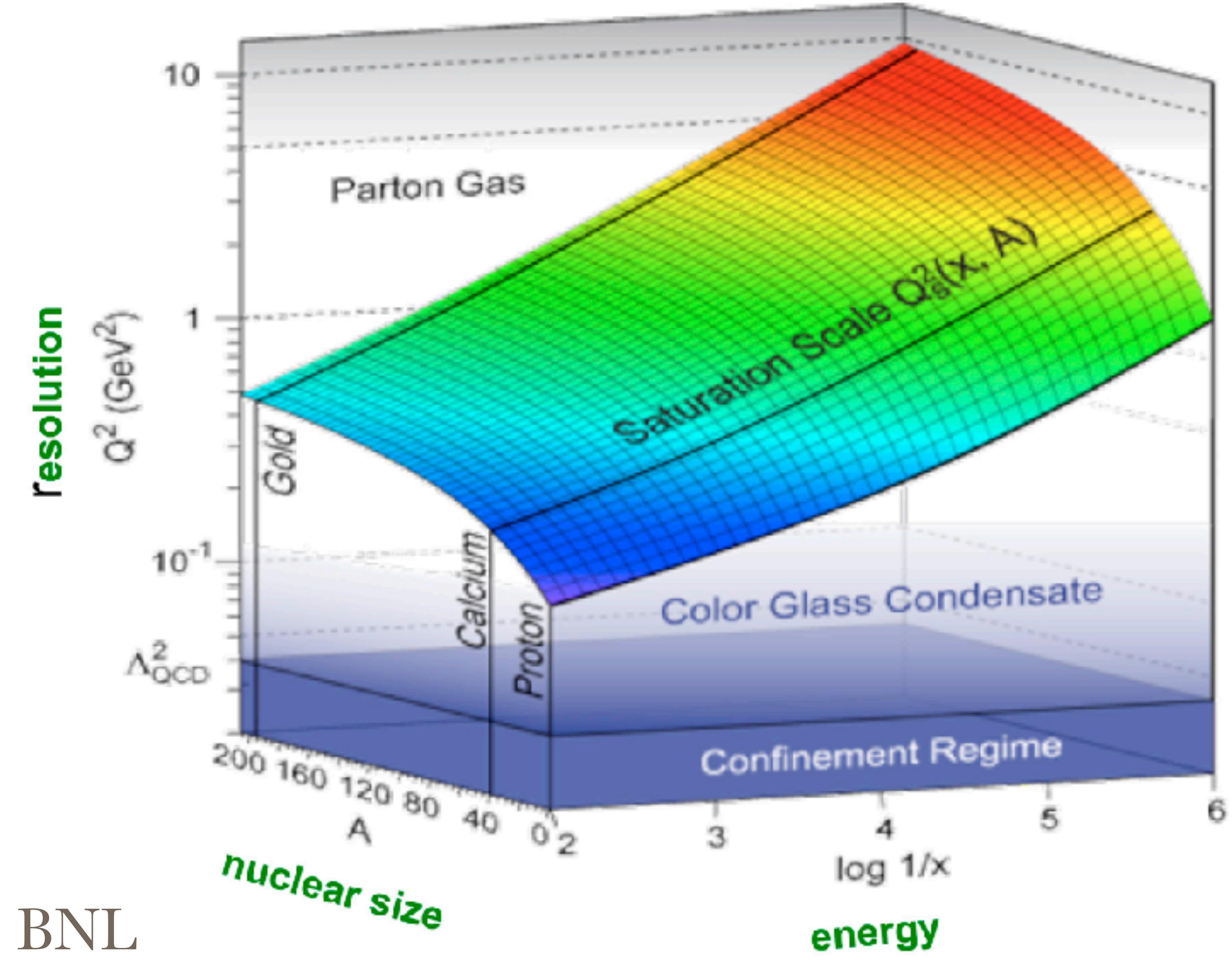
$$\begin{aligned}N_{\text{part}}\left(\mathbf{b}\right) \quad = \quad & A\int\hat{T}_{A}\left(\mathbf{s}\right)\left\{1-\left[1-\hat{T}_{B}\left(\mathbf{s}-\mathbf{b}\right)\sigma_{\text{inel}}^{\text{NN}}\right]^{B}\right\}d^{2}s+\\ & B\int\hat{T}_{B}\left(\mathbf{s}-\mathbf{b}\right)\left\{1-\left[1-\hat{T}_{A}\left(\mathbf{s}\right)\sigma_{\text{inel}}^{\text{NN}}\right]^{A}\right\}d^{2}s,\end{aligned}$$



Connecting to initial conditions

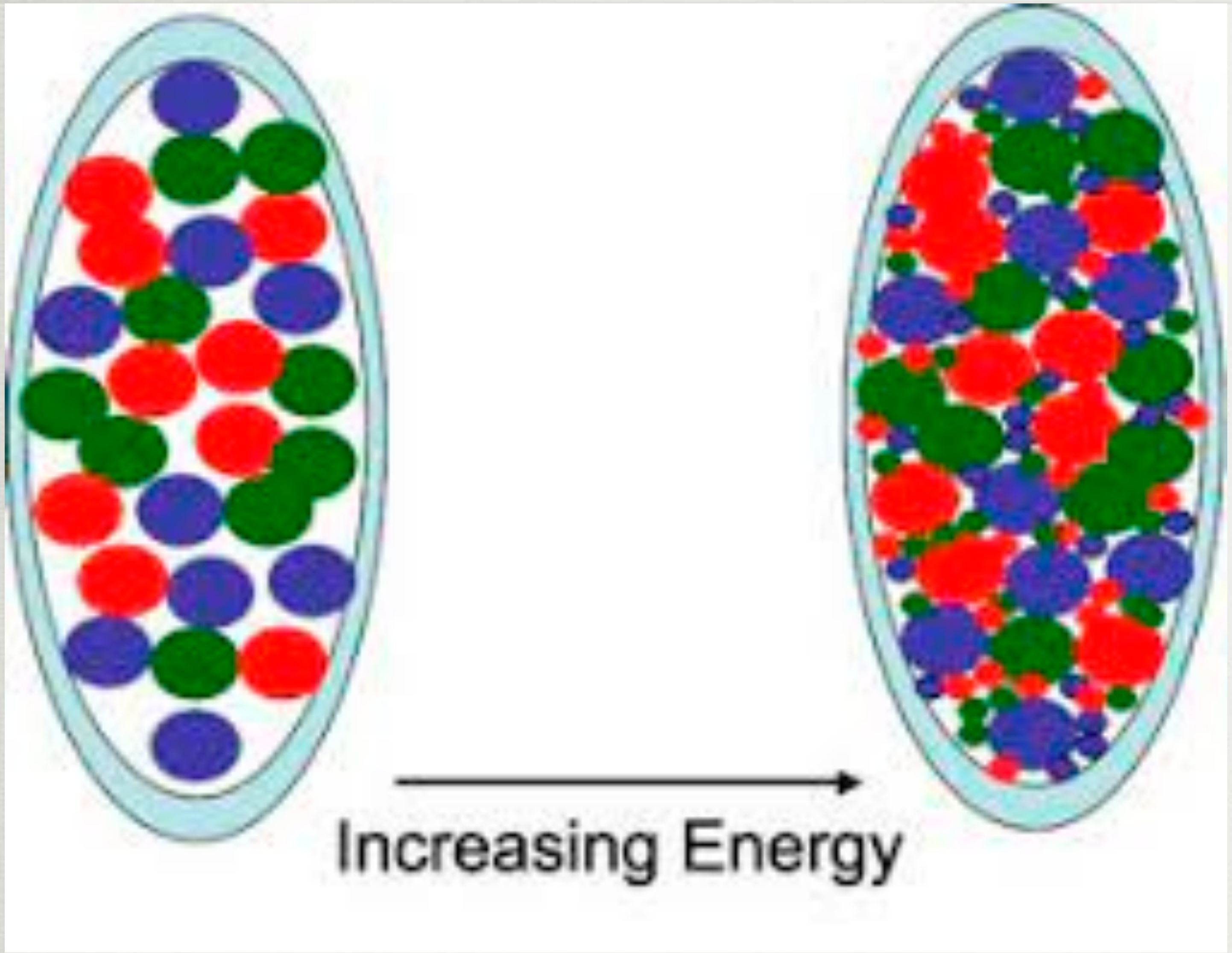
- Not 100% clear how to go from thickness functions to initial energy density
- Commonly used in the field: $\varepsilon \propto T_A + T_B$ (Glauber), $\varepsilon \propto T_A T_B$ (CGC), or $s \propto \sqrt{T_A T_B}$ (Phenomenological)
- Scaling constant needed i.e. $\varepsilon = C T_A T_B$ where C is fitted to measured multiplicity. Unsolved mystery how to find C from first principles.

Initial state “phase diagram”



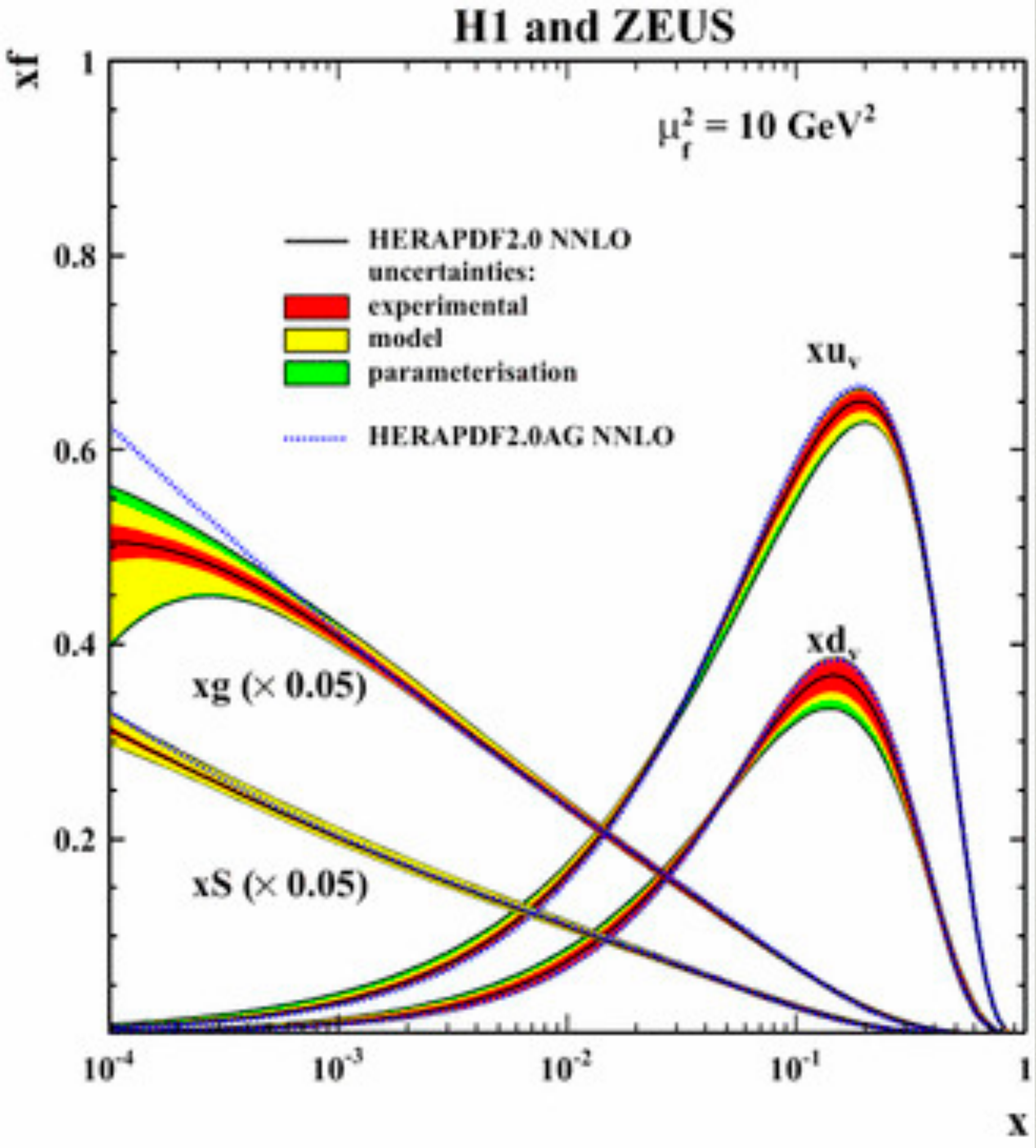
BNL

Gluon Saturation of the nucleus

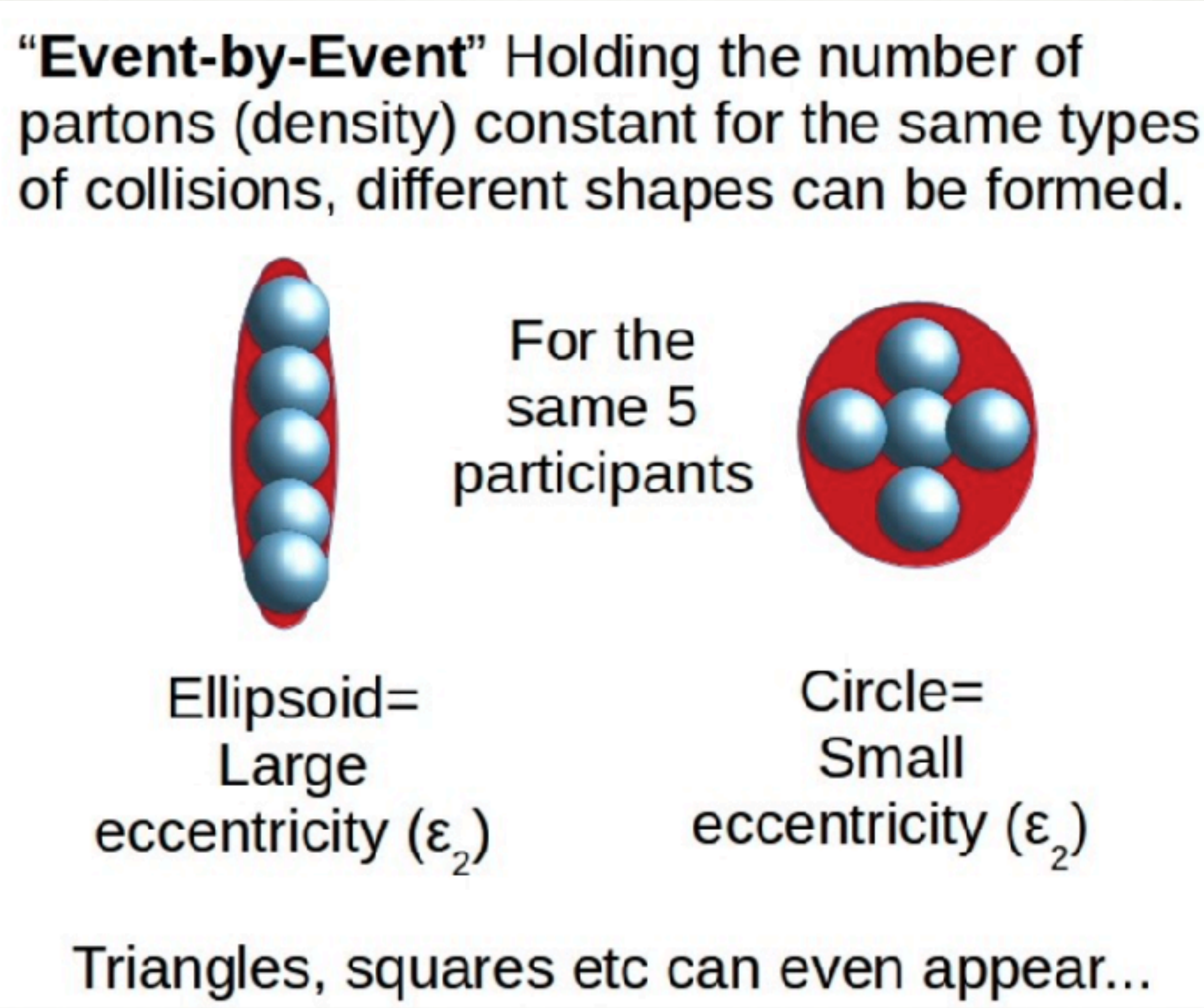


Valence
quarks

Gluons &
Sea quarks



Different shape/fluctuations



“**Event-by-Event**” Holding the number of partons (density) constant for the same types of collisions, different shapes can be formed.

For the same 5 participants

Ellipsoid= Large eccentricity (ε_2)

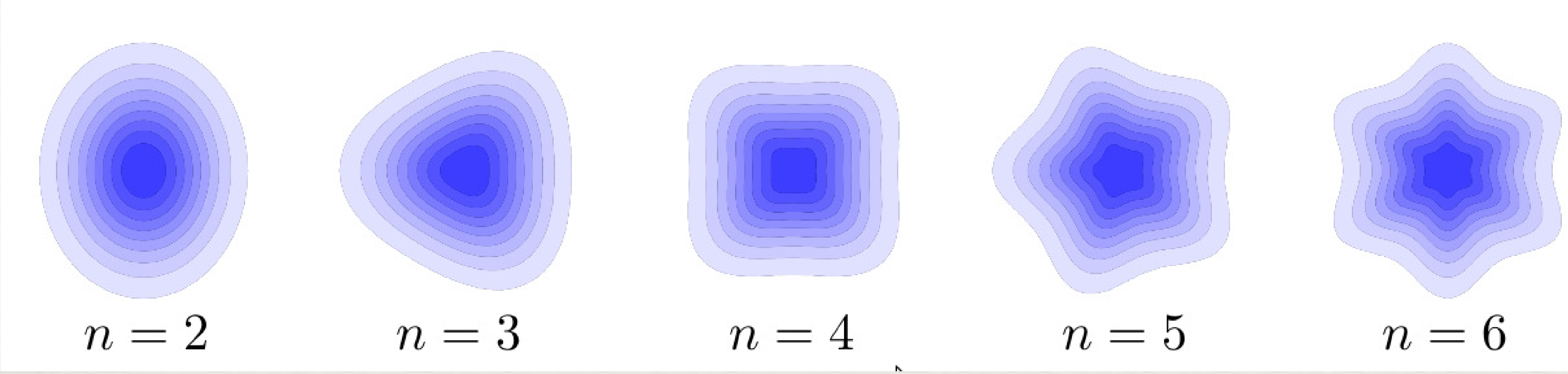
Circle= Small eccentricity (ε_2)

Triangles, squares etc can even appear...

What about other shapes?

$$\varepsilon_{n,m} e^{i\Phi_{n,m}} \equiv - \frac{\{r^m e^{in\phi}\}}{\{r^m\}}.$$

Fourier series….



$n = 2$ $n = 3$ $n = 4$ $n = 5$ $n = 6$

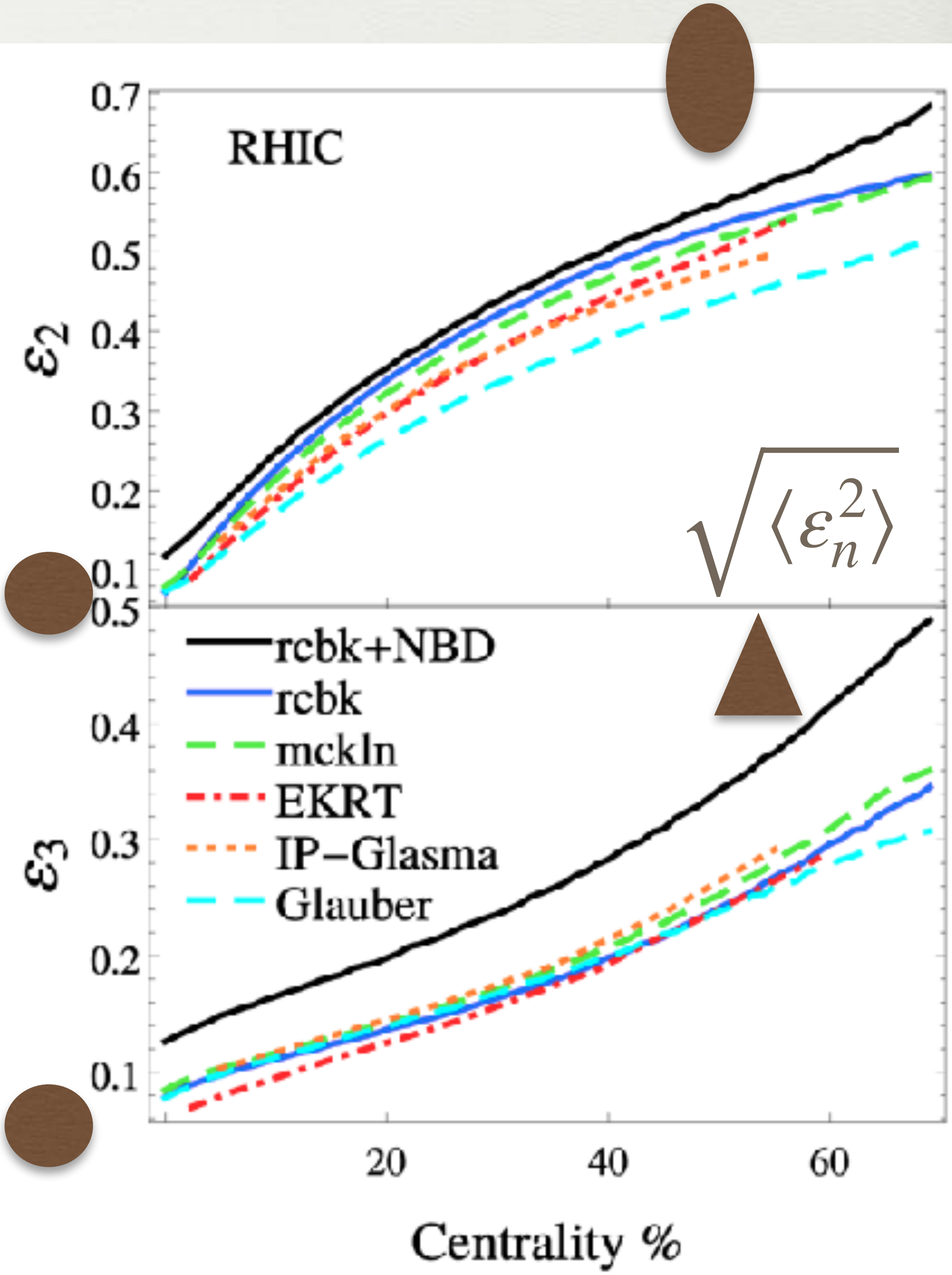
Quantify eccentricities in different models

Eccentricities:

$$\varepsilon_n e^{i\Phi_n} = \frac{\int d^2\mathbf{r}\; r^n\, e^{in\phi}\, \epsilon(\tau_0, \mathbf{r})}{\int d^2\mathbf{r}\; r^n\, \epsilon(\tau_0, \mathbf{r})}$$

Variance in the Eccentricities

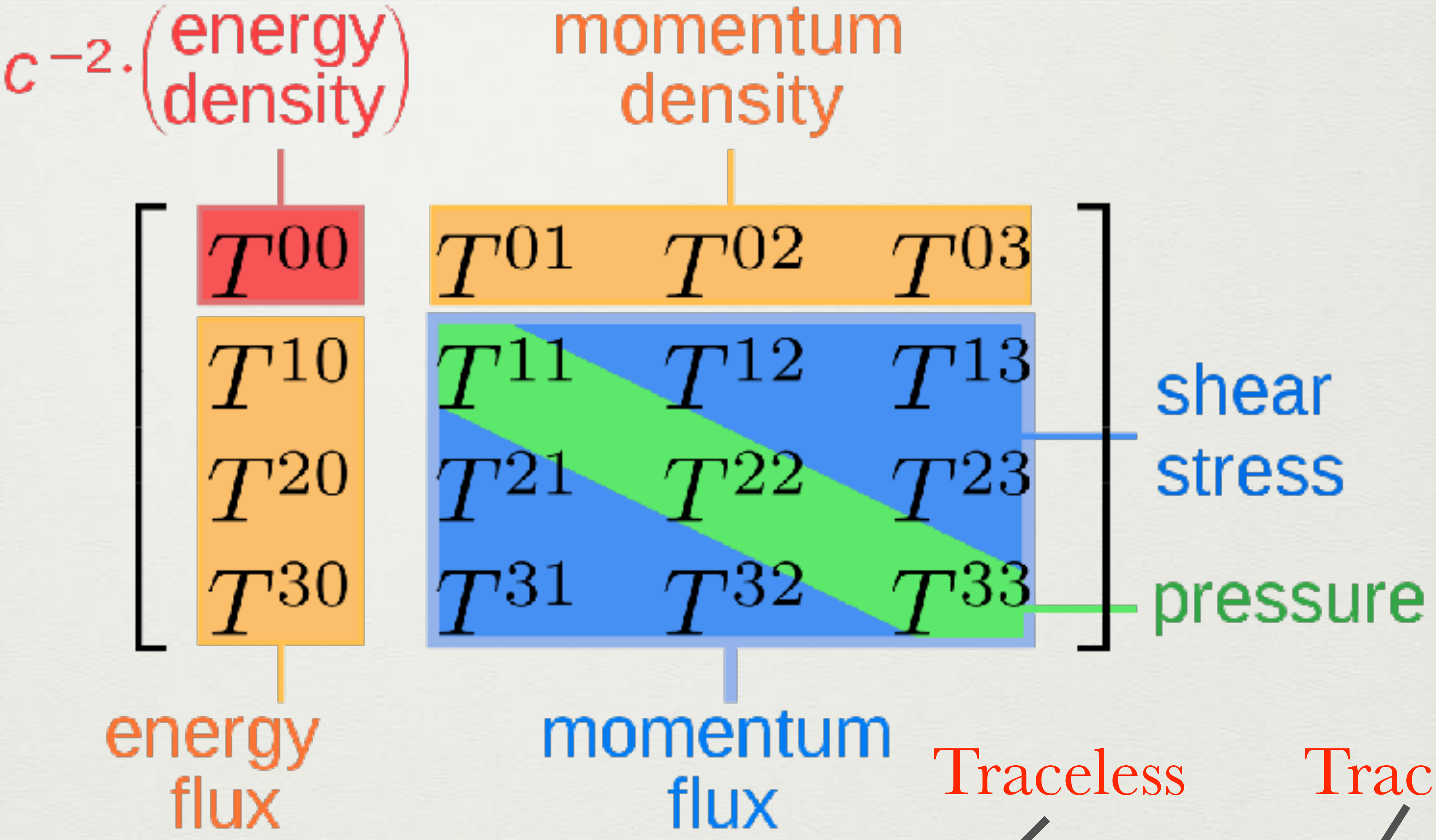
Different initial conditions with varying underlying physical assumptions



$$\sqrt{\langle \varepsilon_n^2 \rangle}$$

Initial conditions out-of-equilibrium

$$T^{\mu\nu} =$$



Traceless

Trace

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}_{visc}$$

$$\Pi^{\mu\nu}_{visc} = \underbrace{\pi^{\mu\nu}}_{shear} + \Delta^{\mu\nu}\underbrace{\Pi}_{bulk}$$

What are people studying today?
Initial State

- Measuring deformations in nuclei, eccentric protons
- Full $T^{\mu\nu}$ and finite baryon density (baryon stopping)
- Signatures of the Color Glass Condensate
- Connections to the Electron Ion Collider
- Correct time evolution: free-stream, kinetic theory, AdS/CFT, classical Yang Mills
- Start time of hydrodynamics?

Fluid Dynamics
Dynamics of Quark Gluon Plasma

Non-relativistic Fluid Dynamics

Navier Stokes Equation: $\rho \frac{\partial v}{\partial t} = -\nabla p - \eta \nabla^2 v$

$\rho \frac{\partial v}{\partial t}$ How quick the fluid changes its velocity

∇p

$\eta \nabla^2 v$ Viscosity- how sticky a substance is

Ideal *relativistic* fluid dynamics

Conservation of Energy and Momentum

$\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu N^\mu = 0$

The *ideal* energy-moment tensor is

$T^{\mu\nu} = (\varepsilon + p)u^\nu u^\nu - p g^{\mu\nu}$ where $u^\mu = \gamma\left(1, \overrightarrow{v}\right)$

We also write:

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$T^{\mu\nu} = \varepsilon u^\nu u^\nu - p\Delta^{\mu\nu}$$

Coordinate System: $x^\mu = \left(\tau, x, y, \eta\right)$ where $\tau = \sqrt{t^2 - z^2}$ and

$$\eta = 0.5 \ln\left(\frac{t+z}{t-z}\right)$$

Solving idea fluid dynamics

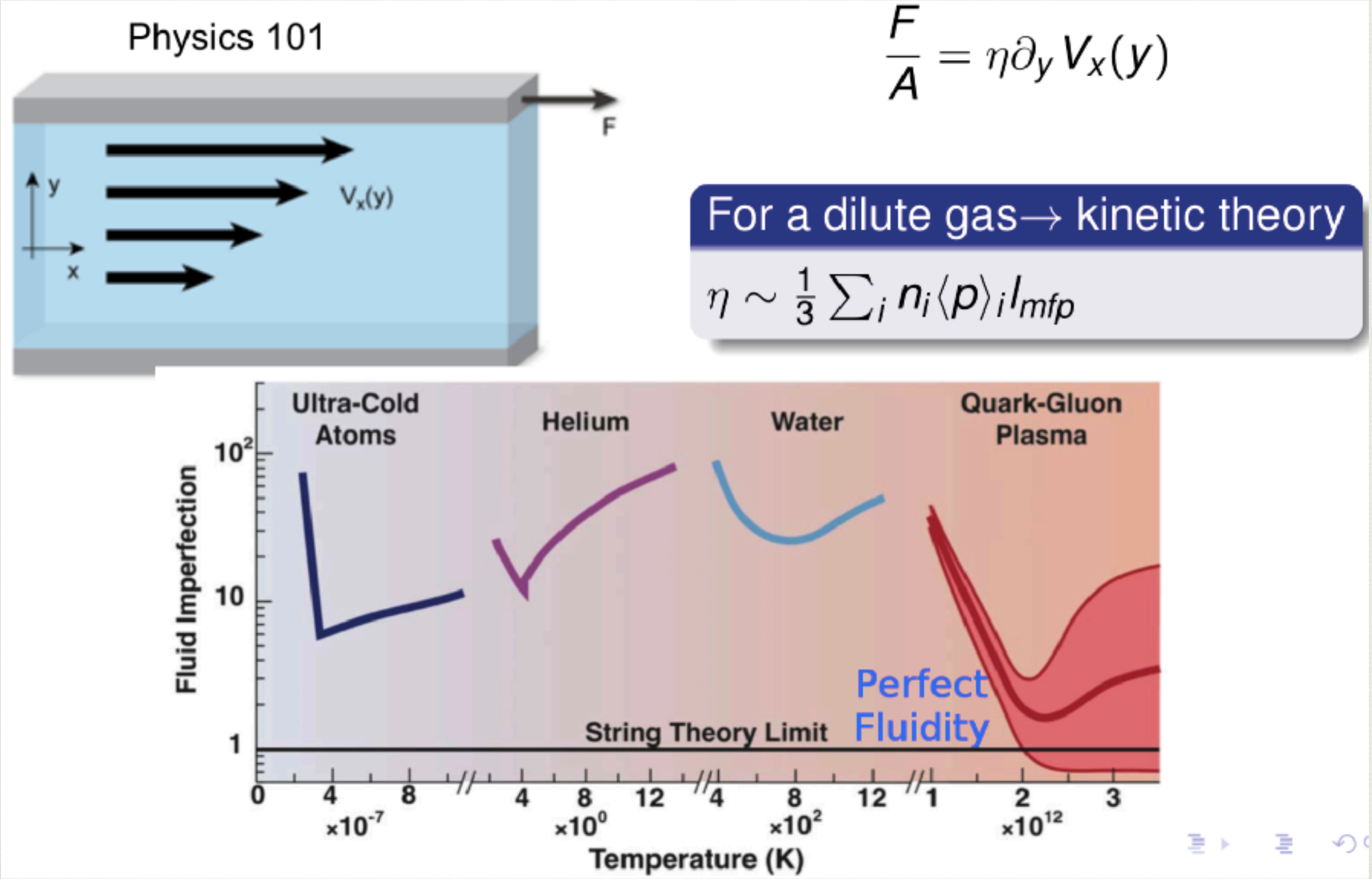
- Project the $T^{\mu\nu}$ in the parallel $u_\nu \partial_\mu T^{\mu\nu} = 0$
  $$\left(\varepsilon + p\right) \partial_\mu u^\mu + u^\mu \partial_\mu \varepsilon = 0$$

  and the perpendicular directions $\Delta^\alpha_\nu \partial_\mu T^{\mu\nu} = 0$
  $$\left(\varepsilon + p\right) u^\mu \partial_\mu u^\alpha - \Delta^{\mu\alpha} \partial_\mu p = 0$$

- Solve in terms of ε, u^μ (hydro variables)

- Find pressure from equation of state $p(\varepsilon)$

Shear viscosity - η/s



Physics 101

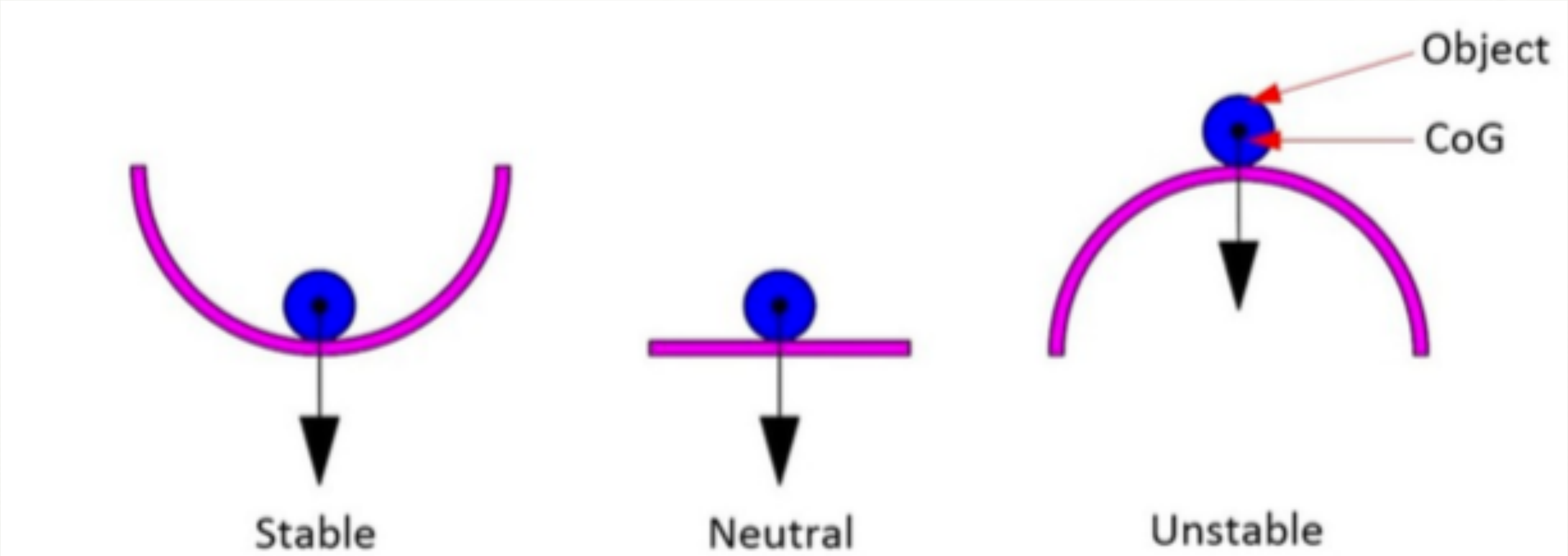
$$\frac{F}{A} = \eta \partial_y V_x(y)$$

For a dilute gas→ kinetic theory

$$\eta \sim \frac{1}{3} \sum_i n_i \langle p \rangle_i l_{mfp}$$

Navier-Stokes equations are acausal and unstable relativistically

Perturb the system and the equations will break down



Requires a finite time for the system to return to equilibrium
Relxation time τ_π

Relativistic Fluid Dynamics

Israel Stewart Annals Phys. 118 (1979) 341-372

Conservation of Energy and Momentum

$\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu N^\mu = 0$

The energy-moment tensor contains a bulk dissipative term Π and the shear stress tensor $\pi^{\mu\nu}$ is

Equation of State

Transport Coefficients

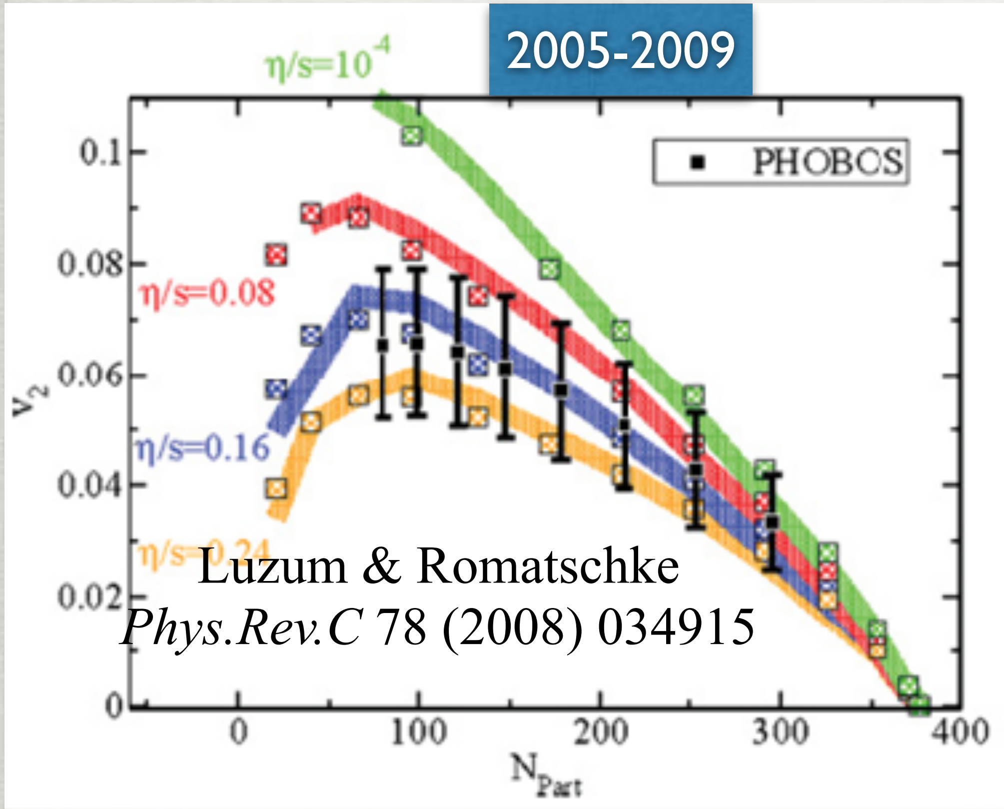
$$T^{\mu\nu} = \varepsilon u^\nu u^\nu - \left(p + \Pi \right) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\boxed{\tau_\pi} \left(\Delta_{\mu\nu\alpha\beta} D\pi^{\alpha\beta} + \frac{4}{3} \pi_{\mu\nu} \theta \right) = \boxed{2\eta} \sigma_{\mu\nu} - \pi_{\mu\nu}$$

$$+\Pi, N^\mu \ \ldots$$

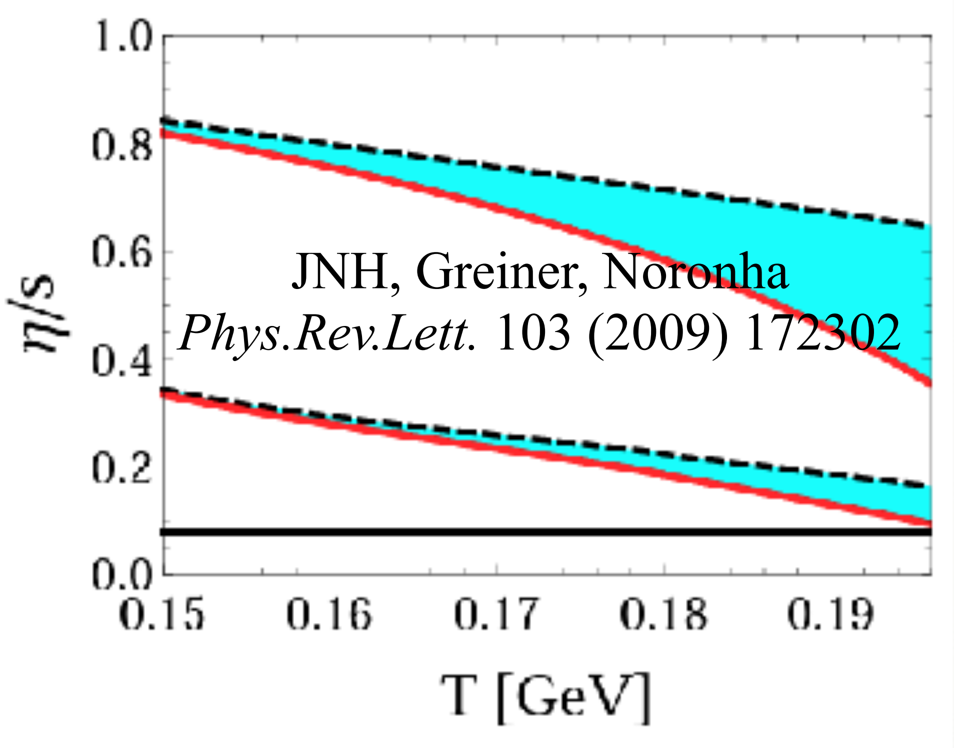
Nearly perfect fluidity

- (see Gajdosova, Tang, Paquet, Schlichting, Seidlitz)

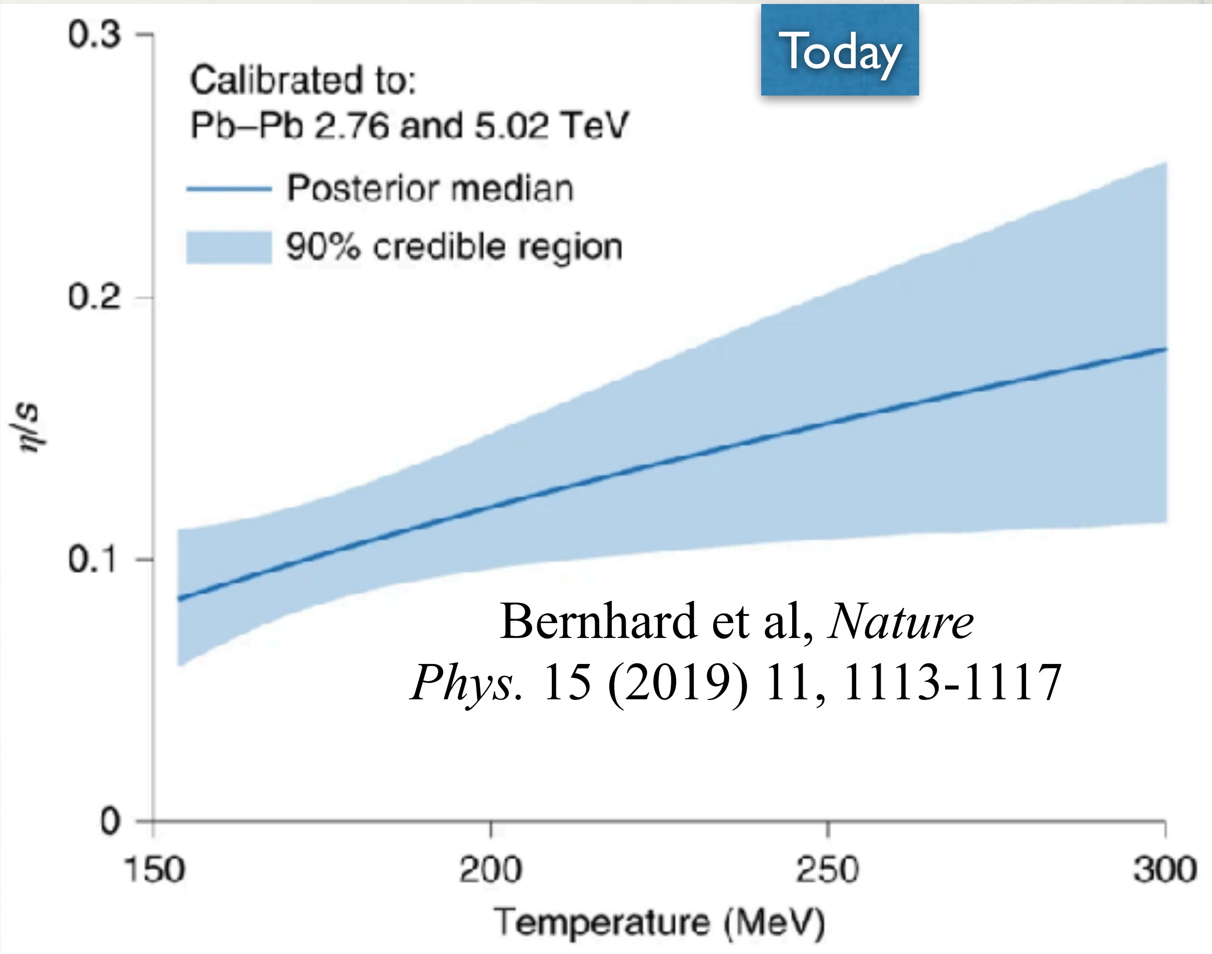


2005-2009

Luzum & Romatschke
Phys.Rev.C 78 (2008) 034915



JNH, Greiner, Noronha
Phys.Rev.Lett. 103 (2009) 172302



Today

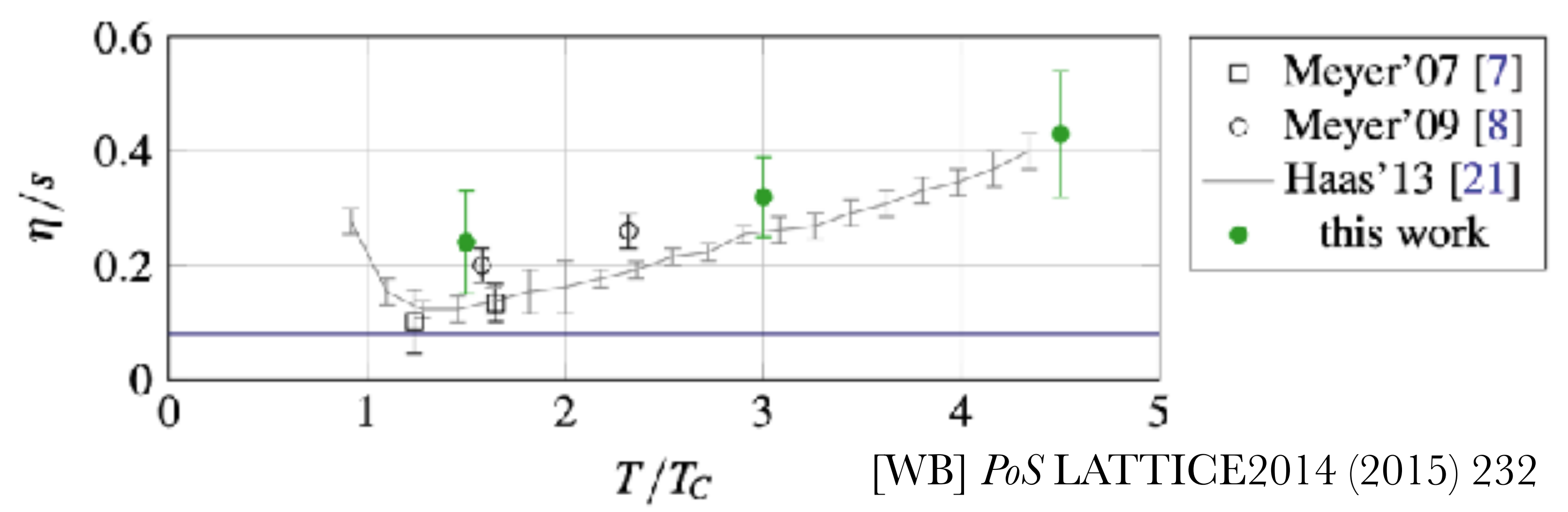
Bernhard et al, *Nature Phys.* 15 (2019) 11, 1113-1117

P. K. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).

$$\eta/s = 1/4\pi$$

Other recent Bayesian analyses:
[JETSCAPE]
Phys.Rev.C 103 (2021) 5, 054904;
Nijs et al,
Phys.Rev.C 103 (2021) 5, 054909

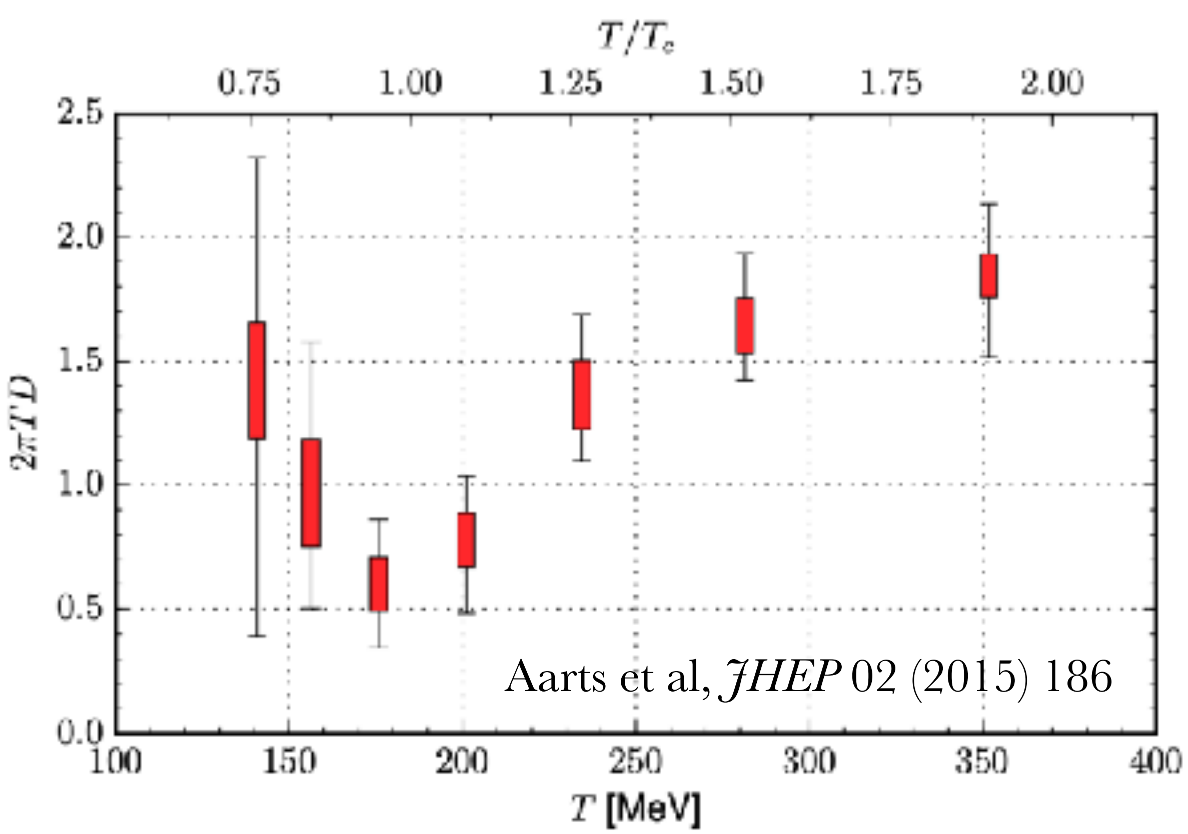
Attempts from QCD



[WB] *PoS* LATTICE2014 (2015) 232

Recent algorithms made for Quantum Computers

[NuQS] *arXiv:2104.02024 [hep-lat]*



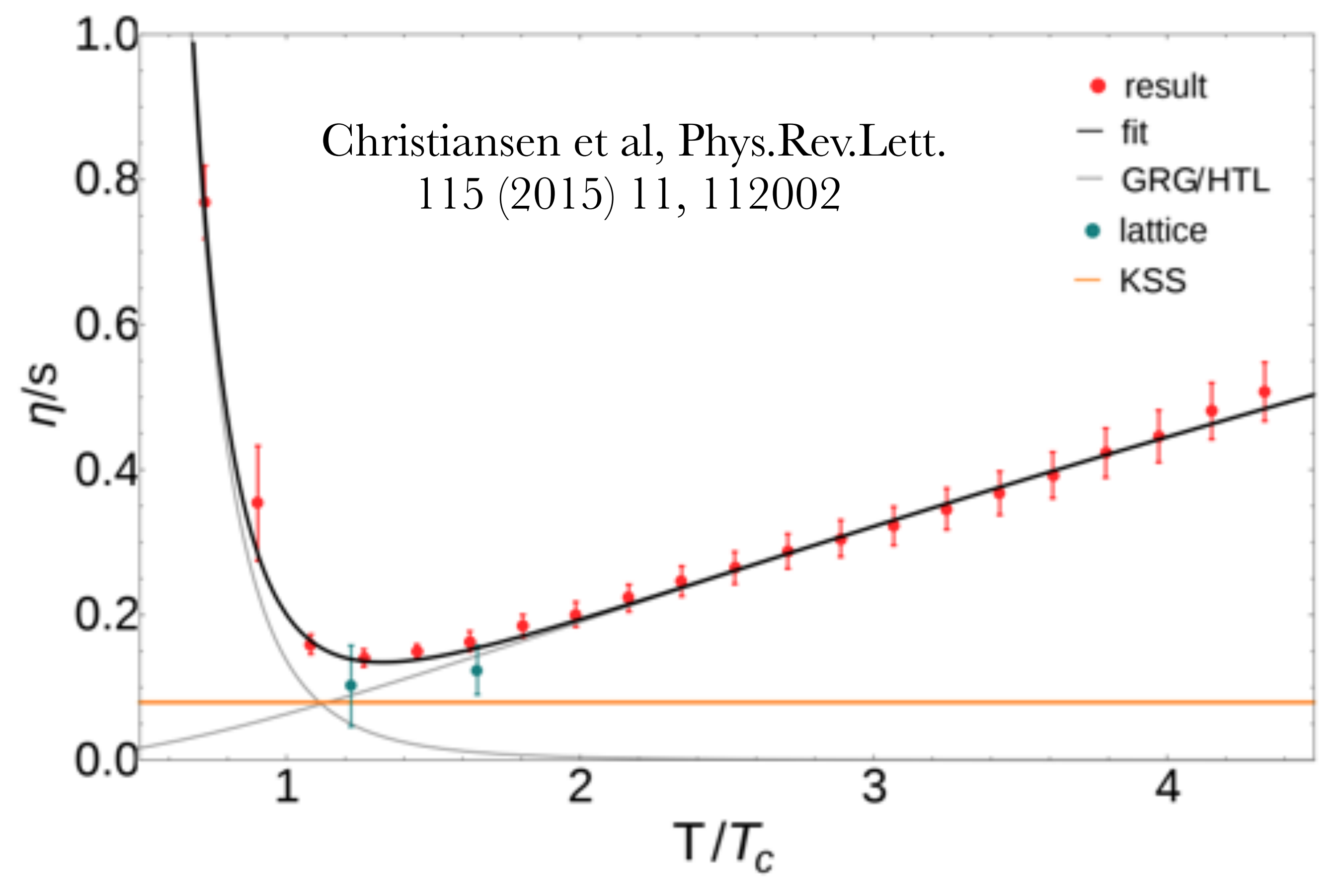
Aarts et al, *JHEP* 02 (2015) 186

Inversion Problem: correlation functions from Lattice QCD in Euclidean time, must convert to Minkowski time.

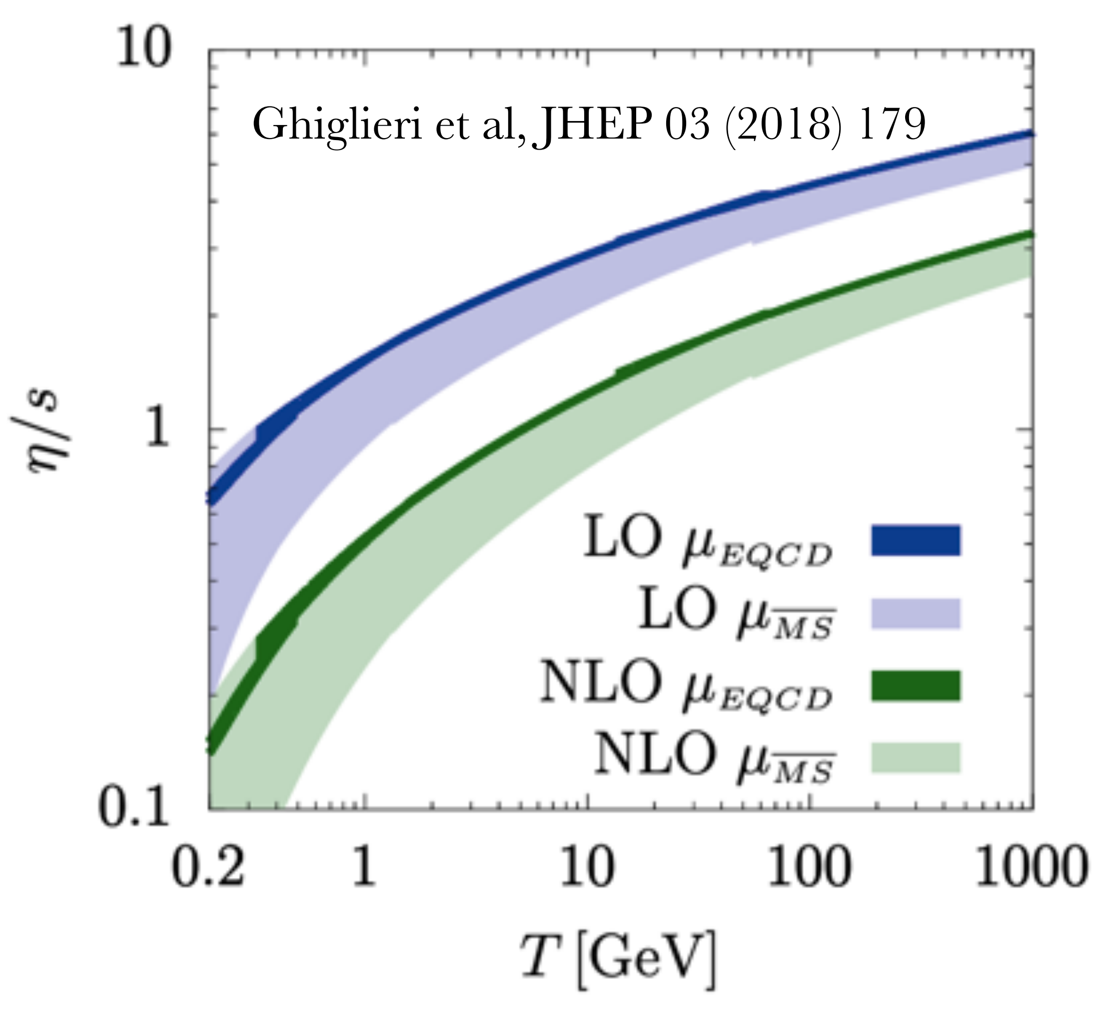
For overview, see Moore 2010.15704

Calculations in certain limits

Pure Yang Mills Theory

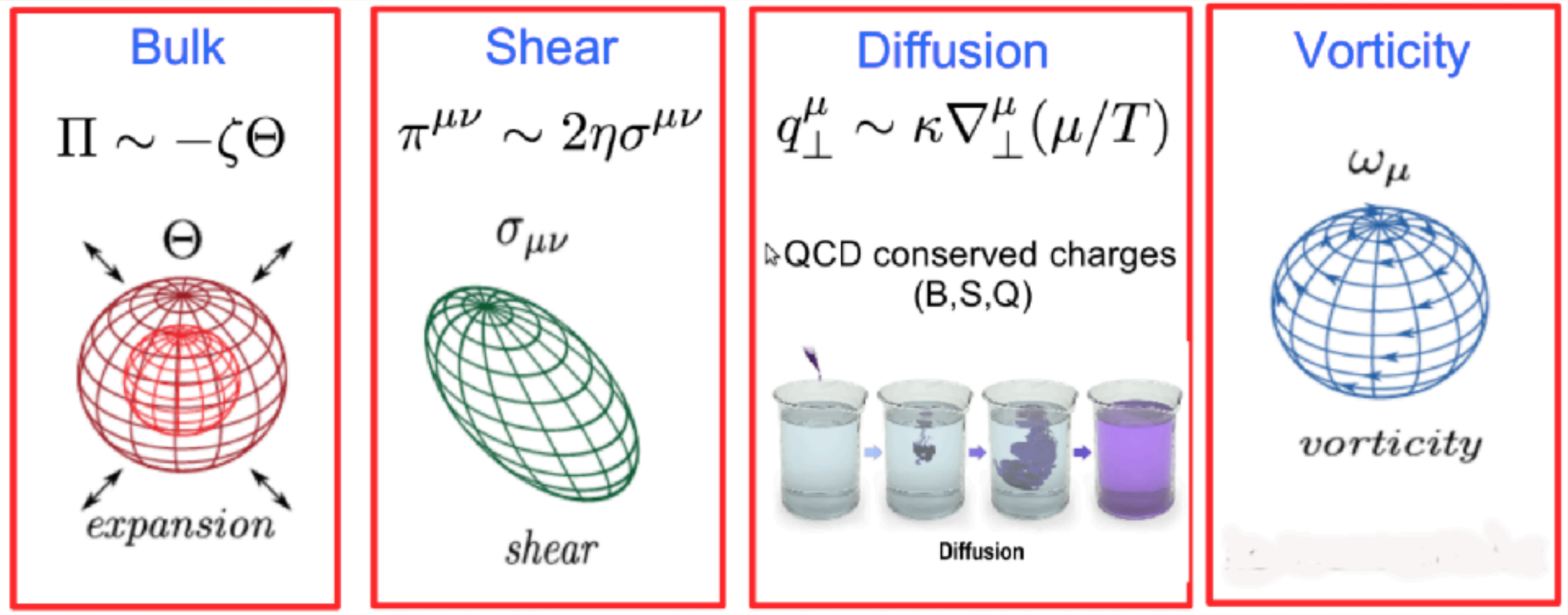

Christiansen et al, Phys.Rev.Lett. 115 (2015) 11, 112002

Perturbative QCD


Ghiglieri et al, JHEP 03 (2018) 179

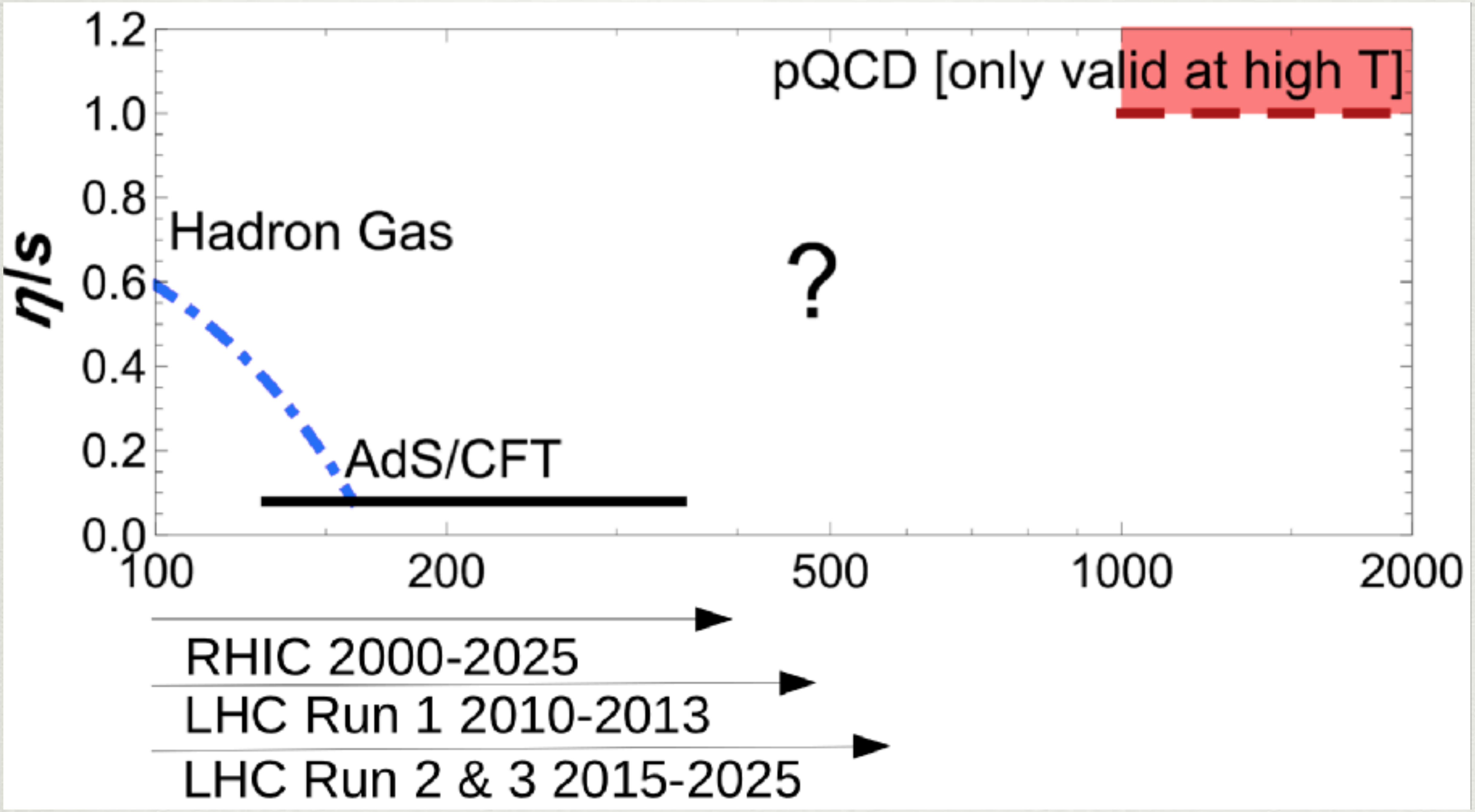
Transport coefficients/viscosities

Transport coefficient: Perturb the fluid from equilibrium- how quickly does it return to equilibrium?



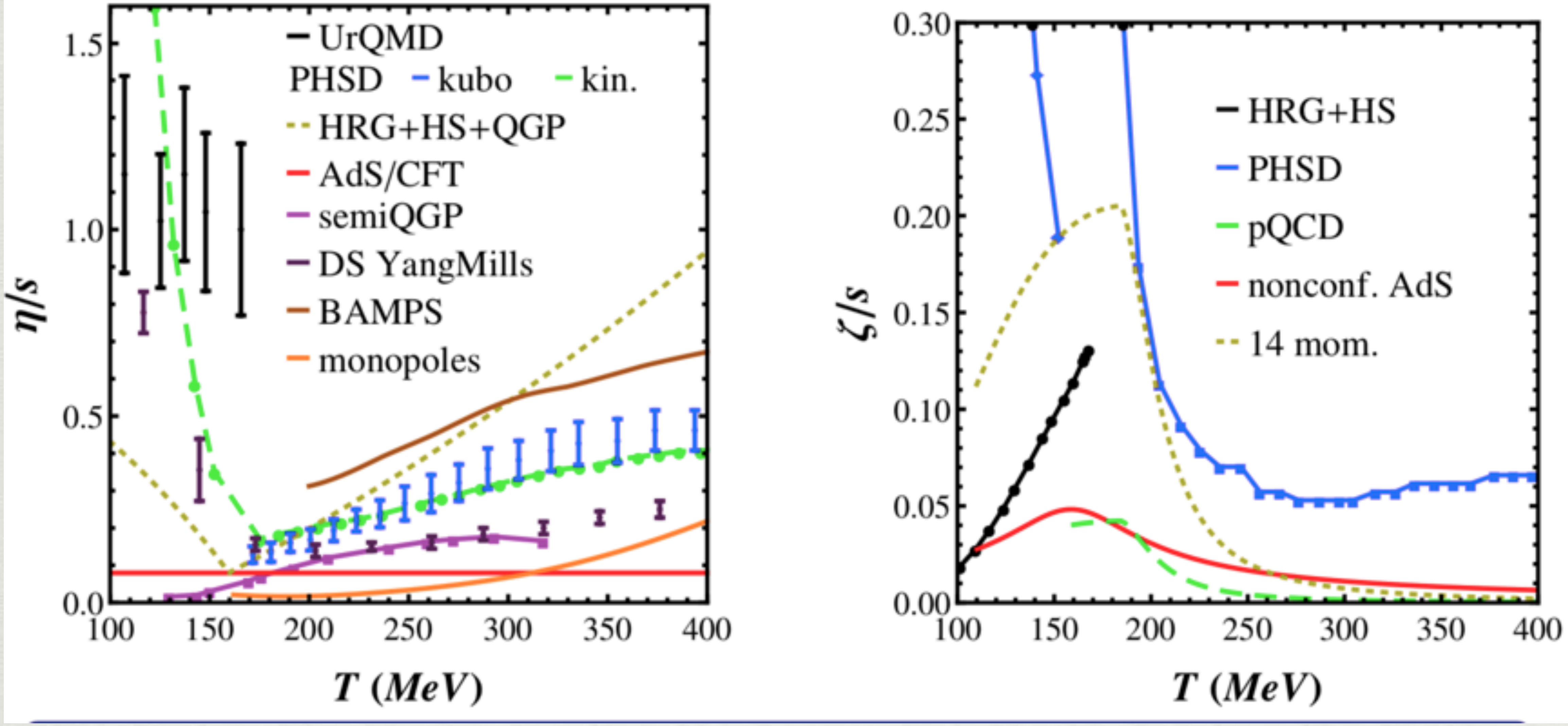
Viscosity - resistance to deformation or “thickness” of liquid

Experimental probes of $\eta/s(T)$



***Hadron Gas**: JNH et al, PRL103(2009)172302; PRC86(2012)024913*
***AdS/CFT:** Kovtun, Son, Starinets PRL94(2005)111601*
***pQCD**: Arnold, Moore, Yaffe JHEP 0011(2000)001 ; JHEP0305(2003)051*

Theoretical calculations of viscosity

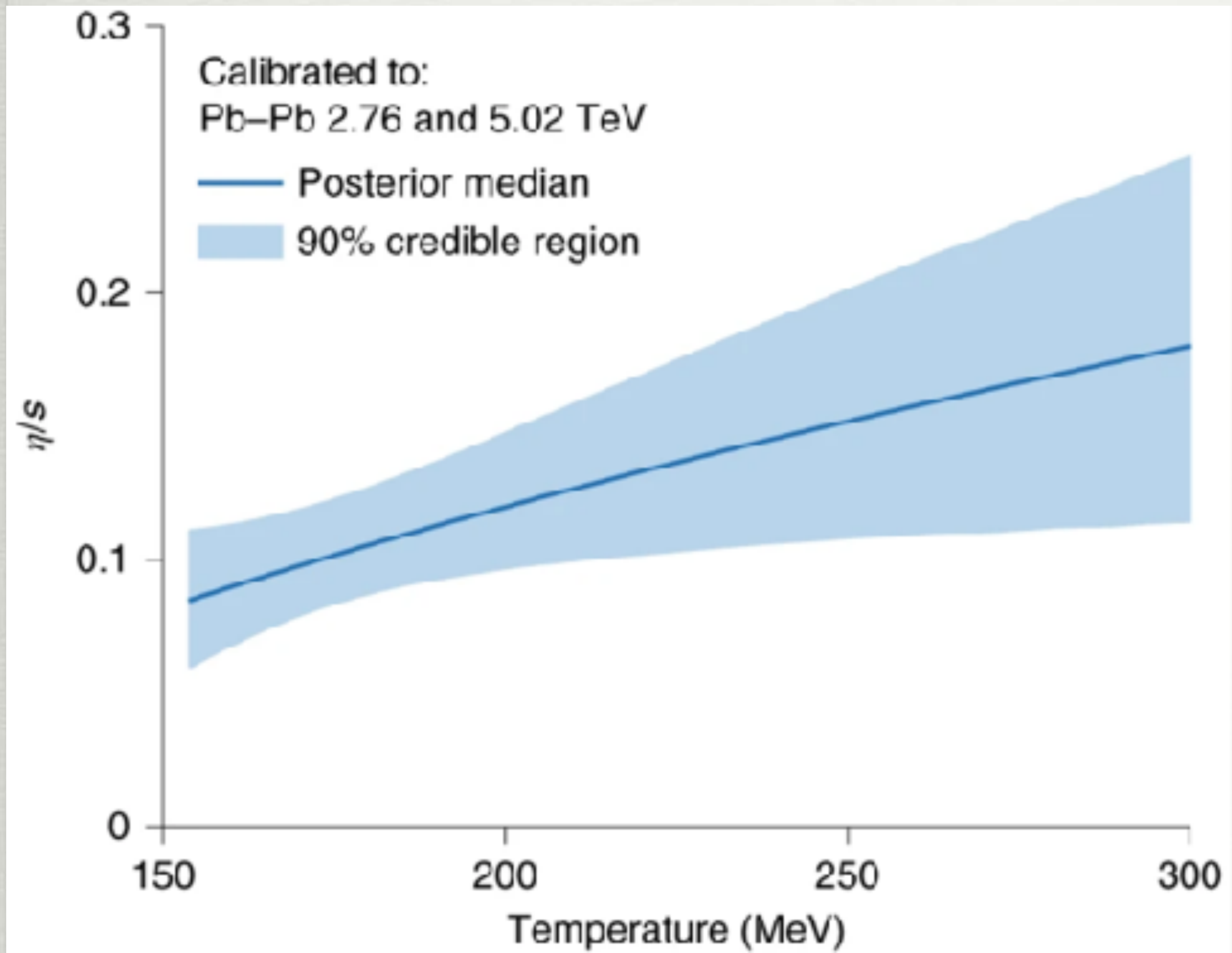


See references in JNH arXiv:1512.06315

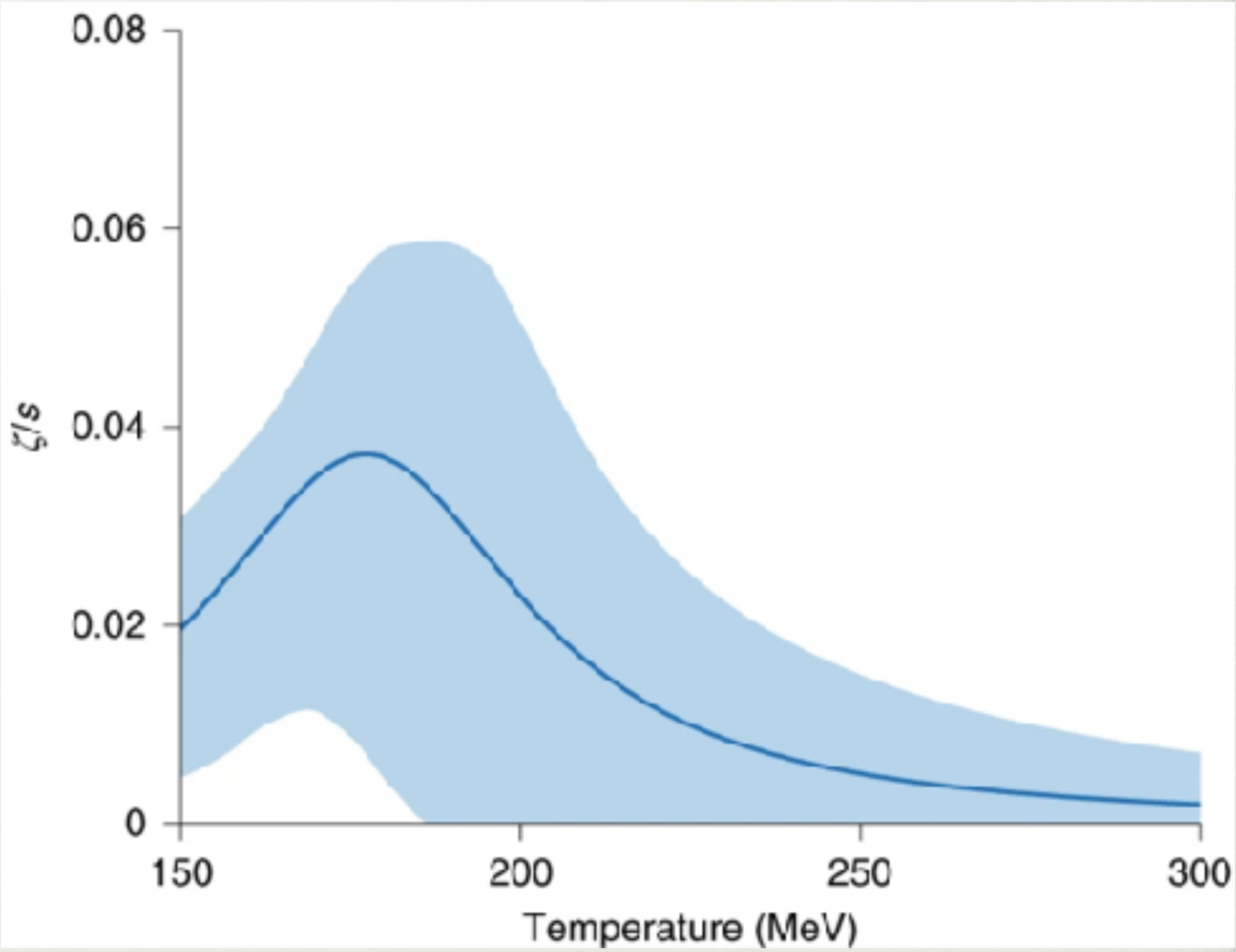
Dip expected: *Phys.Rev.Lett. 97 (2006) 152303, Nucl.Phys. A769 (2006) 71-94, Phys.Rev.Lett. 103 (2009) 172302*

Bayesian analysis (agnostic η/s & ζ/s)

Shear viscosity



Bulk viscosity



Bernhard, Moreland, Bass Nature Phys. 15 (2019) no.11, 1113-1117