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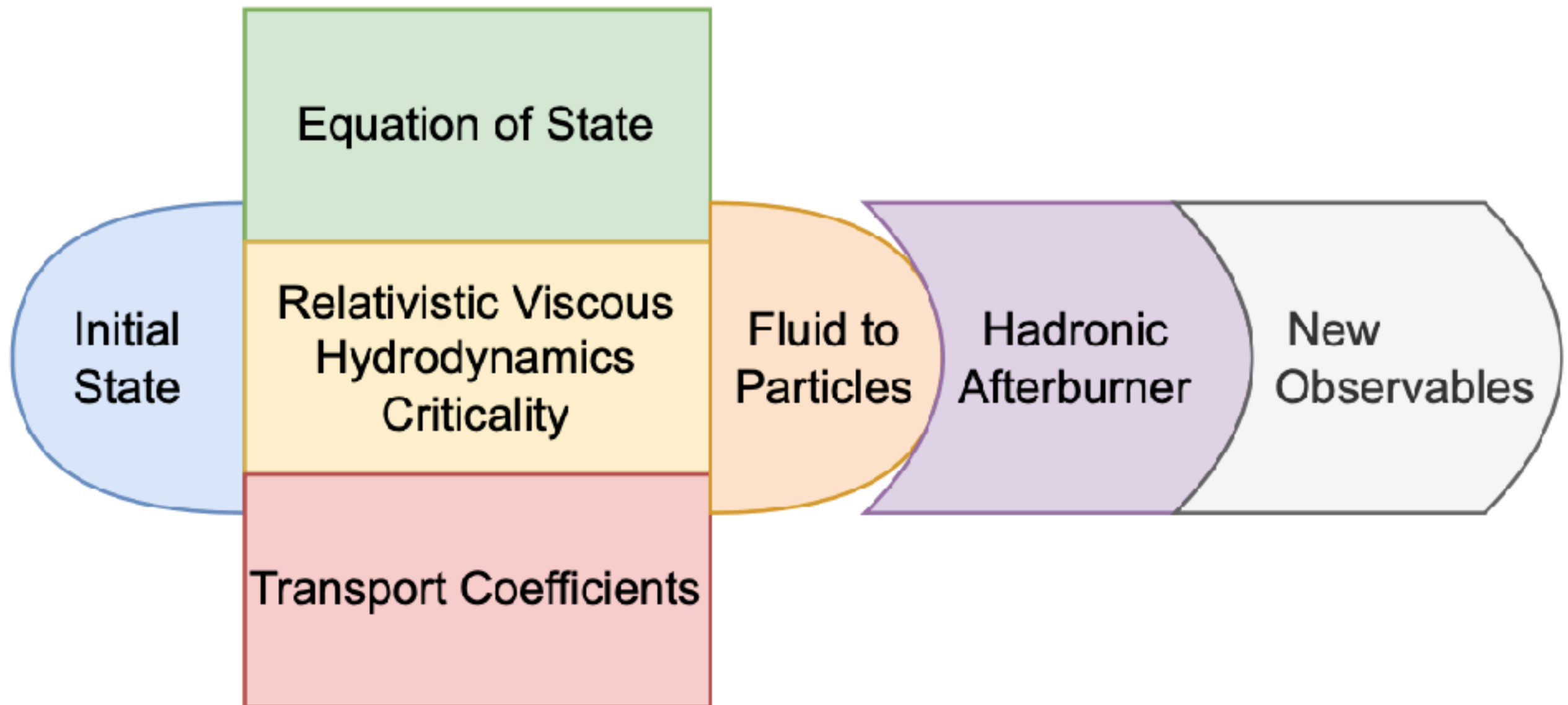
muses



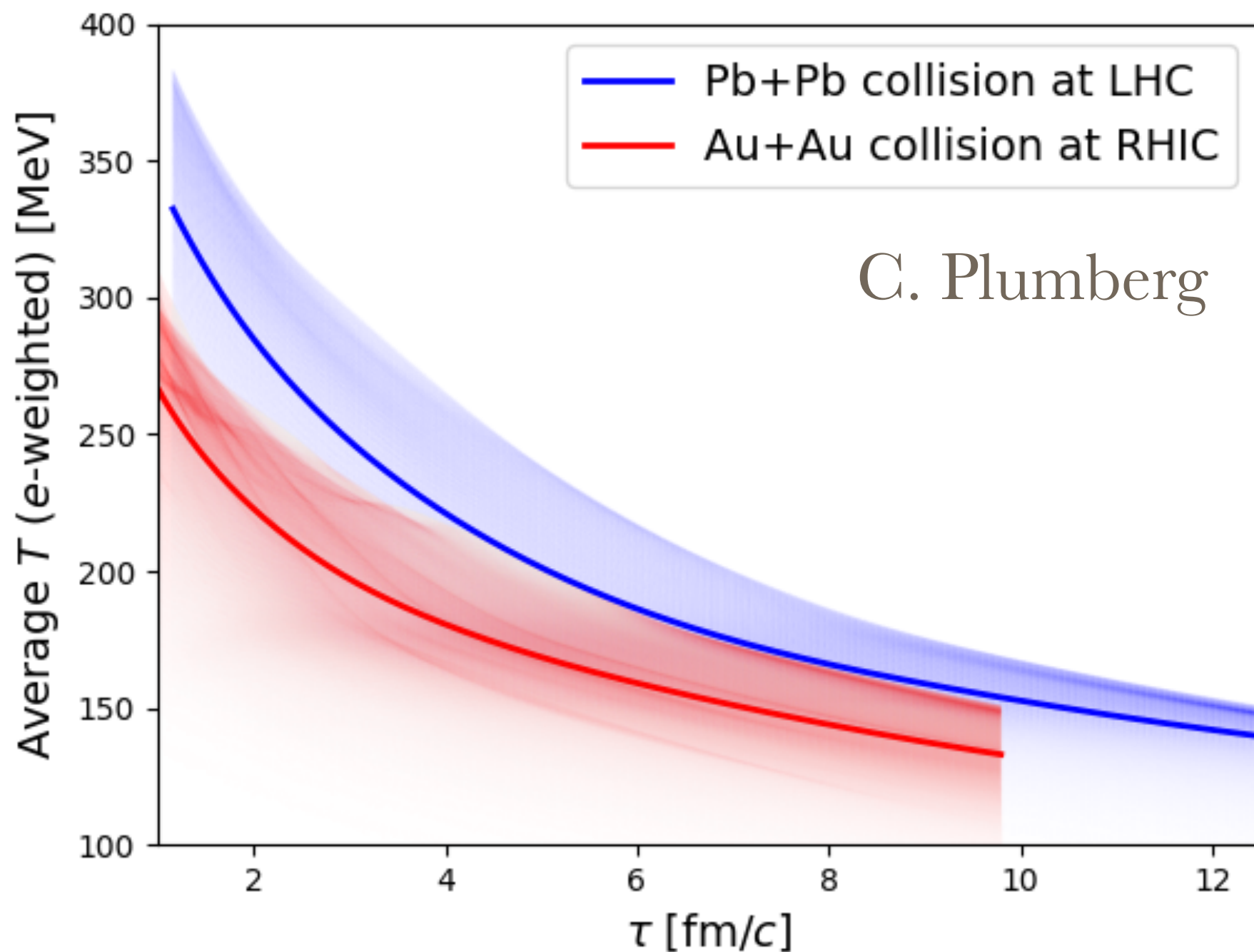
Lecture 3 on Hot QCD Matter: Heavy-Ion Collisions & Future

Jacquelyn Noronha-Hostler
National Nuclear Physics Summer School
MIT 2022

Standard Model of HIC



Average temperature in hydrodynamics



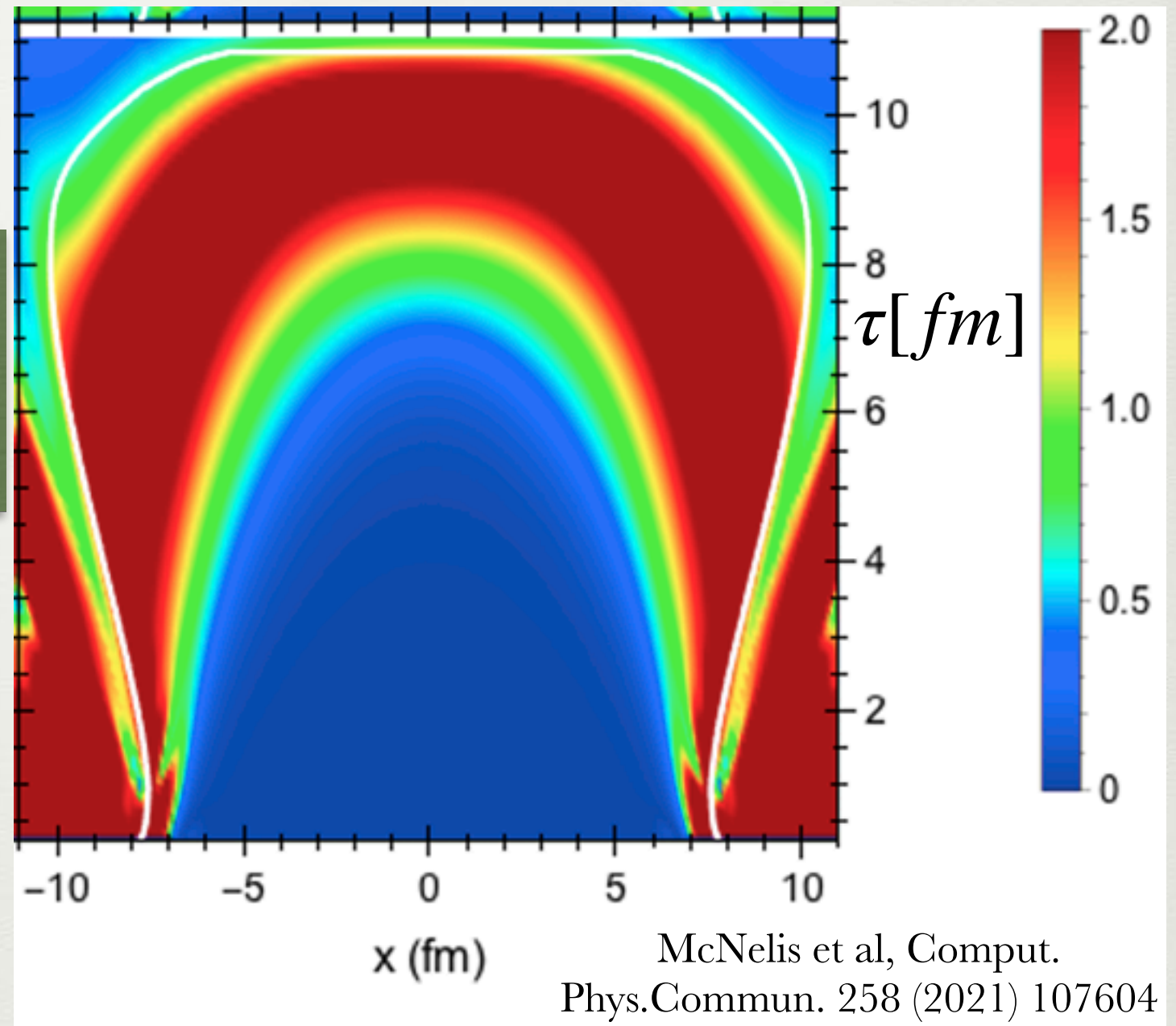
Hypersurface (hadronization)

$$Kn_{\Pi} = \frac{\Pi}{\varepsilon + p}$$

Need to pick a criteria to
switch from fluid to
particles

Temperature, energy
density, Knudsen number,

...



Switching temperature $T_{sw} = 150$

Hadronization: fluid to particles

Cooper-Frye switching fluid to hadrons

Distribution function

$$\left(E_p \frac{dN}{d^3p}\right)_i = d_i \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f_i$$

Degeneracy Hypersurface 4-Momentum

$$f_i = f_{eq,i} + \delta f_{\eta} + \delta f_{\zeta} + \dots$$

$$f_{eq,i} = \frac{1}{e^{m_i/T} + a_i} \quad \text{with} \quad a_i = \mp 1 \quad \text{for mesons/baryons}$$

$$\delta f_{\eta} \propto p^2$$

$$\delta f_{\zeta} \propto ???$$

Hadronic interactions

$$\Omega(2250) \leftrightarrow \Xi\pi K$$

...

$$f_0(1500) \leftrightarrow \pi\pi\pi\pi$$

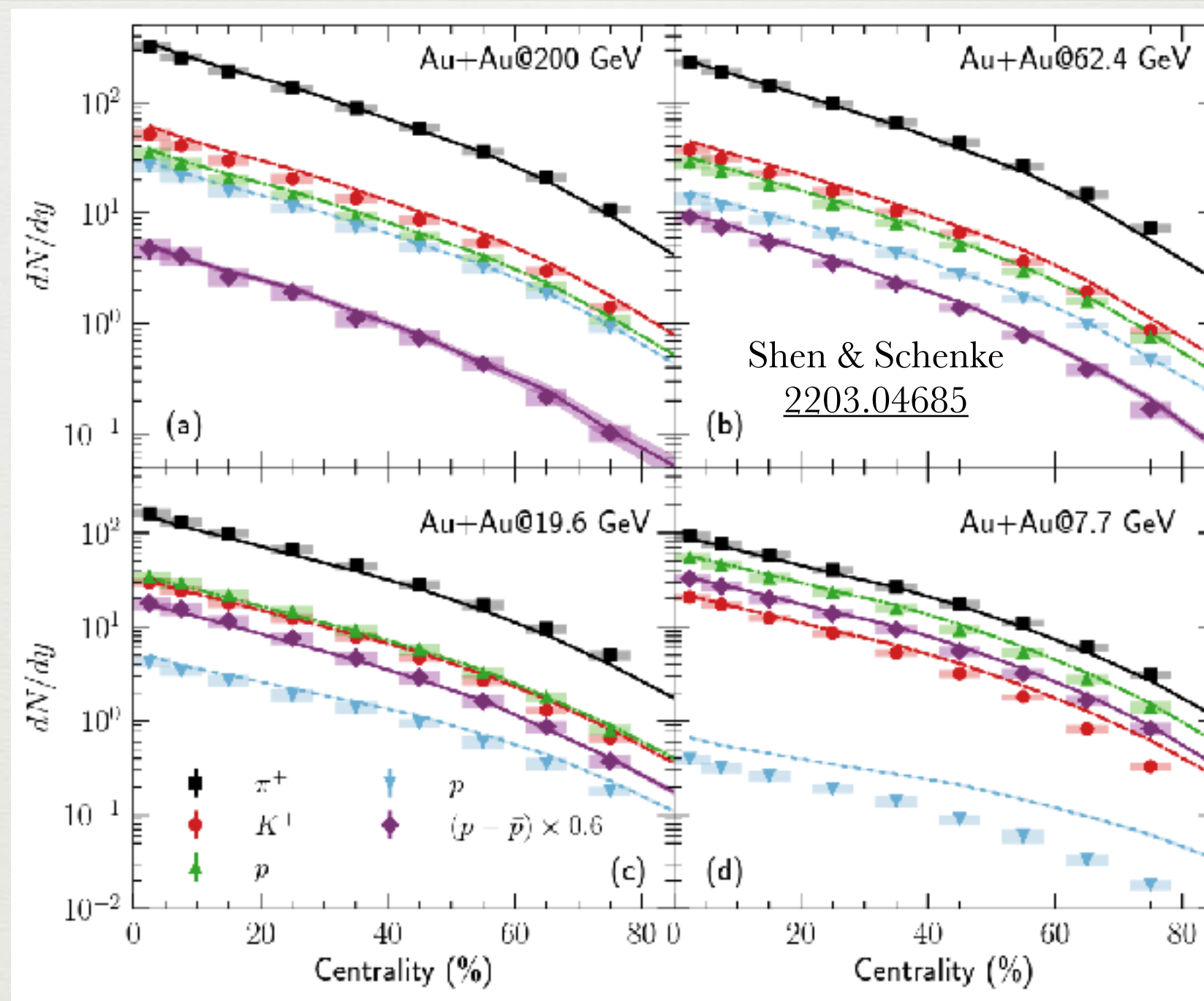
...

$$\rho \leftrightarrow \pi\pi$$

Stable particles: π , K , p , n , Λ , Ξ , Ω

Experiments measure all charged (stable) particles in specific kinematic regions

Multiplicity by identified particles

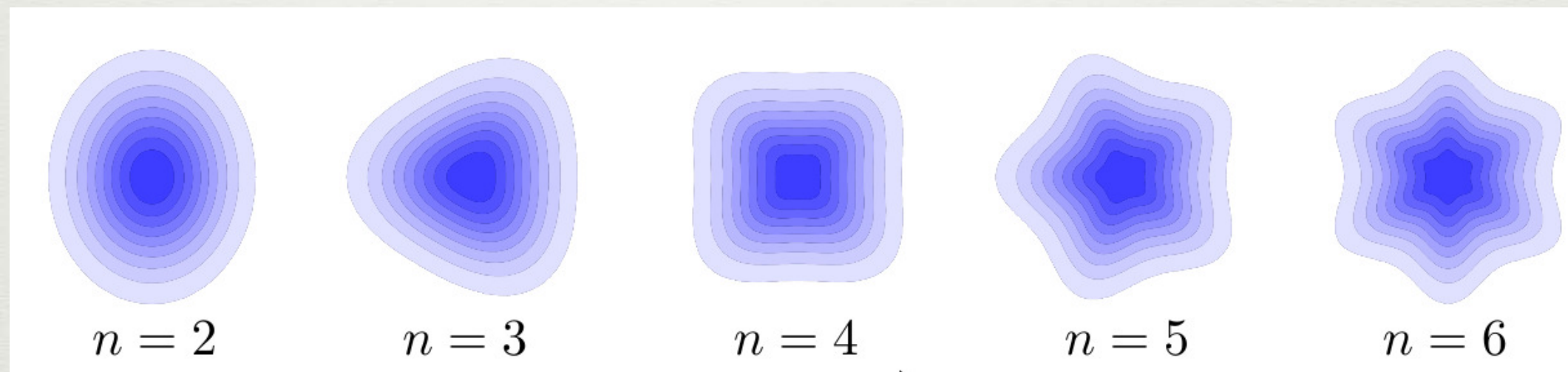


[STAR],Phys.Rev.C 96,044904(2017)

Quantifying flow

The distribution of particles can be written as a Fourier series

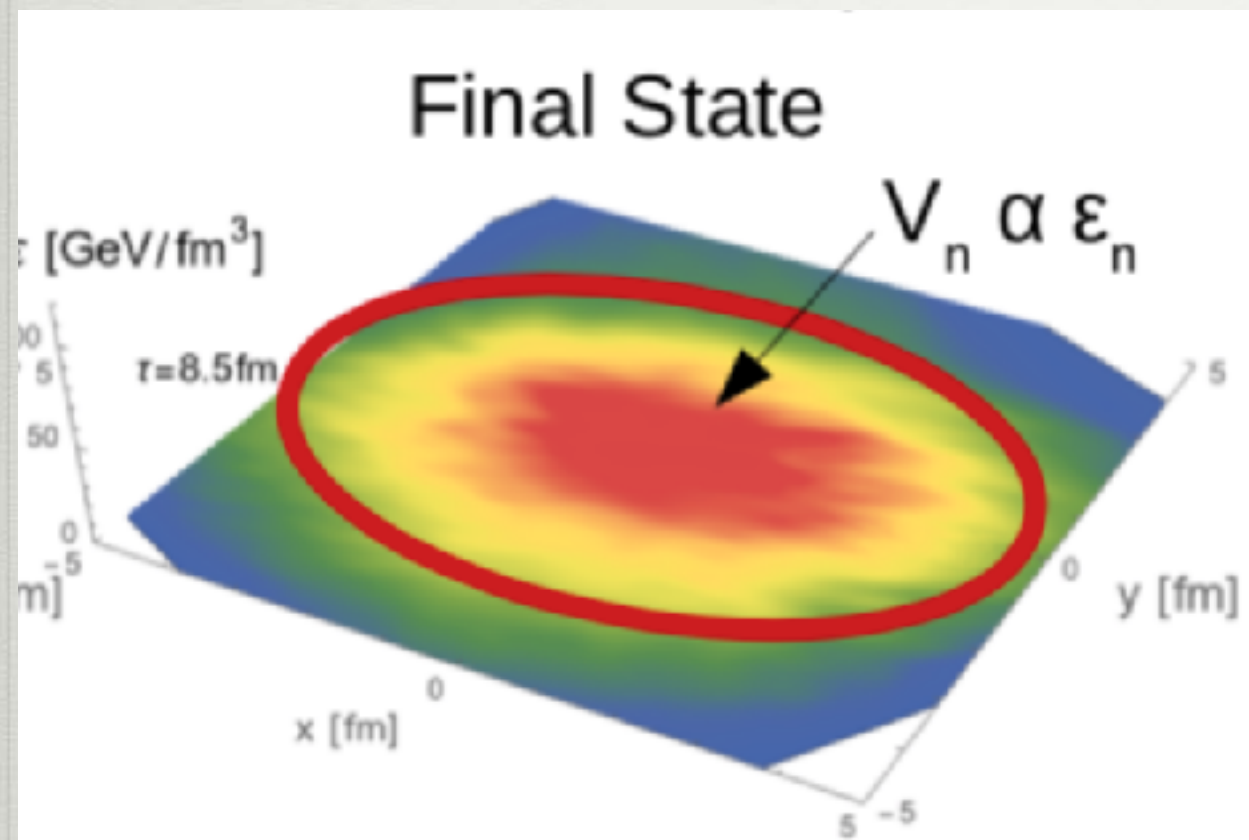
$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left[1 + \sum_n 2v_n \cos [n (\phi - \psi_n)] \right]$$



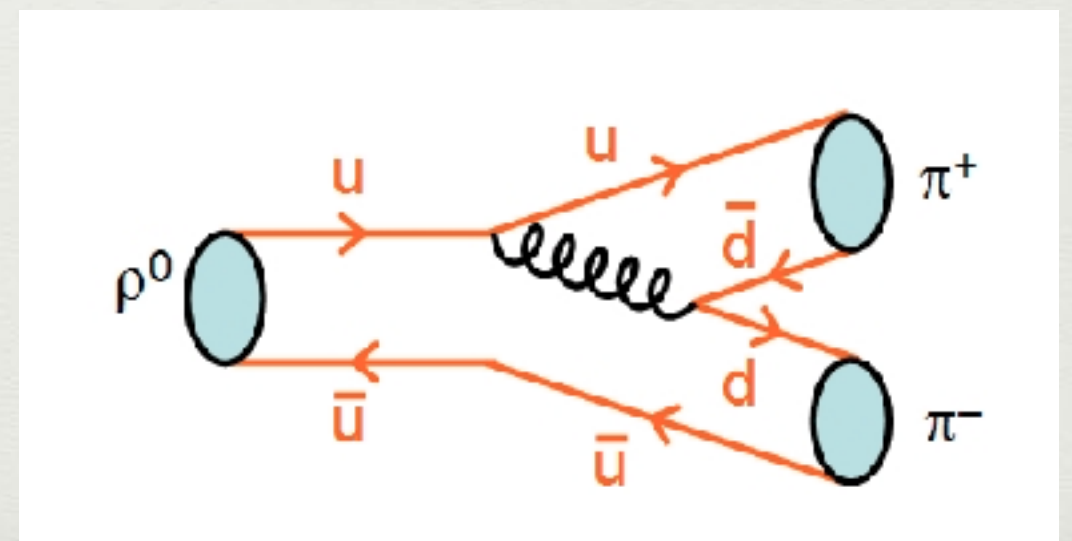
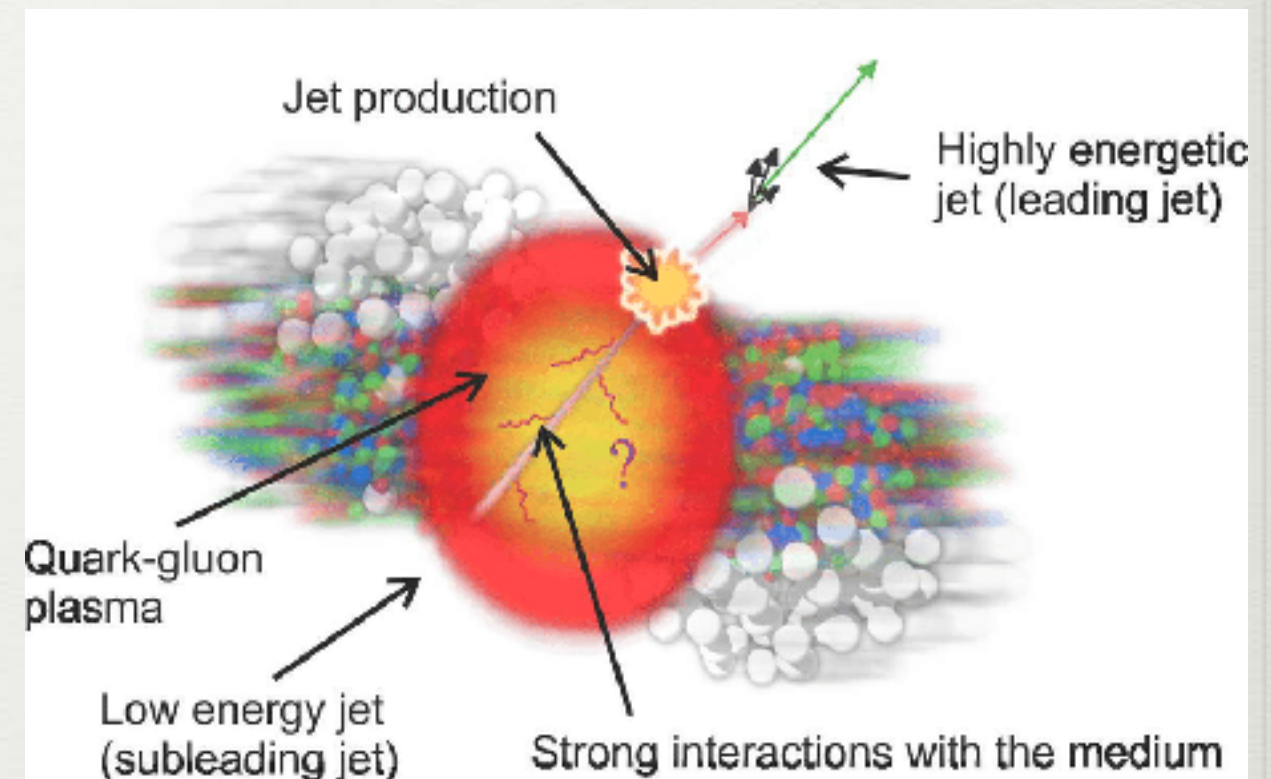
Collective flow: Flow harmonics, $v_n\{m\}$, are calculated by correlating $m=2$ to 8 particles \rightarrow collective behavior

Flow vs. Non-Flow

Flow



Non-Flow



Flow vs non-flow: rapidity cuts

Initial State
correlations

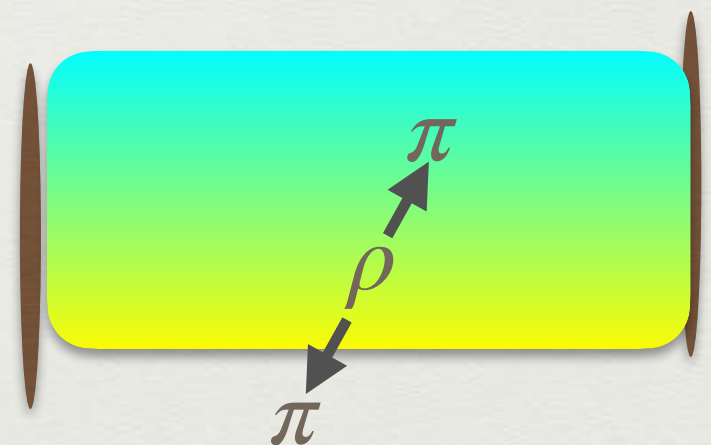
Final State
correlations

Rapidity

Time

Short-range correlations

Long-range correlations



Distribution of pairs in a *single event*:

$$\frac{dN_{pairs}}{d^3p^a d^3p^b} = \underbrace{\frac{dN}{d^3p^a} \frac{dN}{d^3p^b}}_{\text{factorizes}} + \underbrace{\delta_2(p^a, p^b)}_{\text{irreducible}}$$

$$\frac{dN}{d^3p}$$

FLOW: Independently emitted single particle contribution in a *single event*

Azimuthal anisotropies:

$$\delta_{2,n}$$

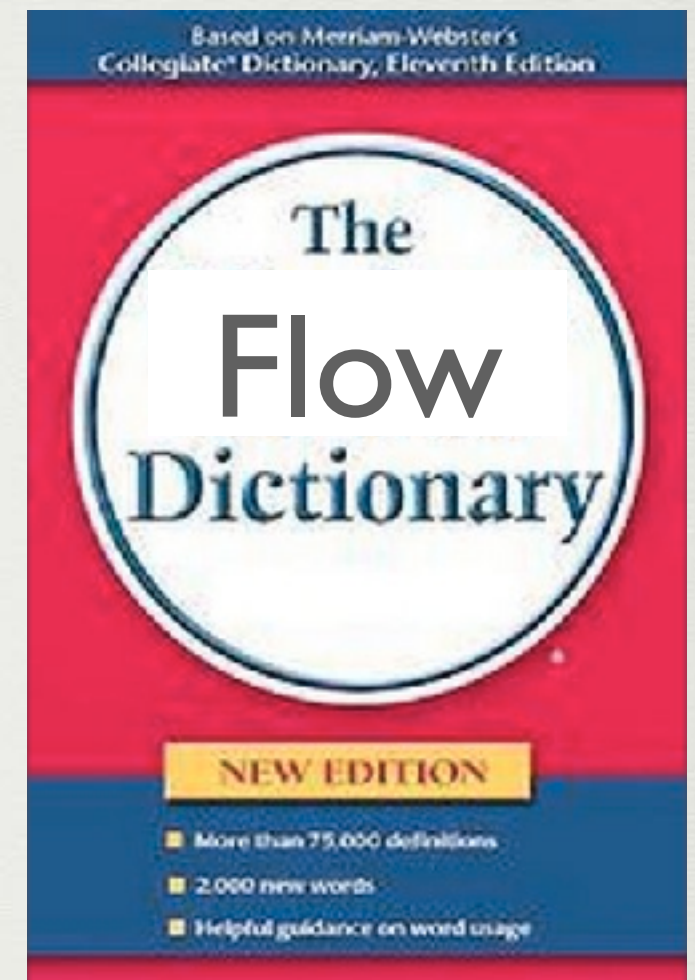
Non-FLOW: Irreducible 2 particle correlations from a *single event*

$$v_n = \langle e^{in\phi} \rangle$$

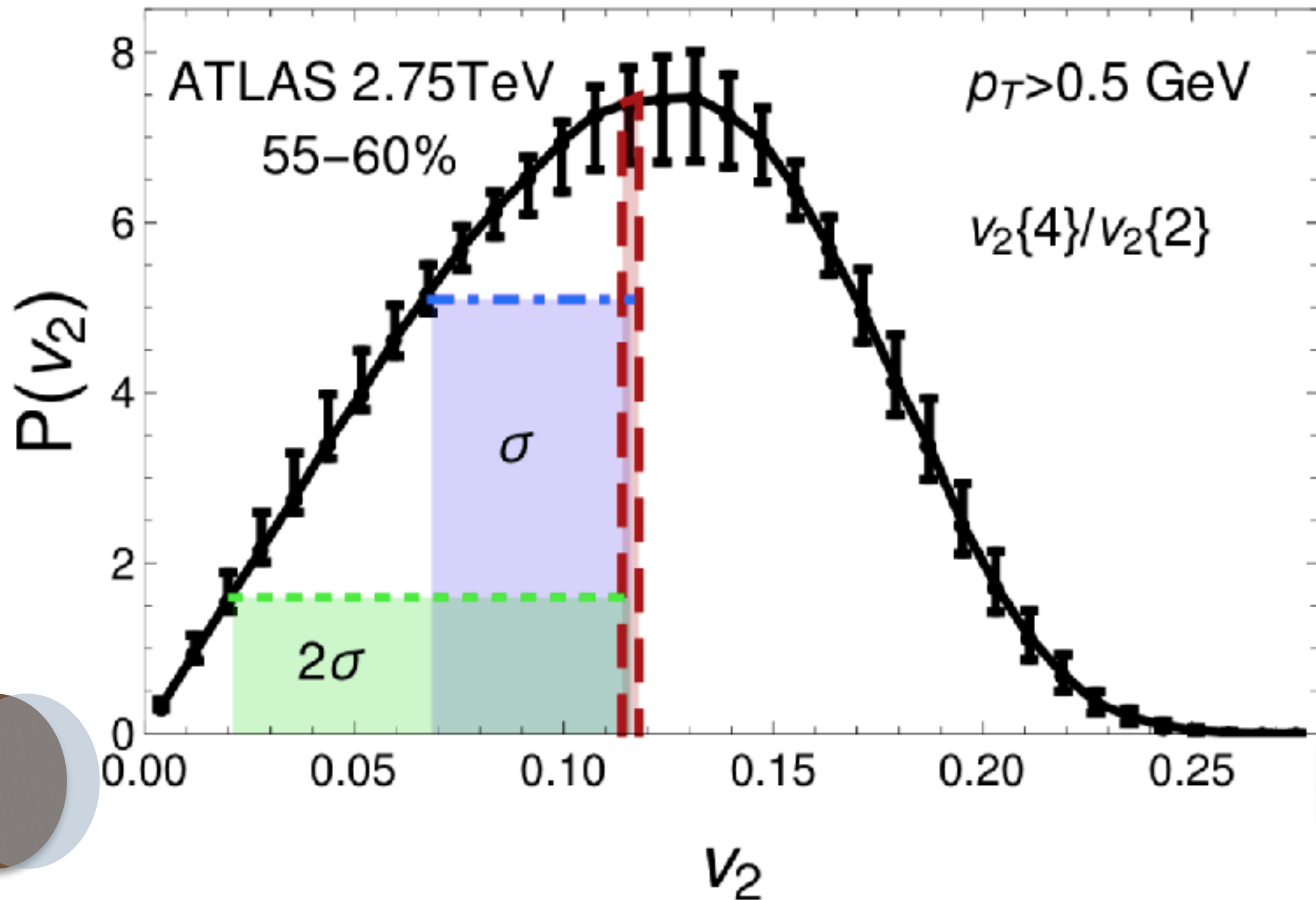
FLOW: Azimuthal anisotropy of a *single event* from independently emitted particles

$$\delta_{2,n}^v \equiv \frac{v_n^2}{N_{pairs}} \int d^3p^a d^3p^b \delta_2(p^a, p^b) \cos n(\phi^a + \phi^b - 2\Psi_n)$$

Correlations between flow and non-flow in a *single event*



Width of v_n distribution at fixed centrality



2 PARTICLE CUMULANT

$$v_n\{2\} = \sqrt{c_n\{2\}}$$

Event averaged quantity

$$c_n\{2\} = \langle \langle e^{in(\phi_1 - \phi_2)} \rangle_{pairs} \rangle_{events}$$

Here I'm explicitly defining that we average first over pairs then events, will generally just use $\langle \dots \rangle$ for both in the future.

Integrated

$$v_n\{2\} = \left(\underbrace{\langle \bar{v}_n^2 \rangle}_{flow} + \underbrace{\langle \delta_{2,n} \rangle}_{non\ flow} \right)^{1/2}$$

Cumulants

Single Event:

$$V_n = v_n e^{in\Psi_n} \equiv \frac{\int d^3p \frac{dN}{d^3p} e^{in\phi_p}}{\int d^3p \frac{dN}{d^3p}},$$

Event Averaging

$$\langle \dots \rangle = \frac{\sum_i^{\text{events}} \text{Re}\{\dots\}_i W(n_s, n_h; p_T)_i}{\sum_i^{\text{events}} W(n_s, n_h; p_T)_i},$$

2-particle correlation

$$c_n\{2\} = \frac{\sum_{j=\text{cent}_{\text{start}}}^{\text{cent}_{\text{end}}} c_{n,j}\{2\} \sum_i^{N_{\text{ev}}^j} W(2,0)_i}{\sum_{j=\text{cent}_{\text{start}}}^{\text{cent}_{\text{end}}} \sum_i^{N_{\text{ev}}^j} W(2,0)_i}$$

4 particle correlation

$$c_n\{4\} = \frac{\sum_{j=\text{cent}_{\text{start}}}^{\text{cent}_{\text{end}}} c_{n,j}\{4\} \sum_i^{N_{\text{ev}}^j} W(4,0)_i}{\sum_{j=\text{cent}_{\text{start}}}^{\text{cent}_{\text{end}}} \sum_i^{N_{\text{ev}}^j} W(4,0)_i}.$$

Statistical error: jackknife resampling

Mean value of an observable x

$$\langle x \rangle = \frac{1}{N} \sum_i^N x_i$$

We want to check that removing one of our sample set won't radically change our results

Mean removing j^{th} sample

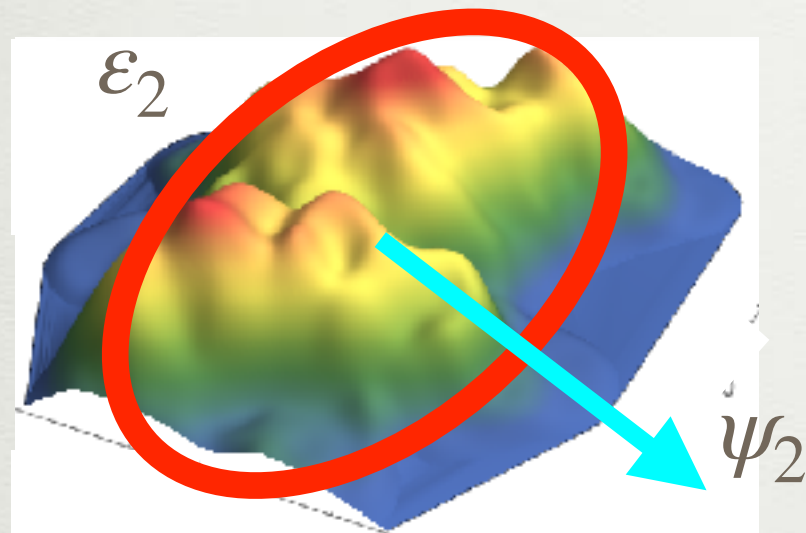
$$\langle x_{ex,j} \rangle = \frac{1}{N-1} \sum_{i,i \neq j} x_i$$

Variance over entire sample set

$$\sigma^2 = \frac{1}{N(N-1)} \sum_j^N \left(\langle x_{ex,j} \rangle - \langle x \rangle \right)^2$$

Quantifying initial and final state

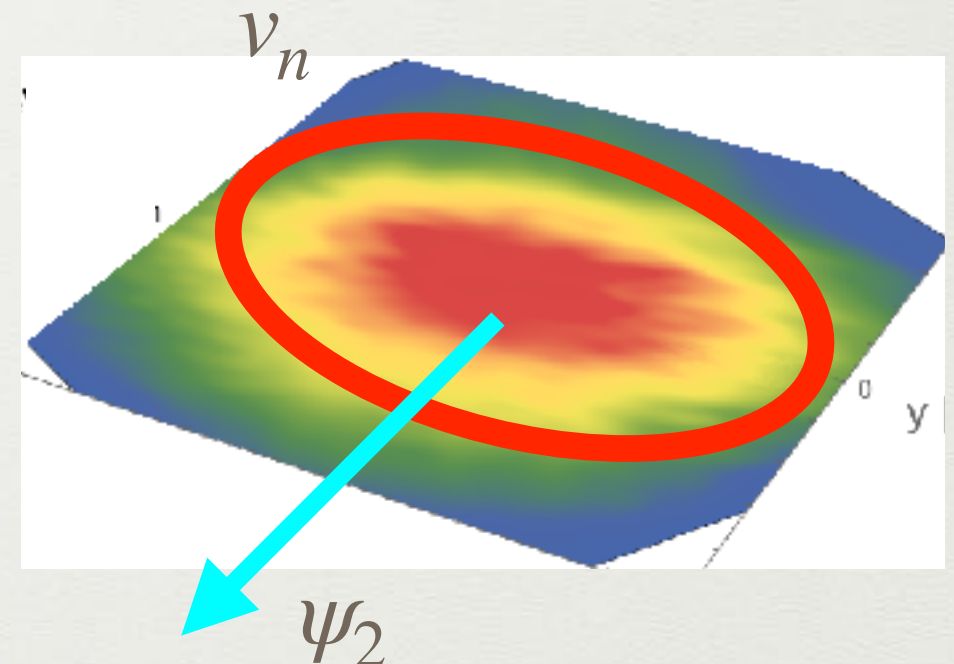
$$\mathcal{E}_n \equiv \varepsilon_n e^{in\Phi_n}$$



Calculated in Coordinate space

Pearson Coefficient

$$V_n \equiv v_n e^{in\psi_n}$$



Measured in Momentum space

$$Q_n = \frac{\text{Re}\langle V_n \mathcal{E}_n^* \rangle}{\langle |V_n|^2 \rangle \langle |\mathcal{E}_n|^2 \rangle}$$

Connecting initial shape \mathcal{E}_n to final flow V_n

Linear response

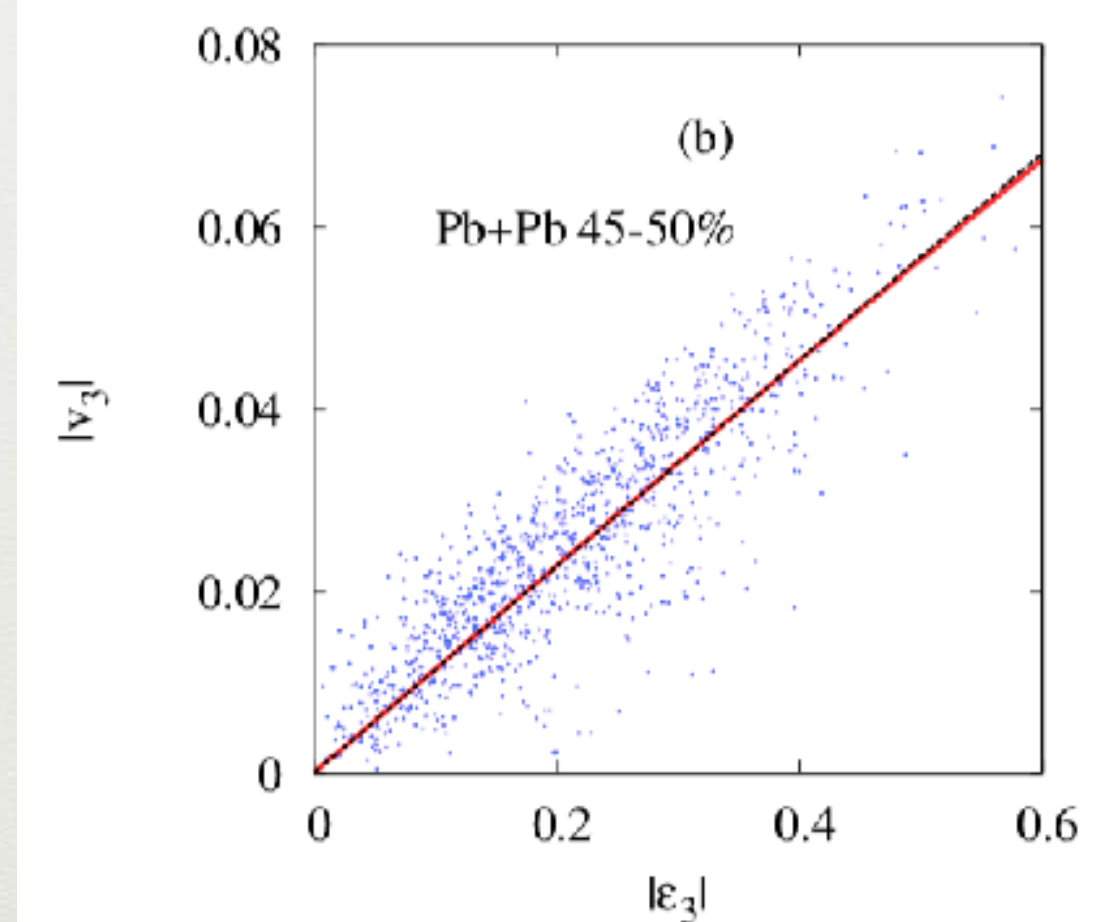
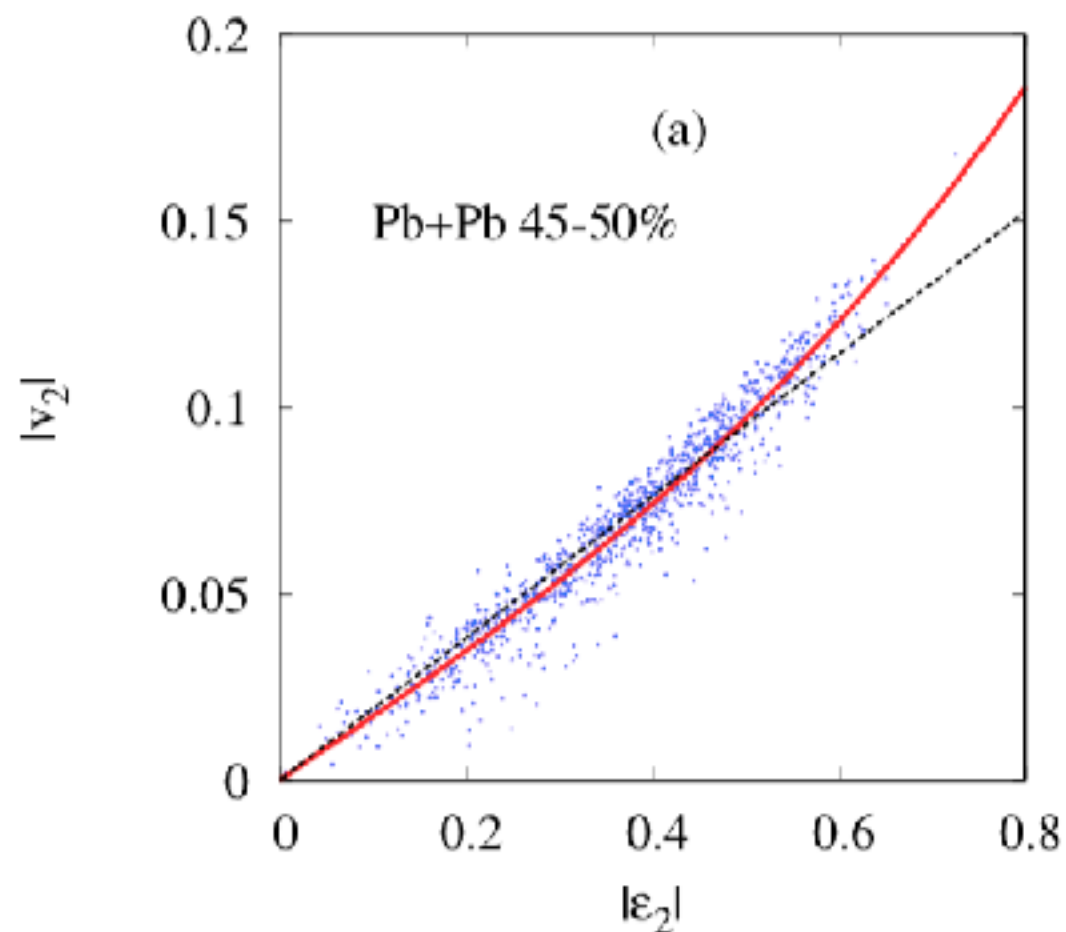
$$V_n^{pred} = \gamma_n \mathcal{E}_n$$

Teaney,Yan,PRC83(2011)064904;Gardim,et al,PRC85(2012)024908;PRC91(2015)3,034902

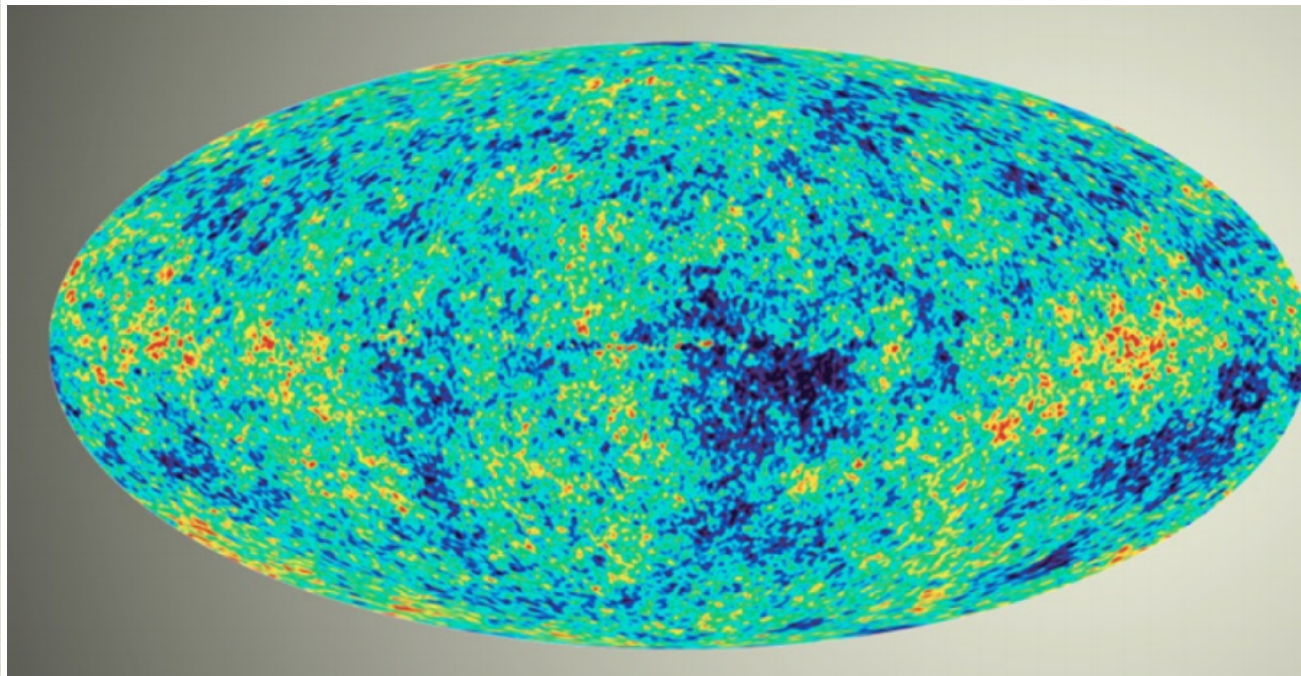
Linear+cubic response

$$V_n^{pred} = \kappa_{1,n} \mathcal{E}_n + \kappa_{2,n} |\mathcal{E}_n|^2 \mathcal{E}_n$$

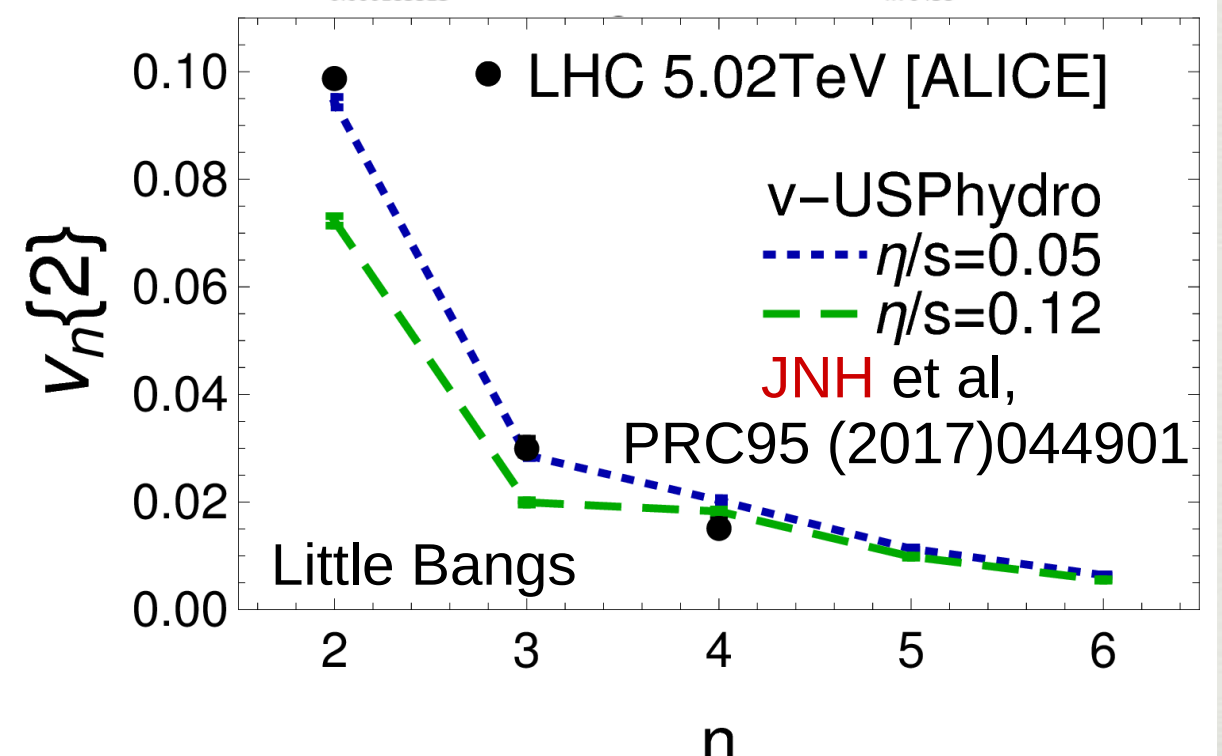
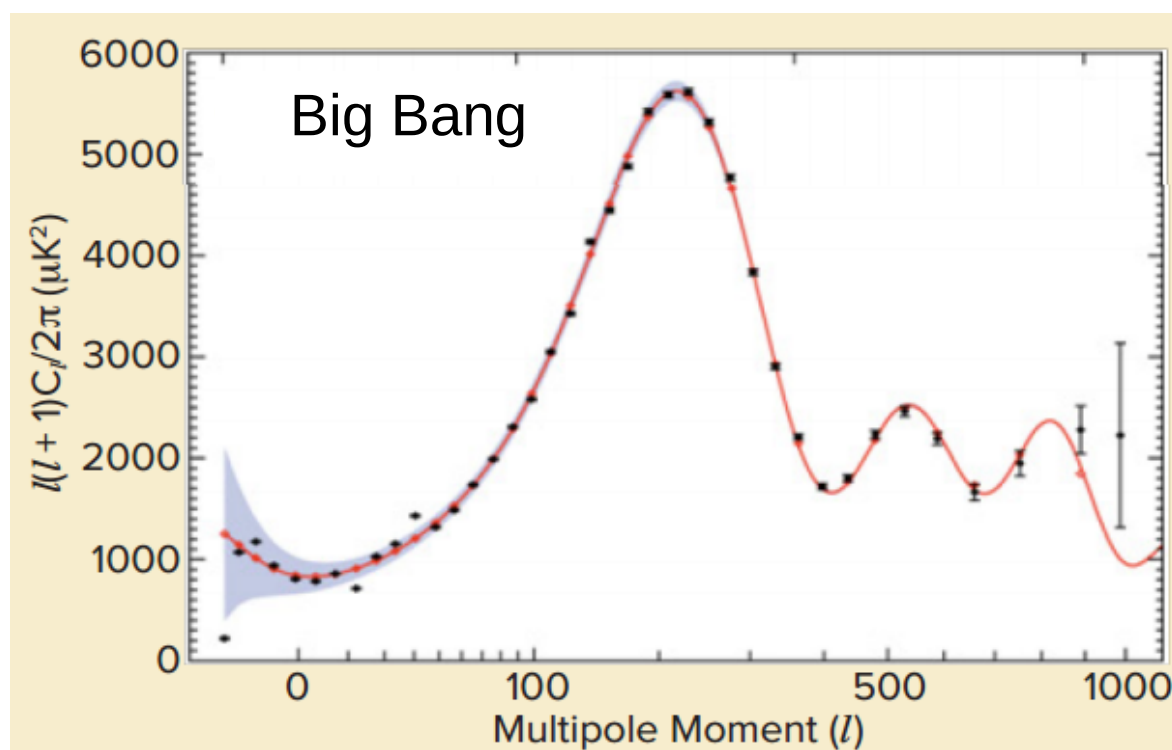
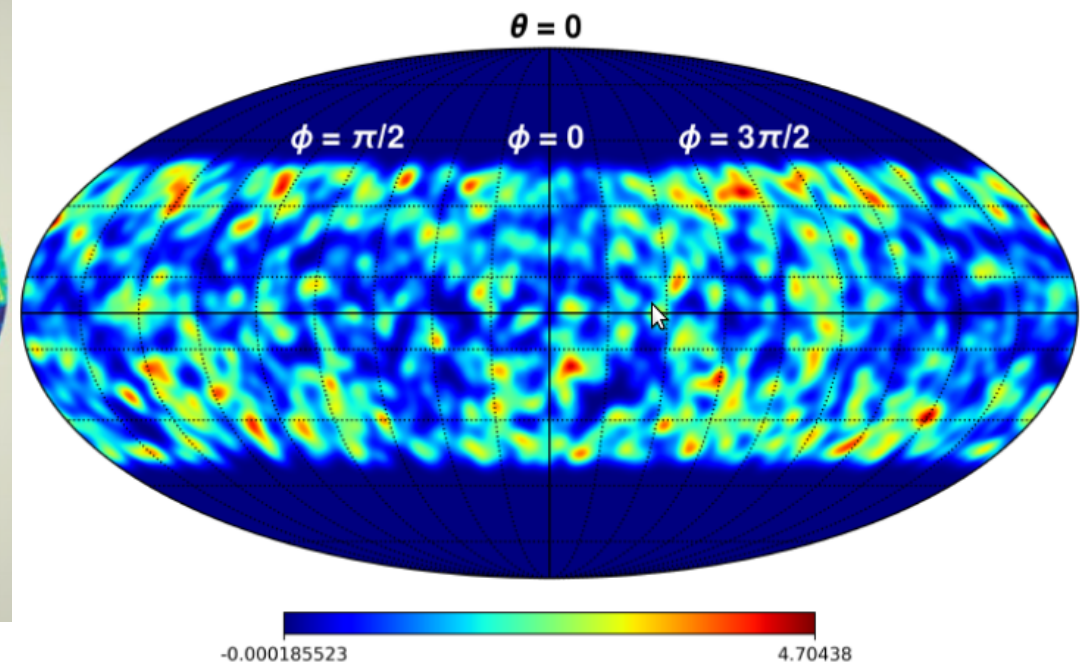
JNH,Yan,Gardim,Ollitrault Phys. Rev. C 93, 014909 (2016)



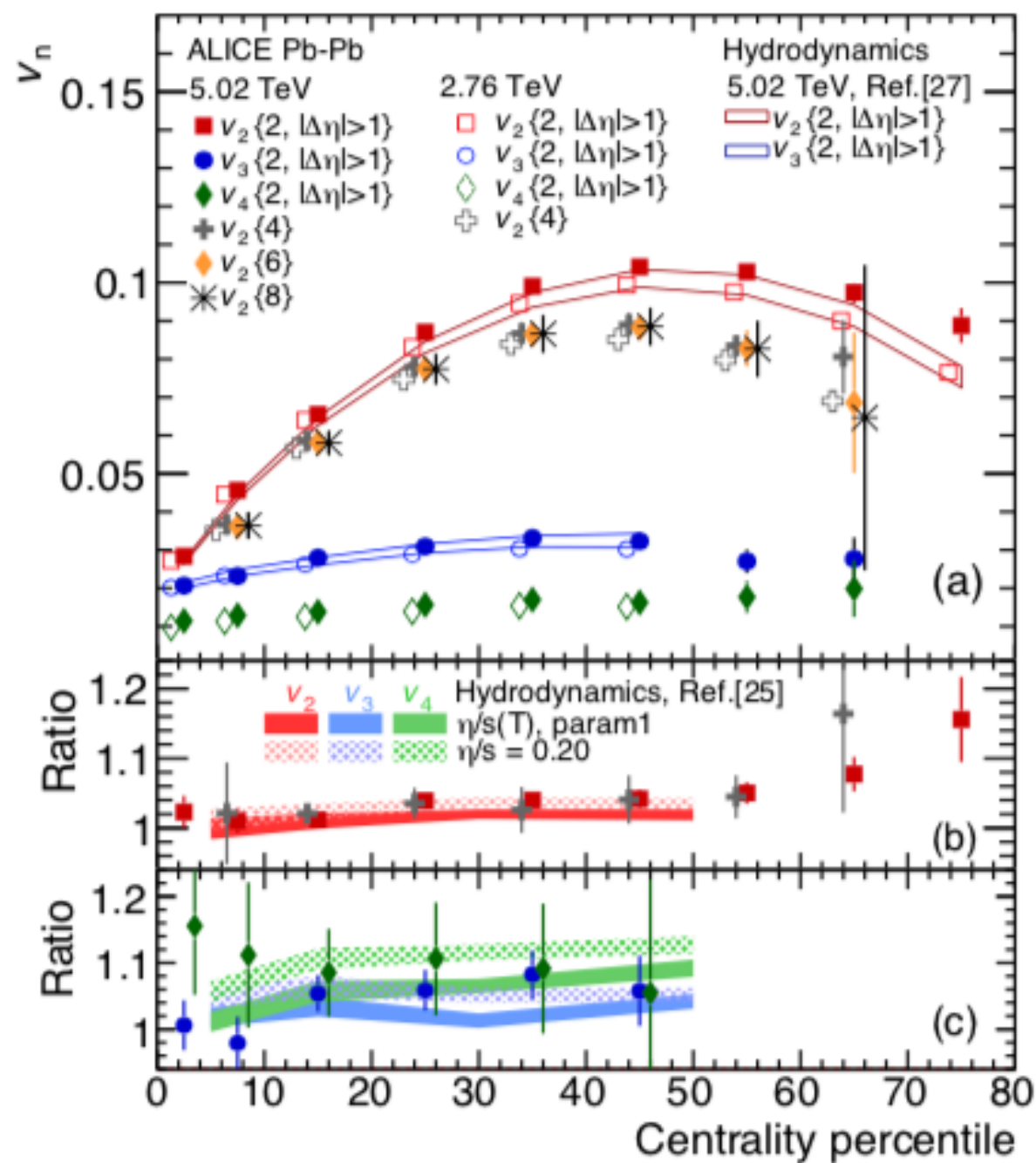
CMB vs. Heavy Ion Collisions



Vieira. Machado et al, Phys.Rev. C99 (2019) no.5, 054910



Precise predictions with hydrodynamics



Hydrodynamic models can successfully make predictions at the $\sim 1\%$ level.

ALICE Phys.Rev.Lett. 116 (2016) no.13, 132302

v-USPhydro predictions: JNH et al, Phys.Rev. C93 (2016) no.3, 034912

EKRT predictions: Niemi et al, Phys. Rev. C 93, 014912 (2016)

When is fluid dynamics applicable?

Large separation of scales
(Knudsen Number)

$$Kn \sim \frac{\text{Small scale}^* (H_2O \text{ molecule})}{\text{Large scale (size of lake)}}$$

$$Kn_{\text{Lake Michigan}} = \frac{3 \cdot 10^{-10} \text{ m}}{500,000 \text{ m}} \sim 10^{-17}$$

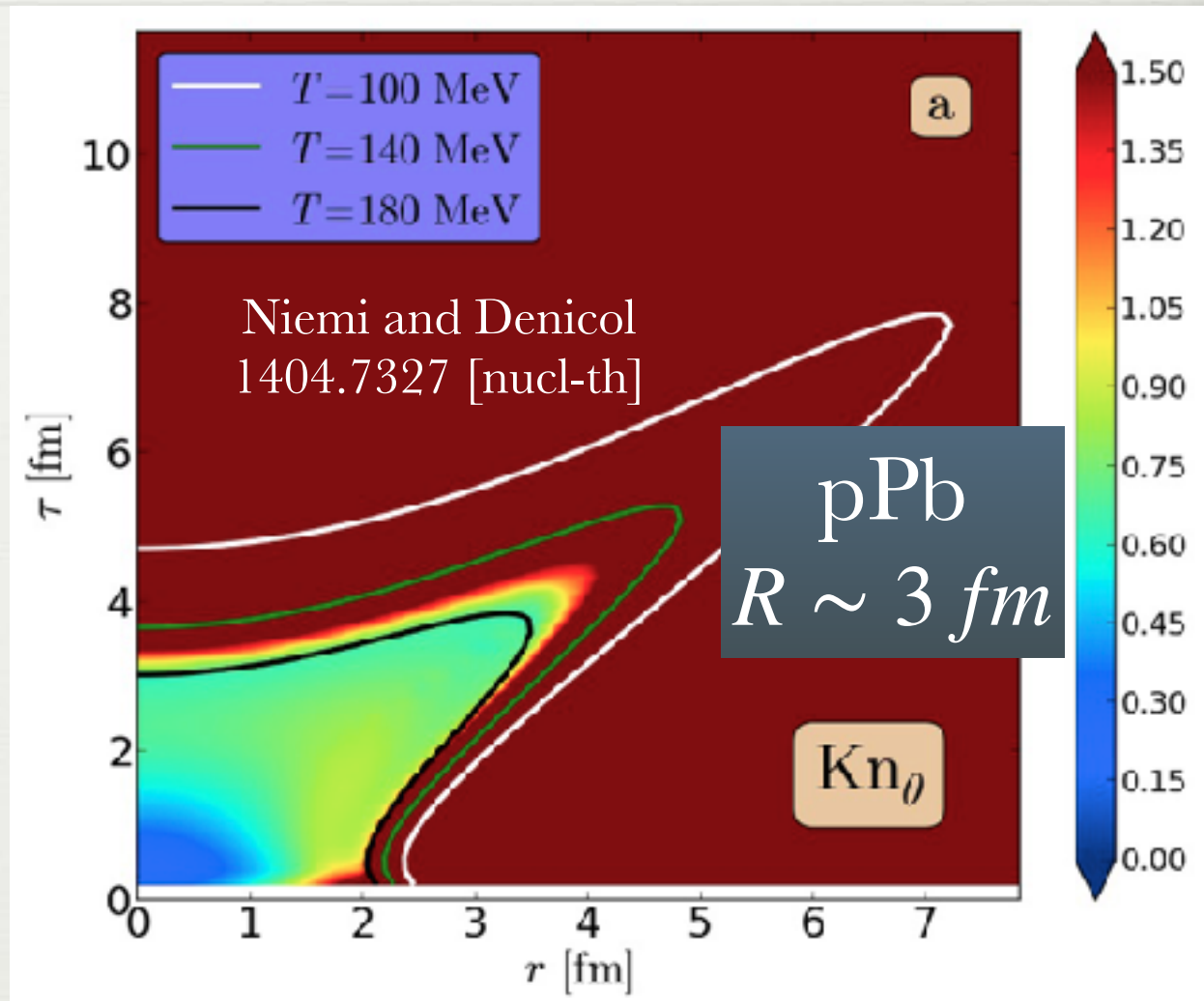
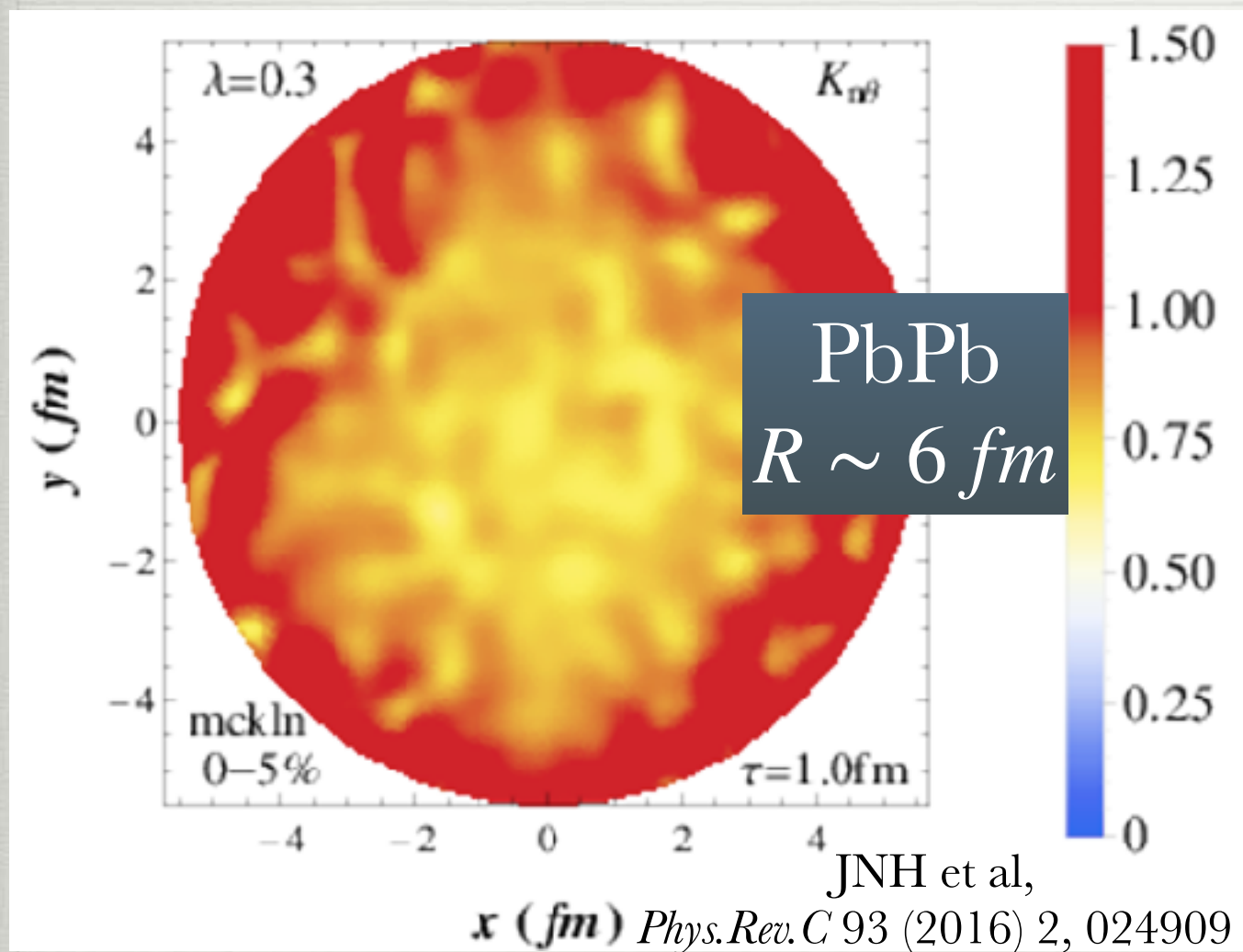
* distance before H_2O hits something

Question: When can you apply fluid dynamics?

Answer: $Kn \ll 1$



Applicability of hydrodynamics when far-from-equilibrium?



$$Kn \sim 1$$

“Large” PbPb systems already begin far-from-equilibrium, small pPb collisions have a small region of applicability

What are people studying today?

Collective flow

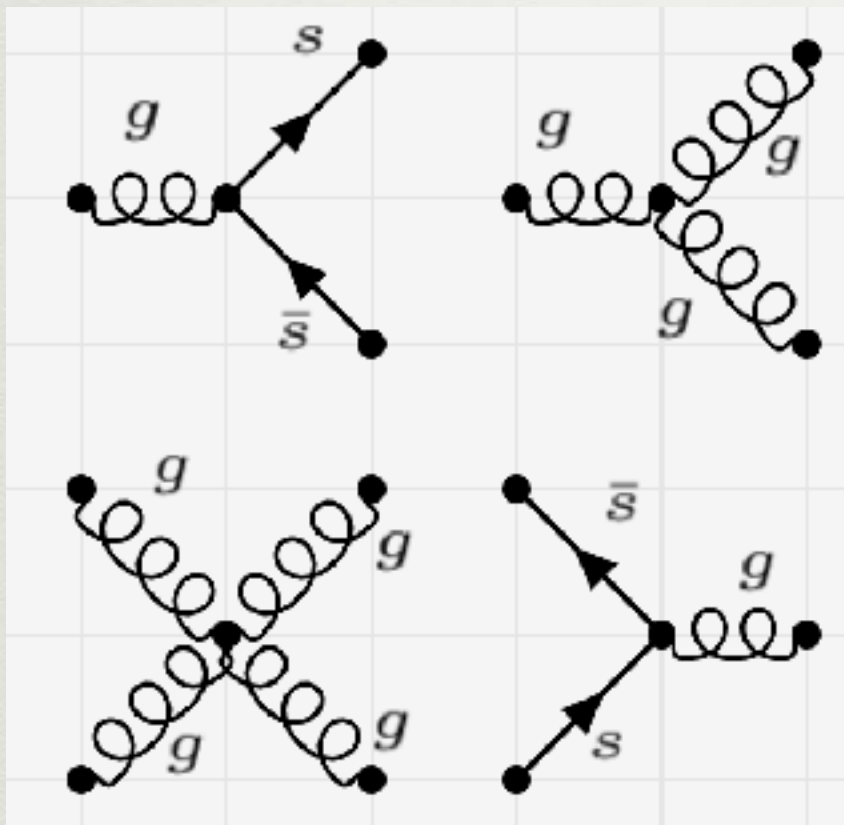
- Far-from-equilibrium relativistic viscous fluid dynamics
- Connecting relativistic viscous fluid dynamics to General Relativity (Neutron Star mergers)
- Applicability of hydrodynamics in small systems
- Bayesian analysis for transport coefficients
- Large baryon densities/EOS for neutron stars

Strangeness Enhancement

Strangeness production: QGP vs HRG

Gluon can efficiently create
 $gg \rightarrow s\bar{s}$, $g \rightarrow s\bar{s}$ pairs

Time scale $\tau \sim 10^{-24}$ s



Heavy resonances decay into
 $X \leftrightarrow K\bar{K}$

Mass of X must be $m_X > 1\text{GeV}$ to
 allow for conservation of energy

Lights resonance $\phi(1080)$

Decay width $\Gamma = 4\text{ MeV}$,

Branching Ratio

$\phi \rightarrow K\bar{K}$ is $Br_{\phi,K\bar{K}} \sim 0.8$

$$\text{Time scale } \tau = \frac{1}{\Gamma_{\phi} Br_{\phi,K\bar{K}}} \sim 10^{-22} \text{ s}$$

Strangeness in thermal equilibrium

Strangeness neutrality $\Delta S = N_S - N_{\bar{S}} = 0$

However, $N_S \neq 0$ and $N_{\bar{S}} \neq 0$

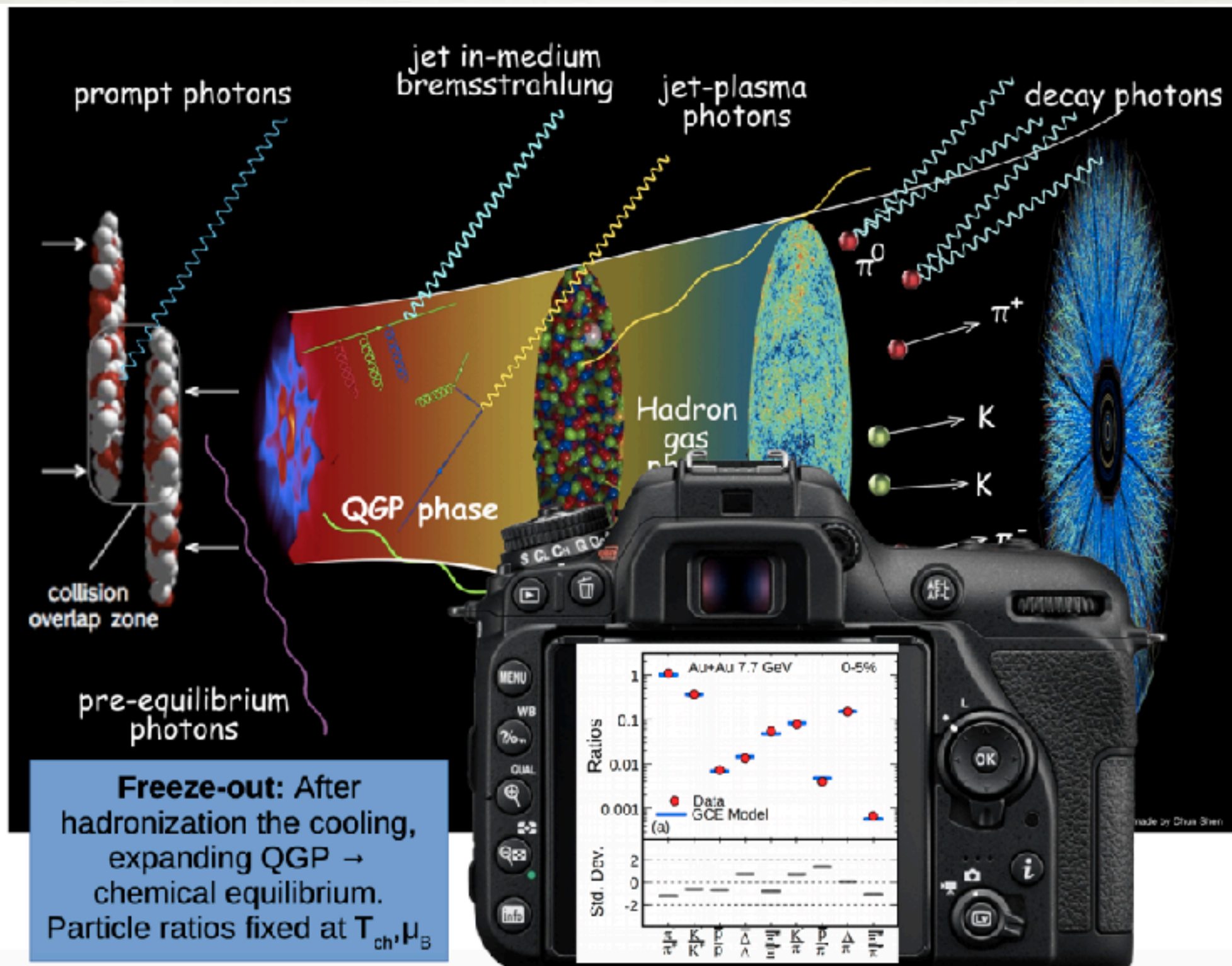
$$p(T, \vec{\mu}) = \frac{T}{V} \sum_i \ln Z_i(T, \vec{\mu}_i)$$

$$\ln Z_i(T, \vec{\mu}) \sim \frac{d_i}{2\pi} \left(\frac{m_i}{T} \right)^2 \sum_{k=1}^{\infty} \frac{(-1)^{(|B_i|-1)(k+1)}}{k^2} K_2 \left(\frac{km_i}{T} \right) \cosh [k \vec{\mu}_i / T]$$

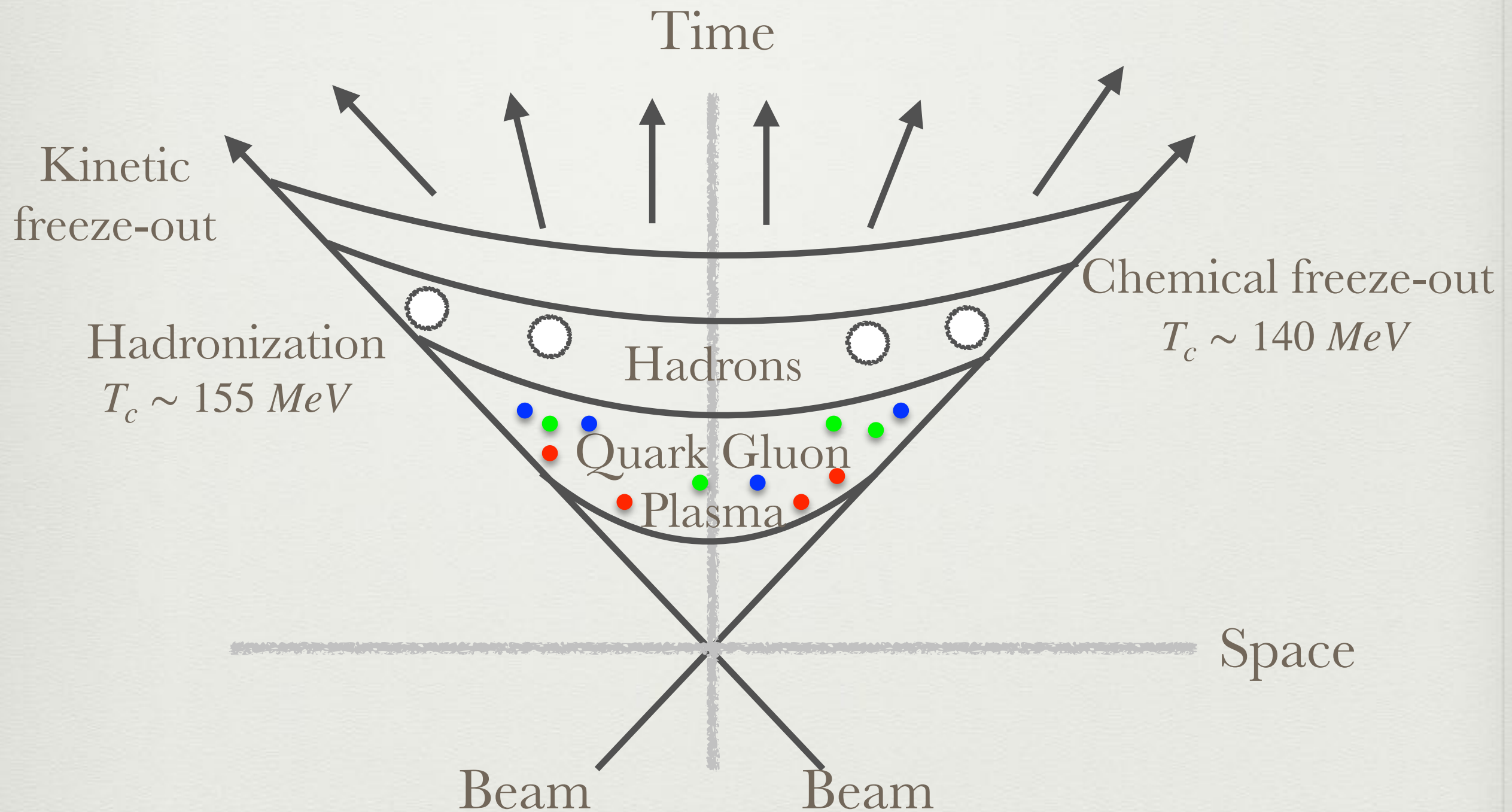
Where $\vec{\mu}_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$

Note, $k = 1$ is a decent approximation

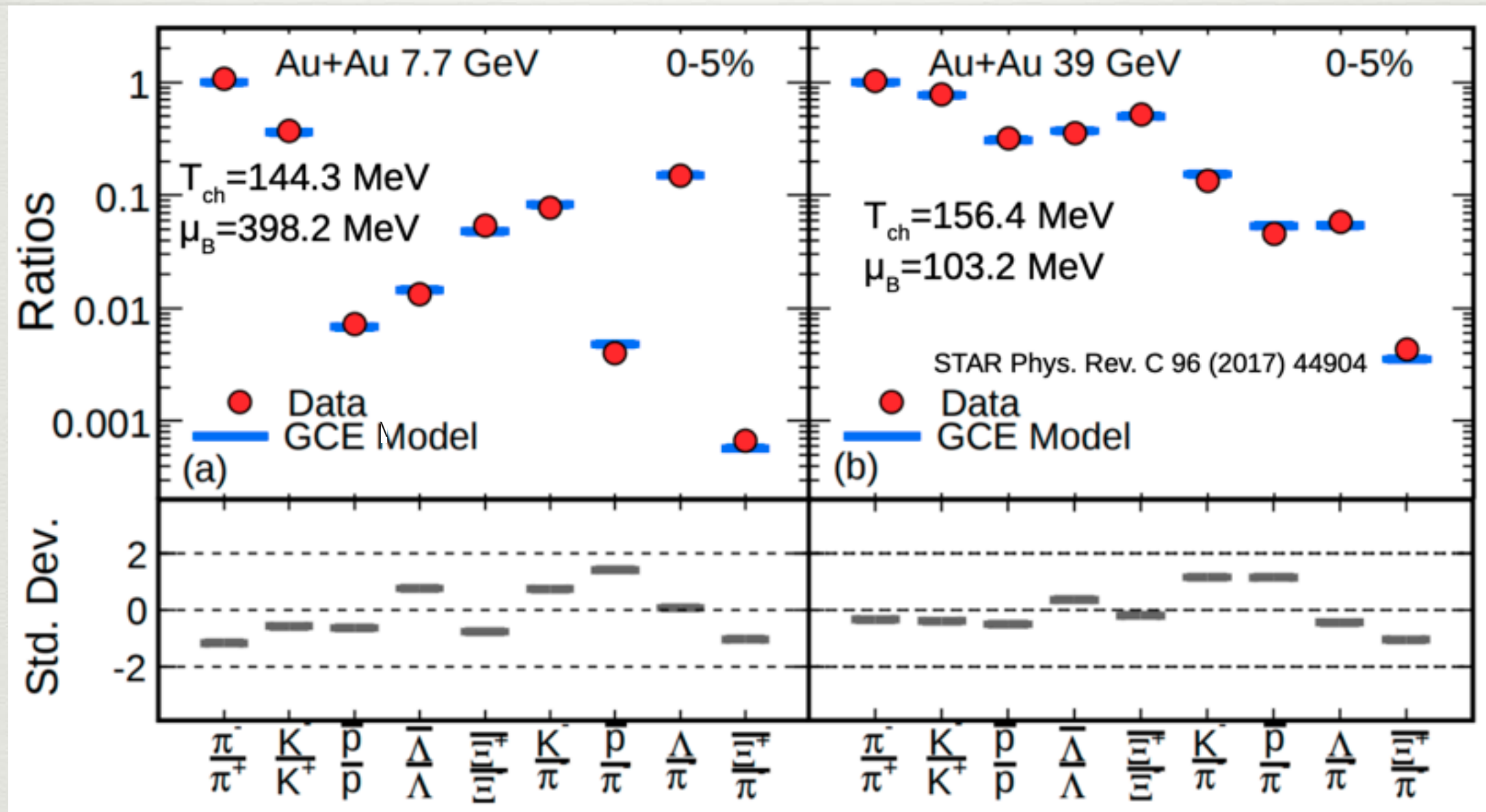
Snapshot of the temperature



Hadronization and freeze-out



Thermal models: extracting freeze-out temperatures



- Assume particles are in thermal and chemical equilibrium
- Calculate ratios of particles in HRG across T and μ_B (volume cancels)
- Extract temperature at freeze-out

Strangeness in thermal equilibrium

Take a gas of K ($B = 0, S = \pm 1$) vs π ($B = 0, S = 0$) at $\vec{\mu} = 0$

$$p(T, \vec{\mu})_K / T^4 = \frac{1}{\pi^2} \left(\frac{m_K}{T} \right)^2 K_2 \left(\frac{m_K}{T} \right)$$

$$p(T, \vec{\mu})_\pi / T^4 = \frac{3}{2\pi^2} \left(\frac{m_\pi}{T} \right)^2 K_2 \left(\frac{m_\pi}{T} \right)$$

Freeze-out $T \sim 150$ MeV, check ratio!

From HRG $N_K/N_\pi = 0.18$

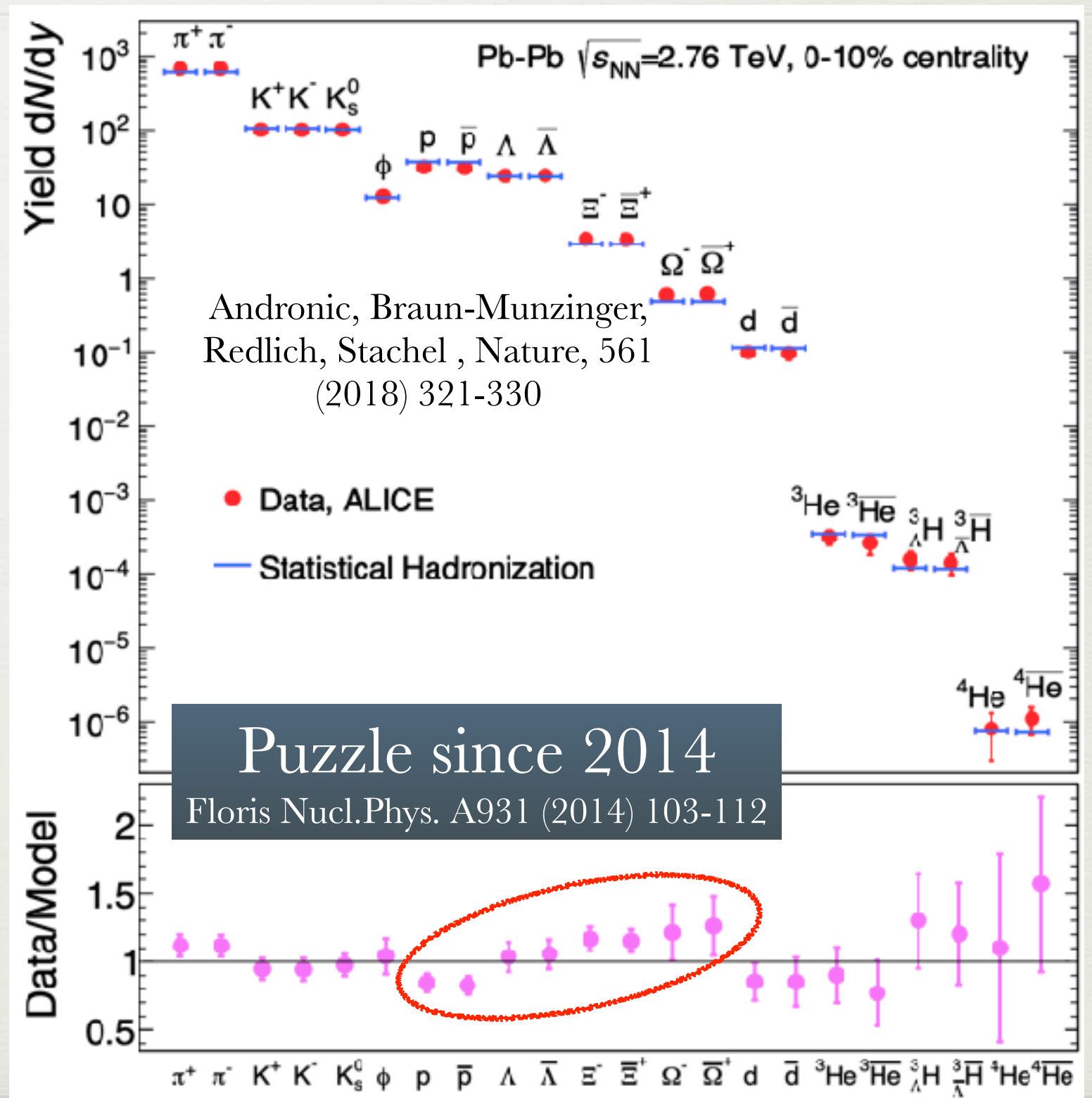
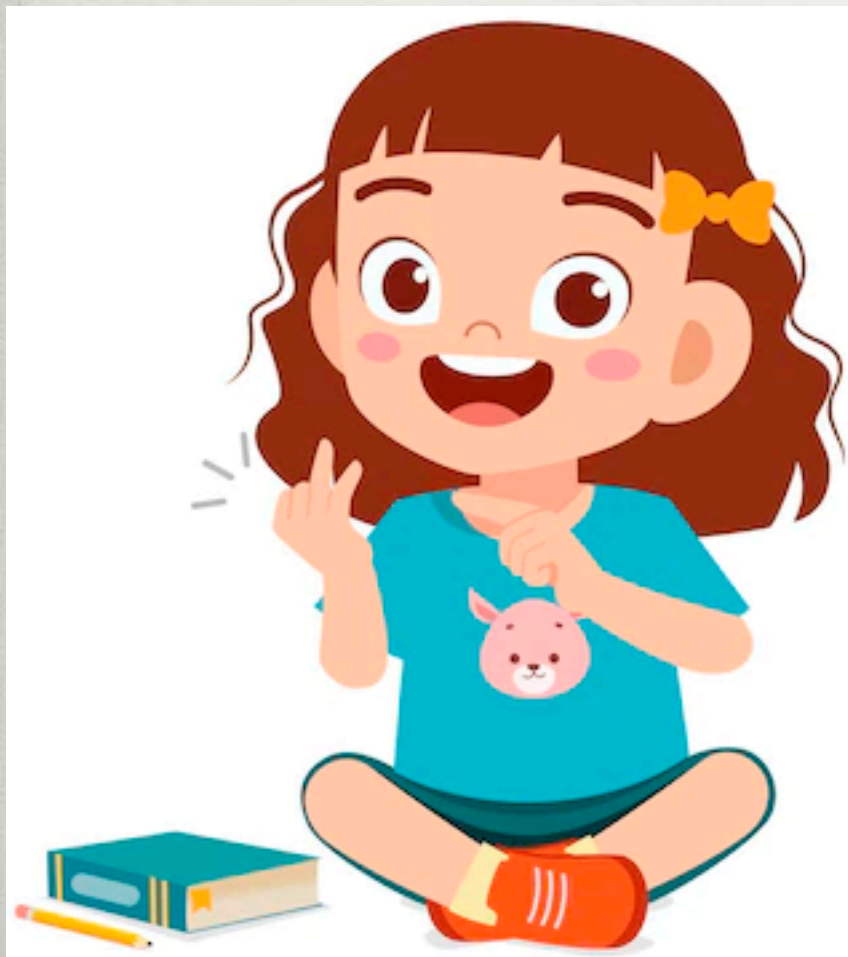
From ALICE $N_K/N_\pi = 0.16$

[ALICE] *Phys.Rev.C* 101 (2020) 4, 044907

Quick estimate, strangeness appears to be thermal equilibrium!

Measuring Strangeness Enhancement

Count up strange particles, are strange quarks thermalized?

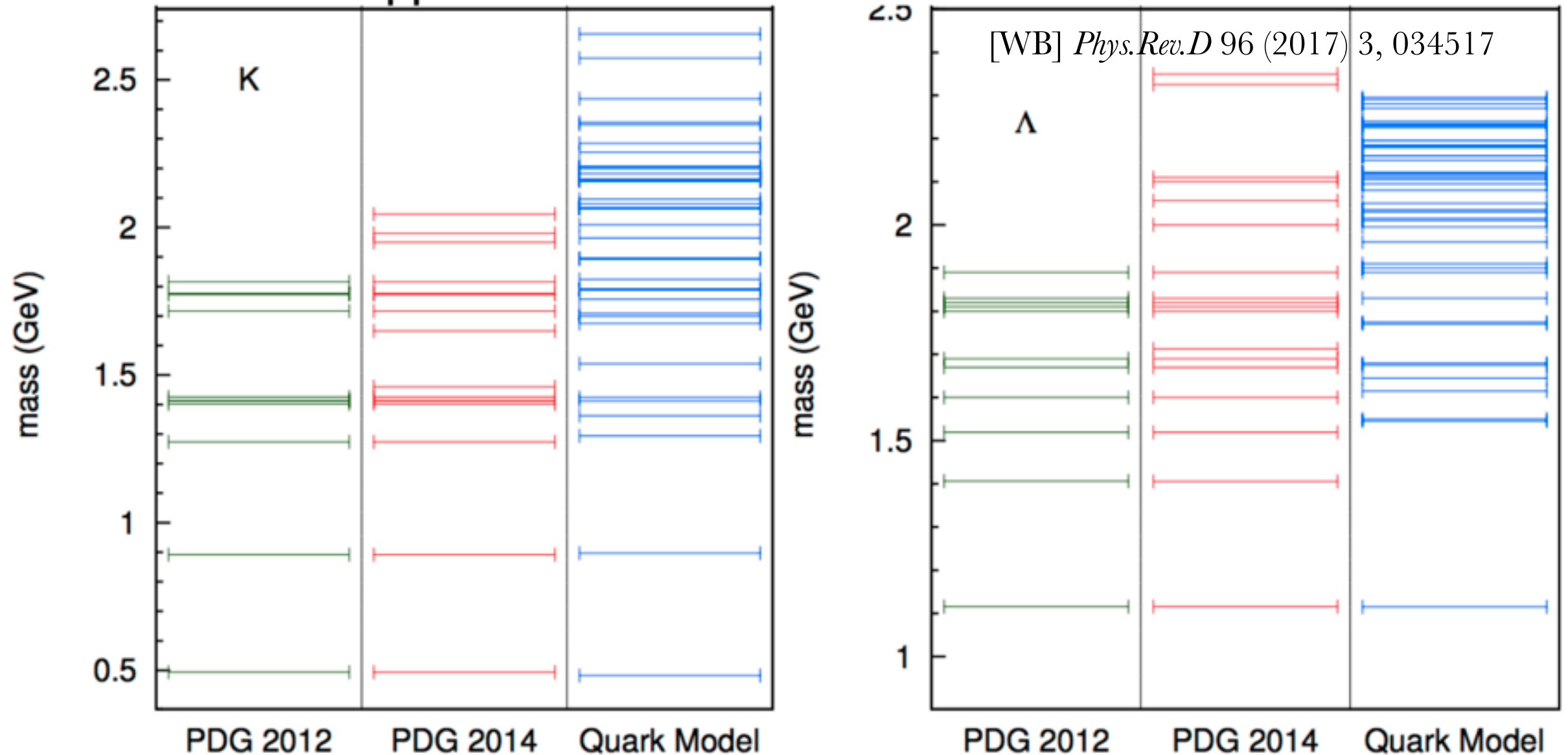


Proposed solutions to proton-to-pion puzzle

- Missing resonances
- Specific hadronic interactions (e.g. $B\bar{B}$ annihilation)
- Flavor hierarchy (different light and strange freeze-out temperatures)
- Dynamical freeze-out
- All of the above?

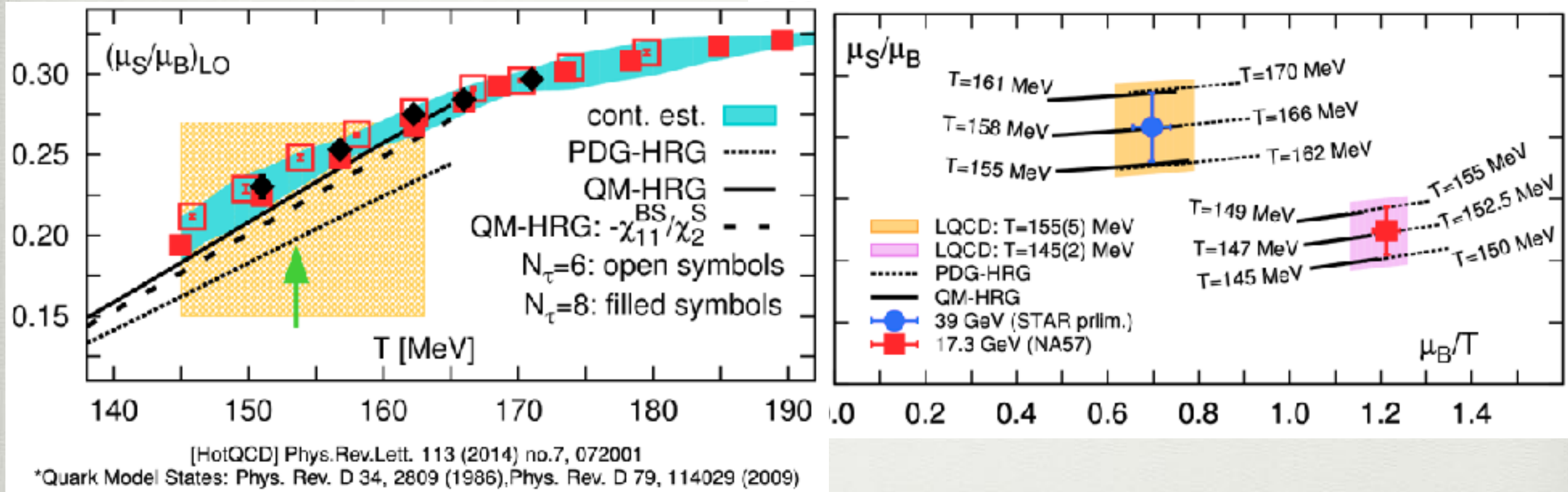
2014 Particle Data Group, many new states

□ New states appear in the 2014 version of the PDG



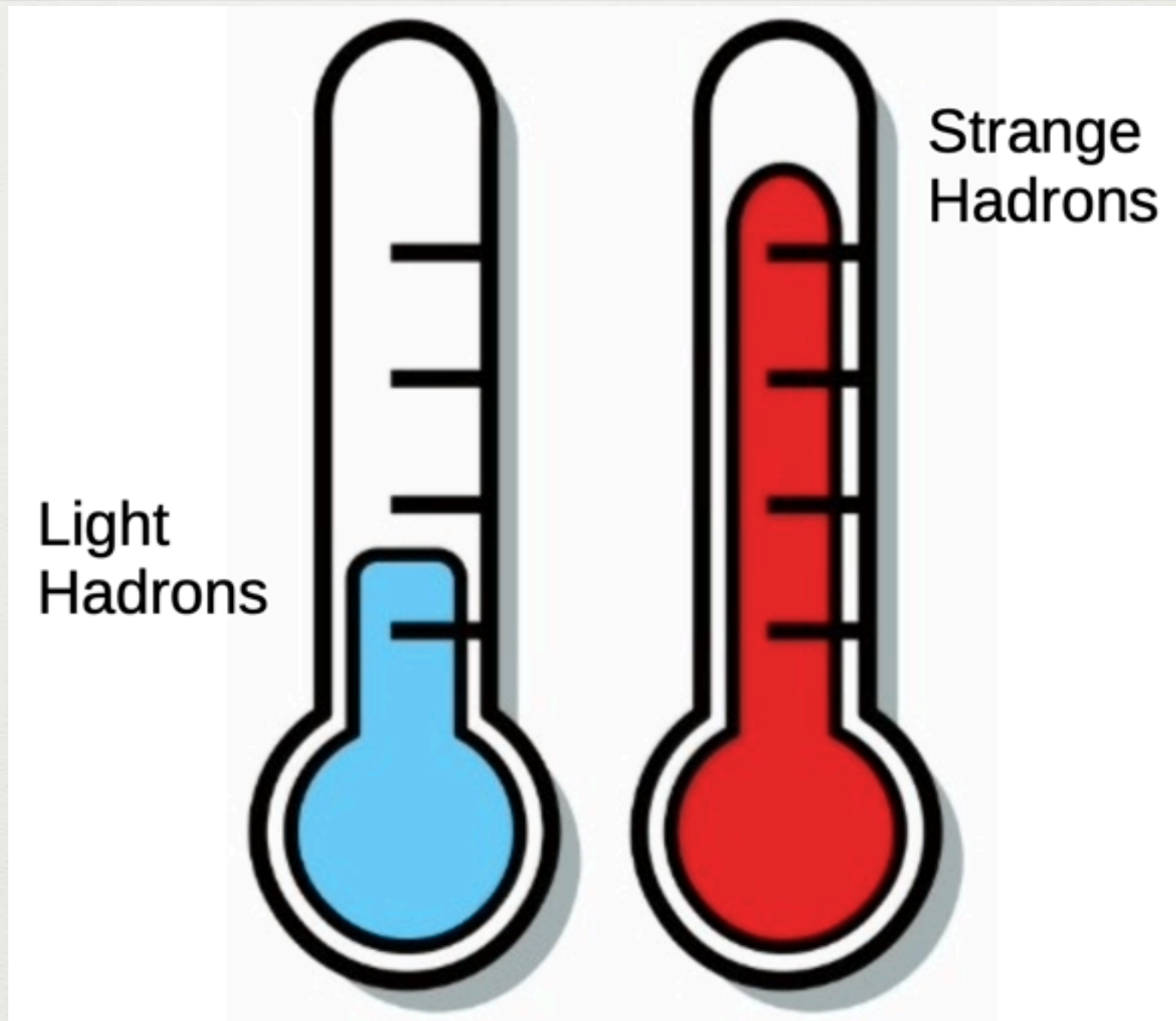
However, many more predicted from quark model

2014 Missing strange baryons?

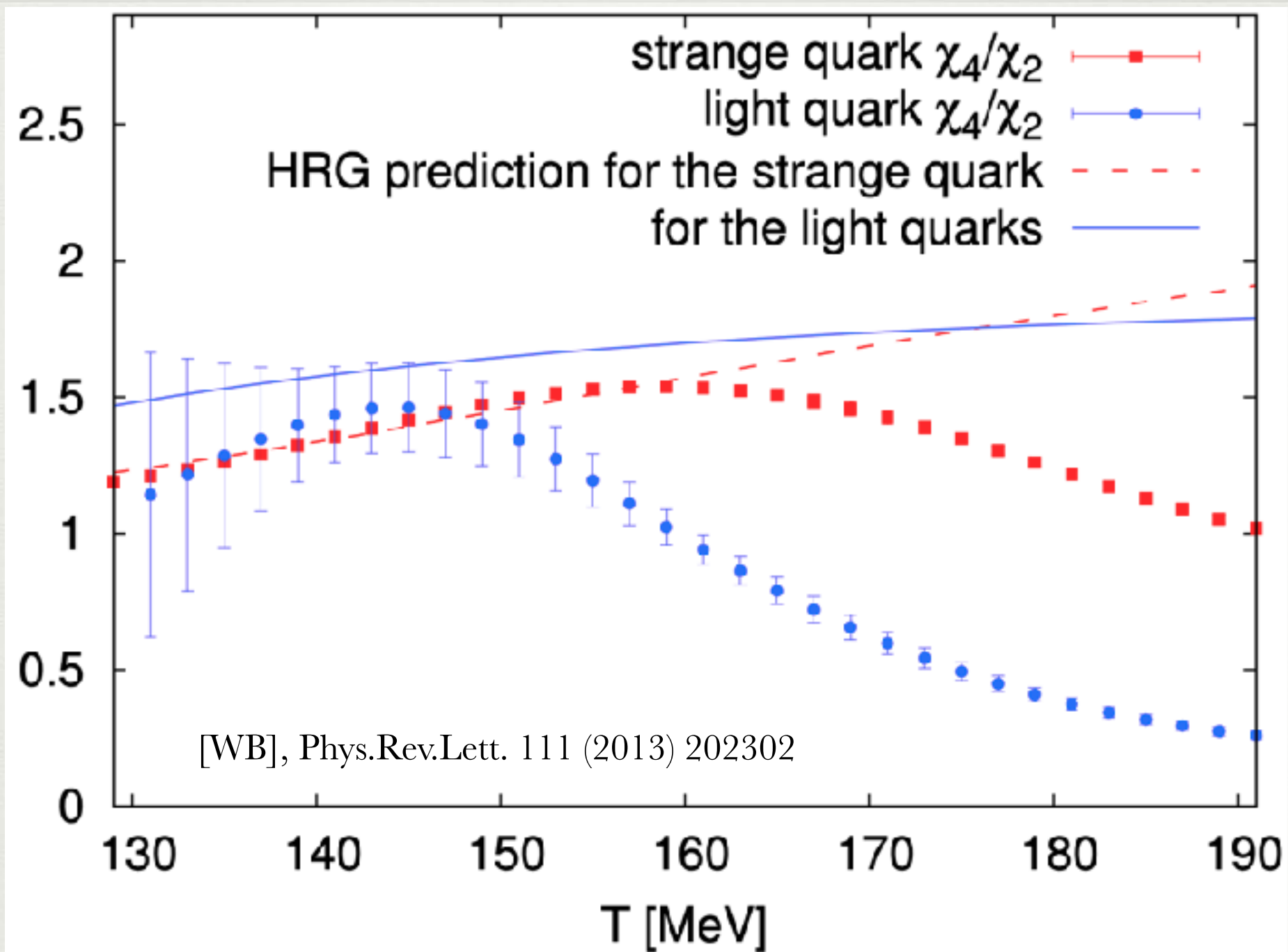


- Experimentally measured particles (Particle Data Group) not enough to understand strange baryons to strange hadrons
- More resonance \Downarrow freeze-out temperature (better for protons)

Flavor hierarchy: heavy particles freeze-out at higher T



Strange quarks $T_s \sim 165 \text{ MeV}$
Light quarks $T_L \sim 145 \text{ MeV}$



Partial Pressures (hadrons only)

For an ideal hadron resonance gas, we assume that the pressure can be written as:

$$P_{tot} = P_{S=0,B=0} + P_{S=\pm 1,B=0} + P_{S=0,B=\pm 1} + P_{S=\pm 1,B=\mp 1} + P_{S=\pm 2,B=\mp 1} + P_{S=\pm 3,B=\mp 1}$$

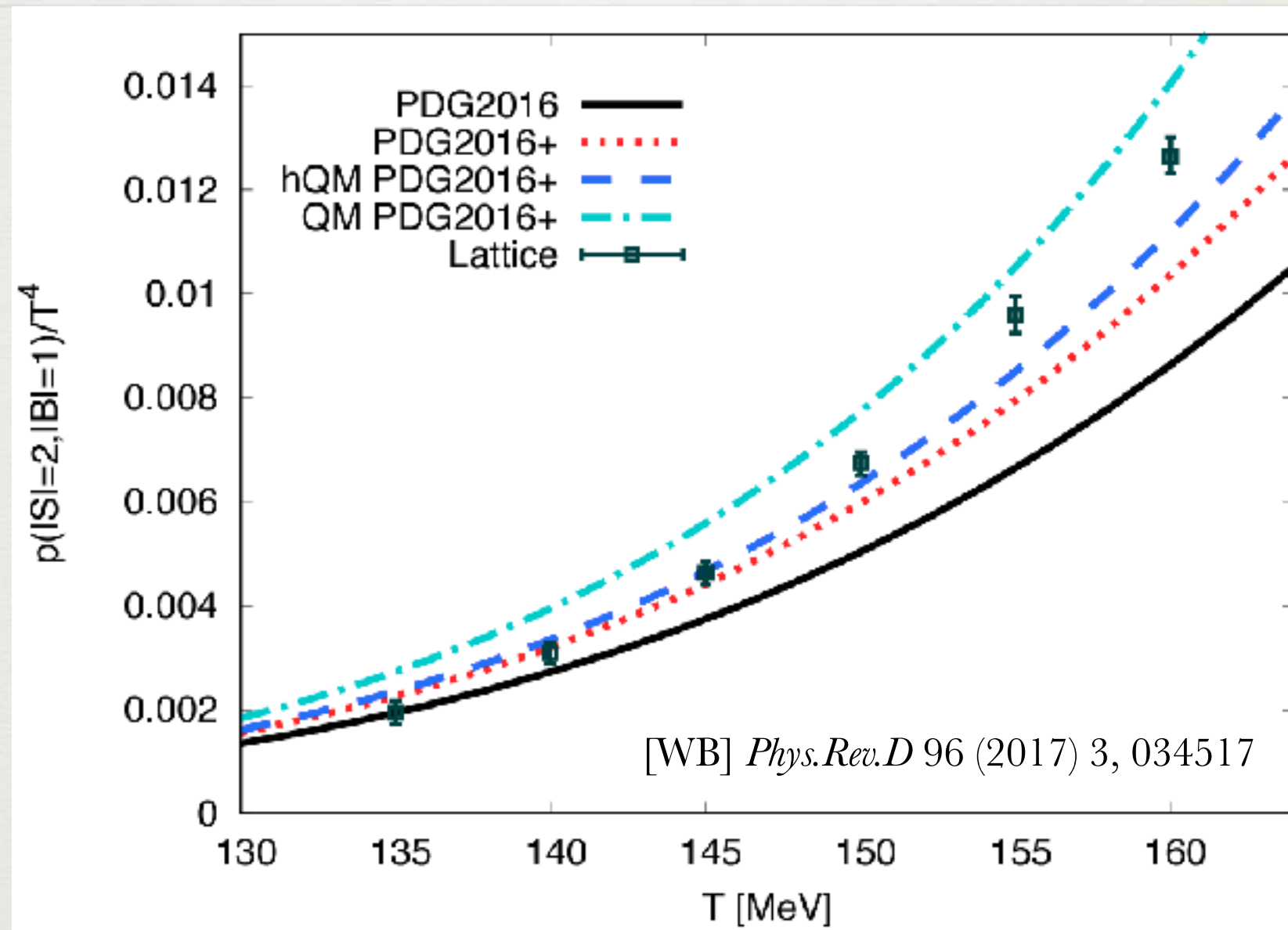
It's often easier to write in terms of the stable hadrons

$$P_{tot} = P_{\pi} + P_K + P_{p,n} + P_{\Lambda,\Sigma} + P_{\Xi} + P_{\Omega}$$

For example,

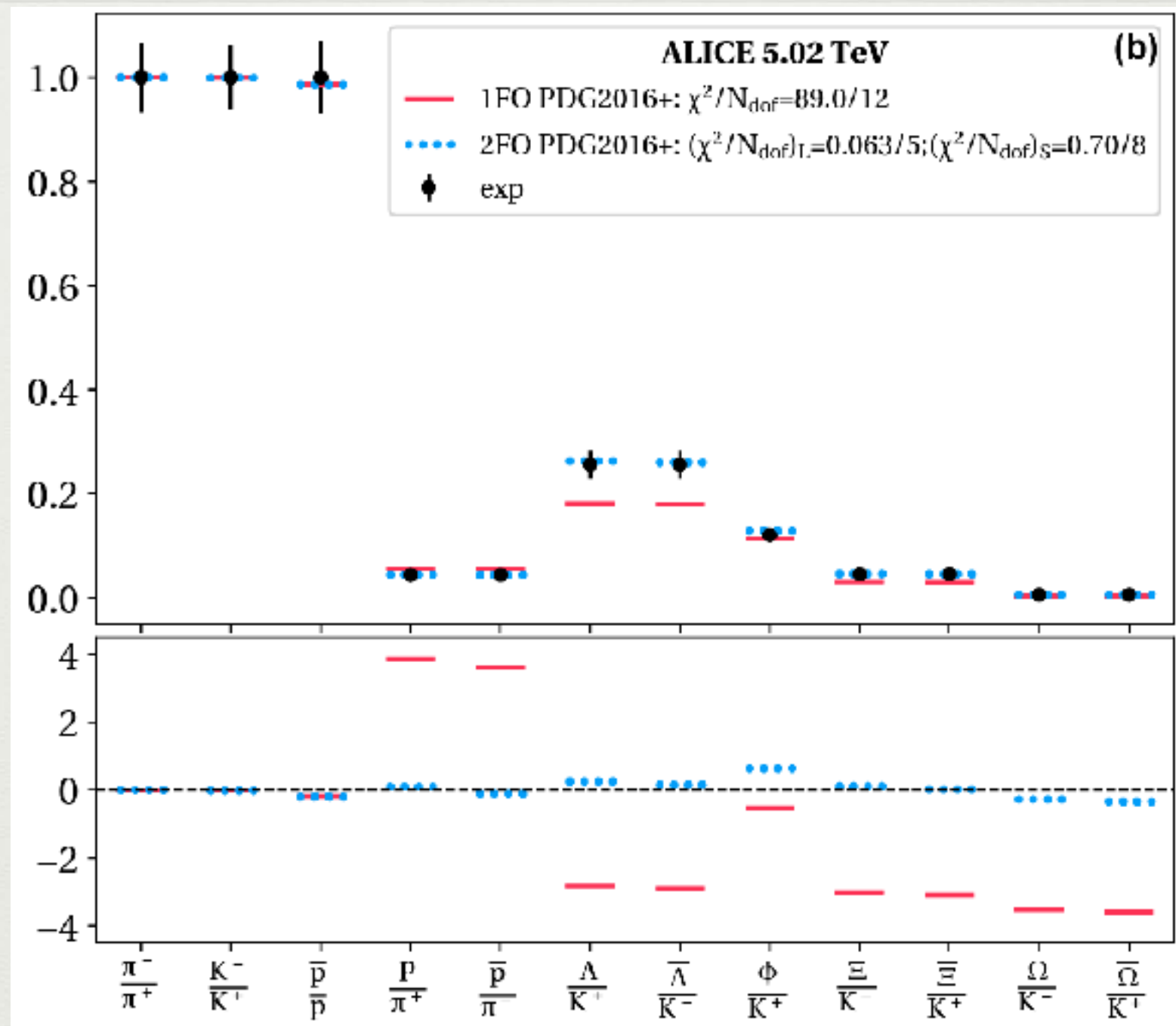
$$P_{\Xi} = A \cosh (\mu_B/T - 2\mu_S/T)$$

Partial pressure and missing states



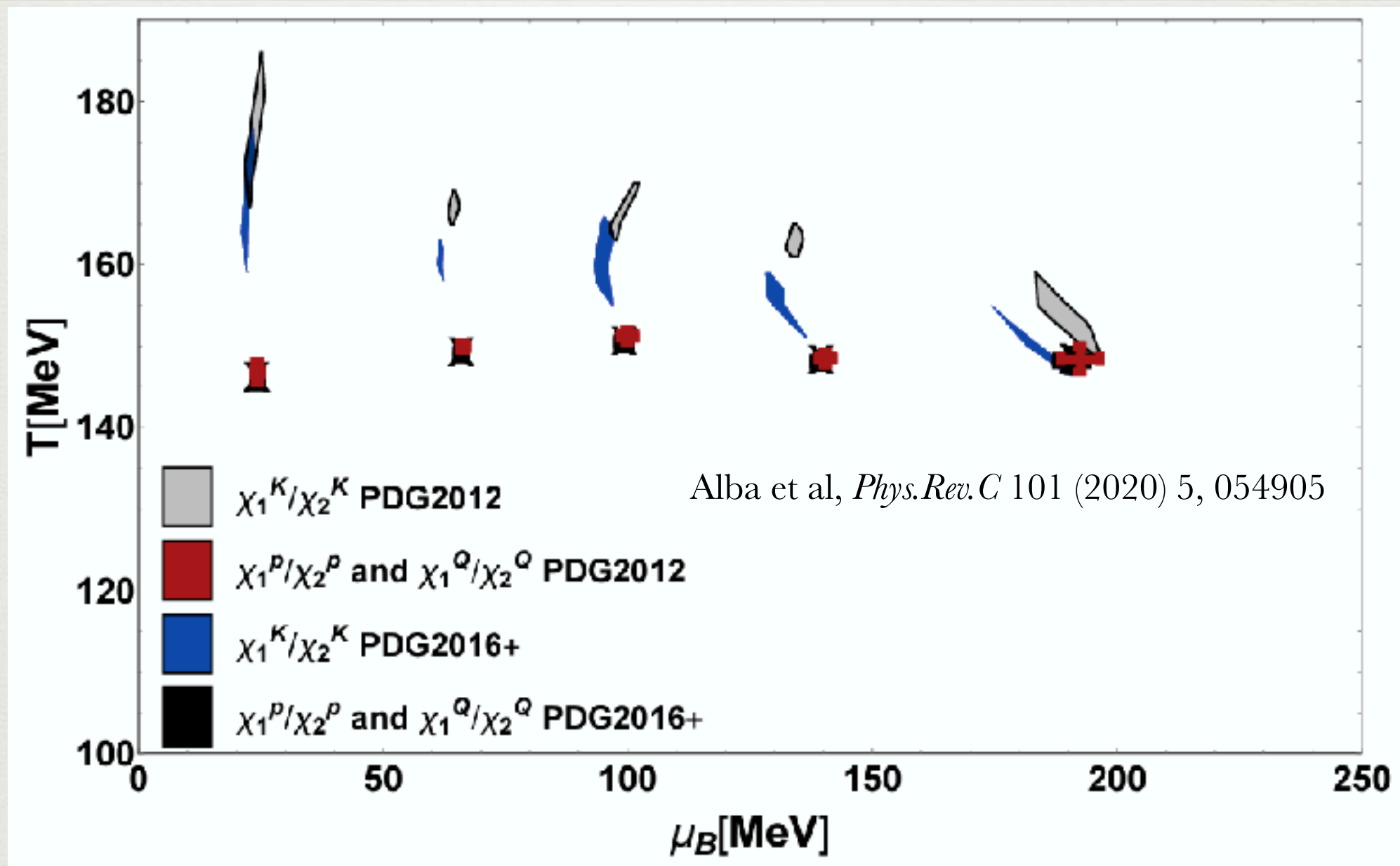
From comparisons to lattice QCD, even most uncertain PDG states needed

Do the extra states solve the proton-to-pion puzzle?



2 freeze-out temperatures needed even for extra resonances

Extracted light vs. strange freeze-out T



Flavor hierarchy appears to be needed to reproduce fluctuations results

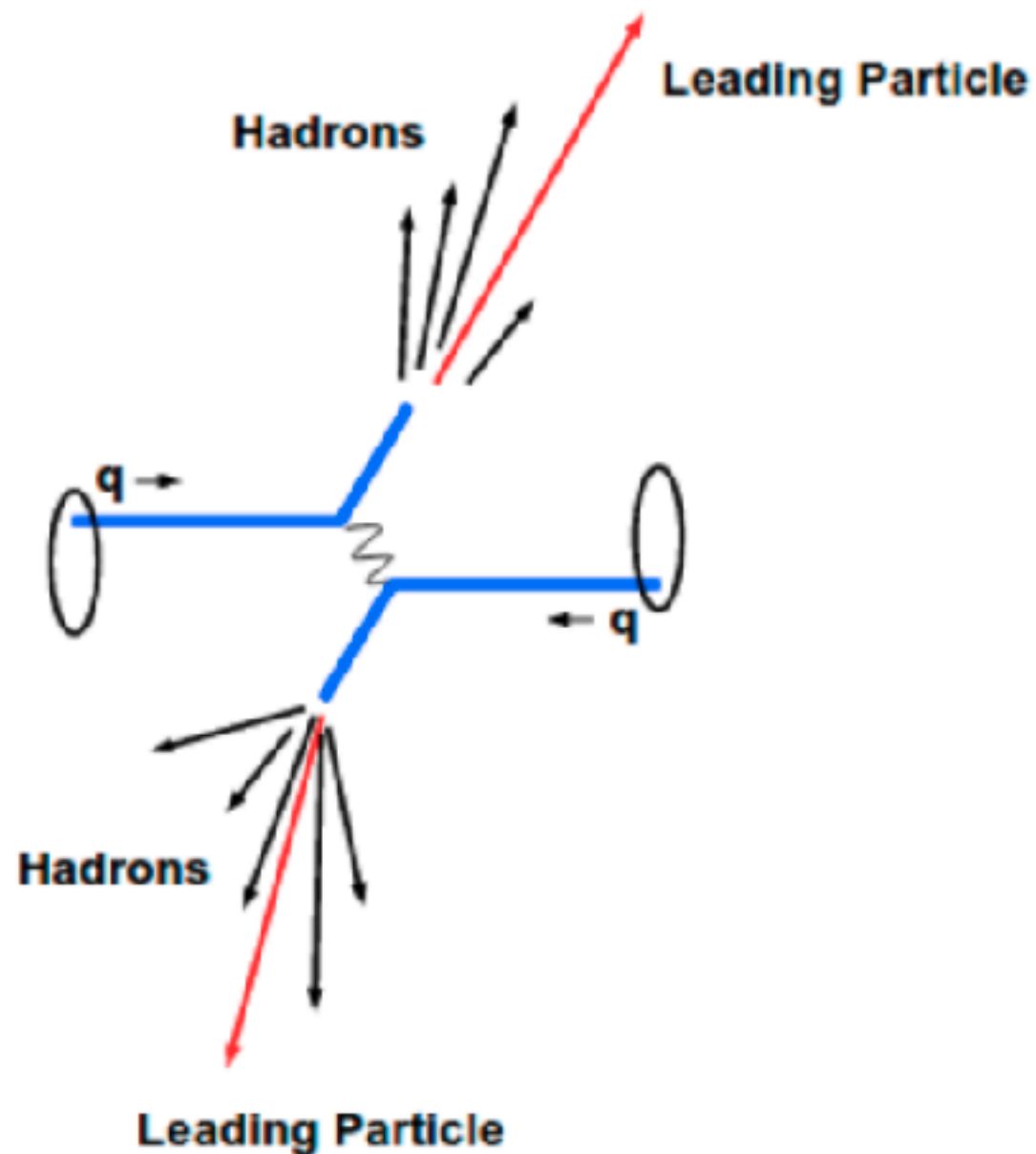
What are people studying today?

Strangeness

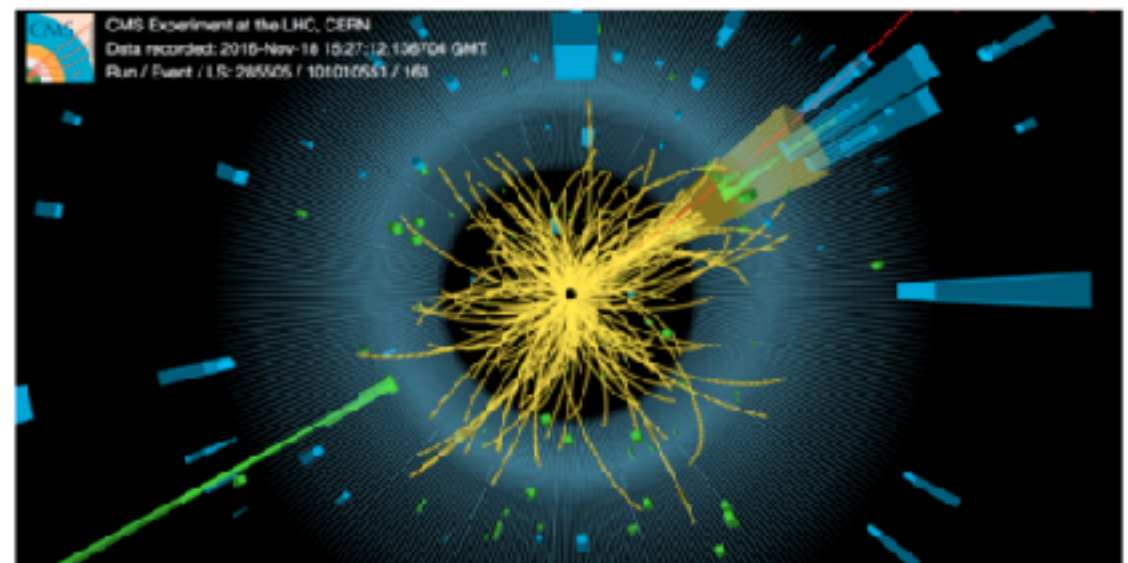
- Missing strange particles? Lattice QCD vs. HRG
- Repulsive vs. attractive interactions in HRG
- Collective flow of strange particles
- Strangeness in small systems
- Hypernuclei
- Λ polarization

Jets and Heavy flavor

Hard probes



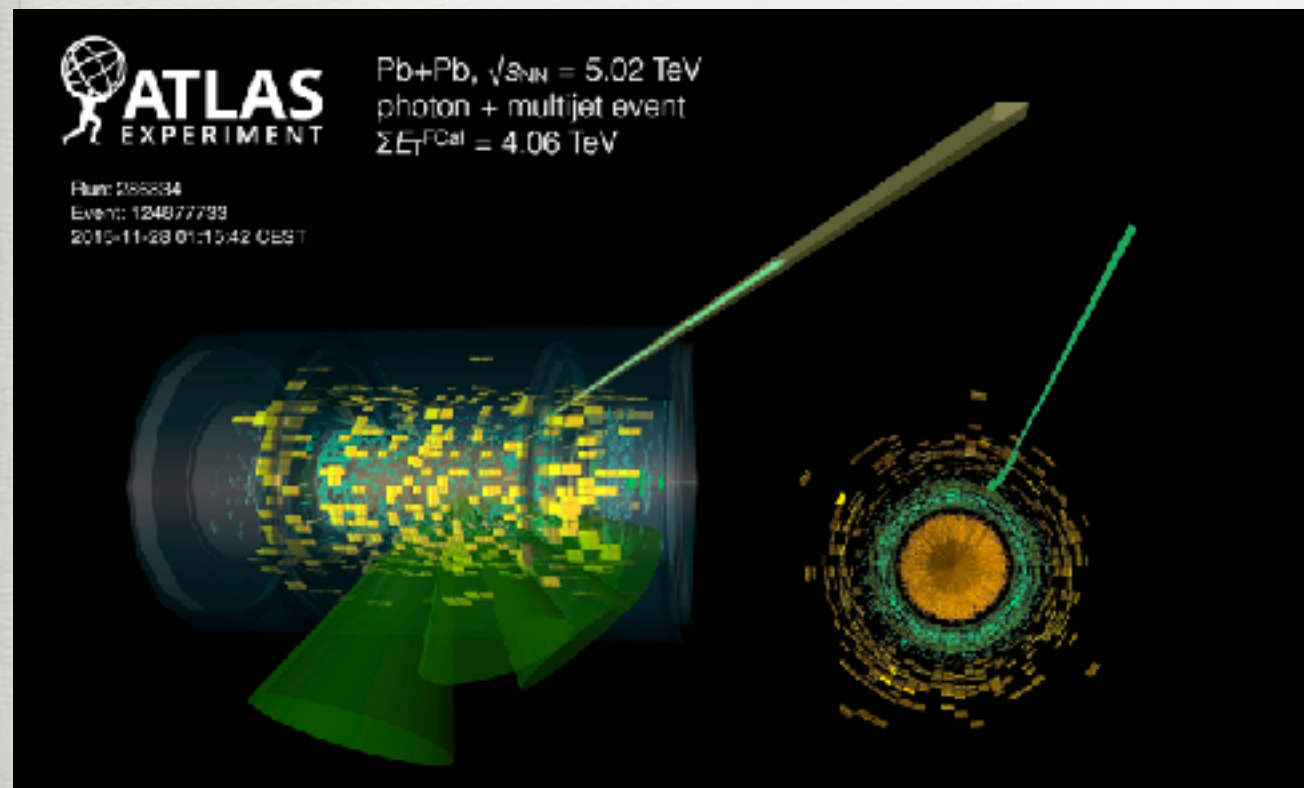
- In the initial collision, hard scattering occurs between colored partons (quarks or gluons)
- Due to large momentum exchange (large Q^2), pQCD cross-sections are used



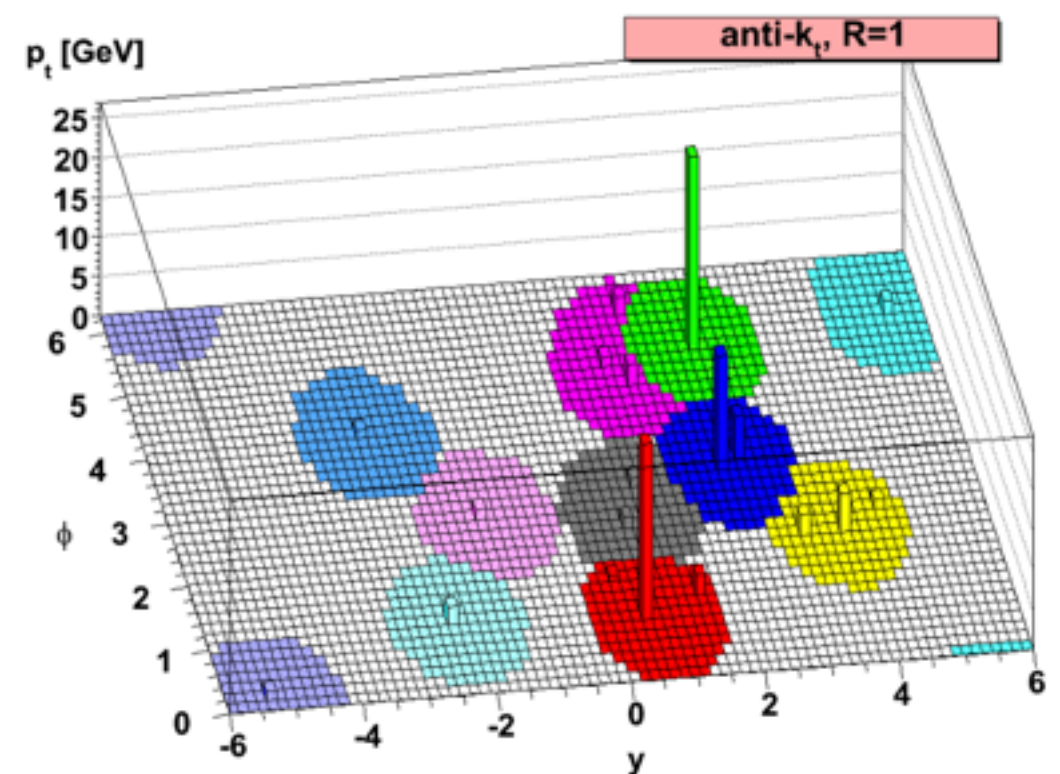
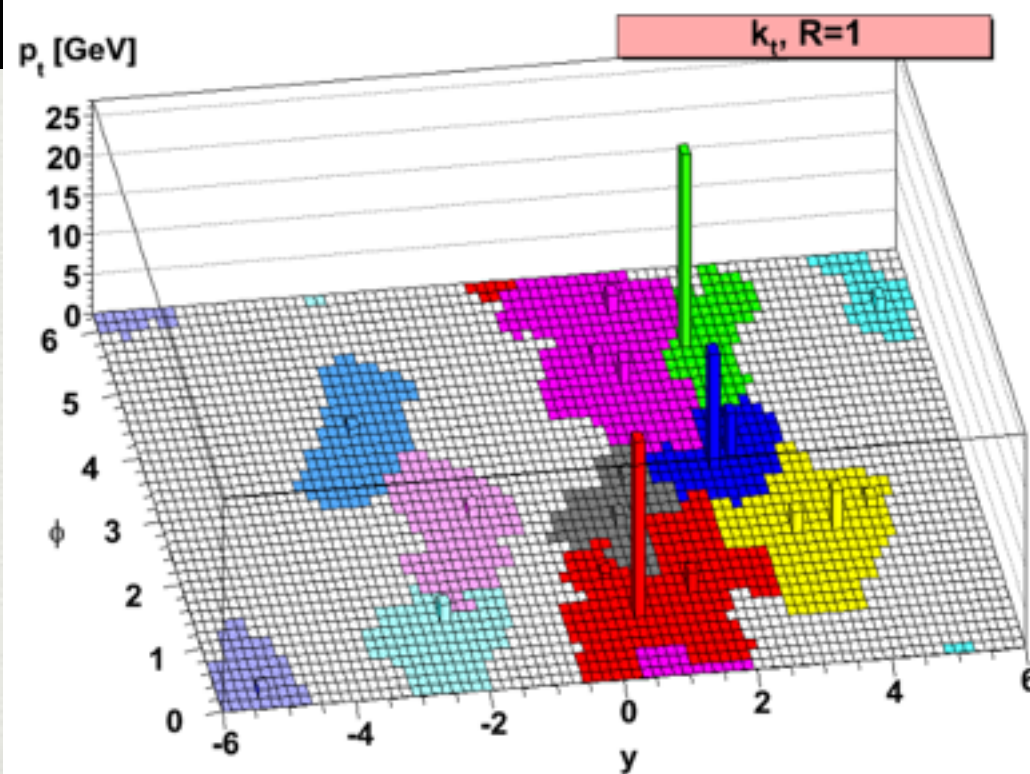
Properties of QCD jets

- Produced from hard scattering of either quarks or gluons
- Produced back-to-back (momentum conservation) with 2 prongs
- Jets get bumped around in the medium, lose energy
- Eventually jets fragment (create a shower of particles) that are measured

Seeing vs understanding jets

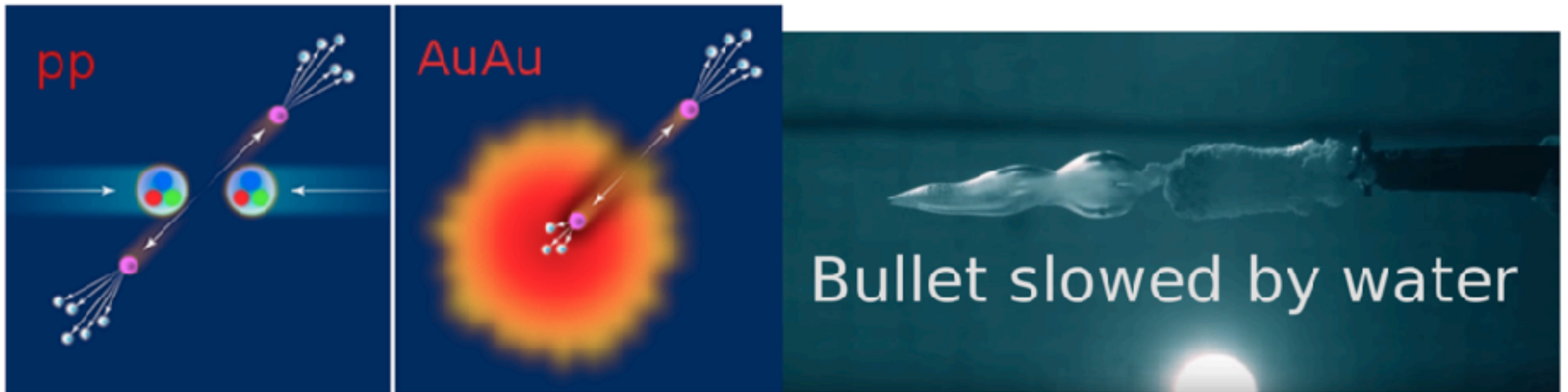


We can clearly see jets in event displays but it's not trivial how we define them.



Suppression of hard/heavy probes

Back-to-back jets are produced in the initial stages of heavy-ion collisions



Jets shooting through a liquid

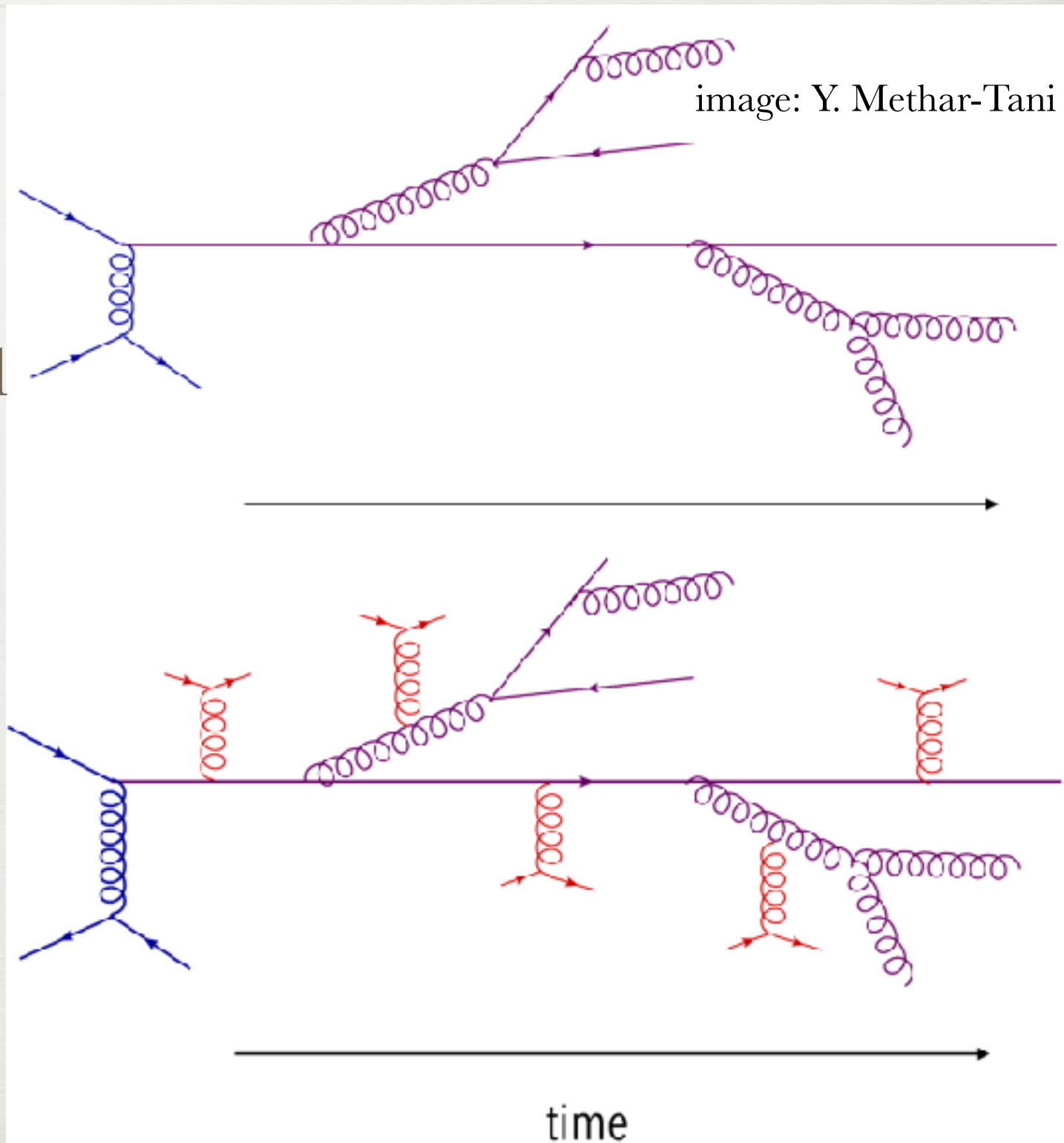
Jets are quenched in heavy-ion collisions compared to proton-proton collisions due to the Quark Gluon Plasma liquid!

Jets in vacuum vs medium

Vacuum (pp collisions)

Use pp as reference

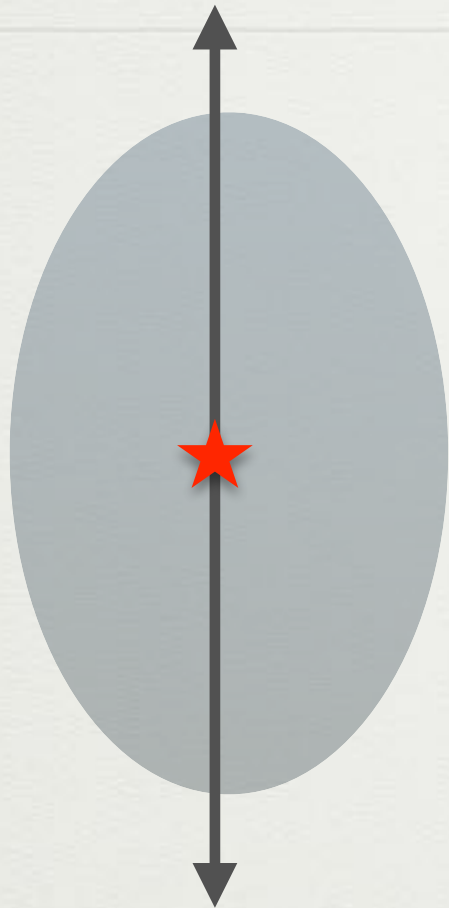
Maybe no QGP? or very small



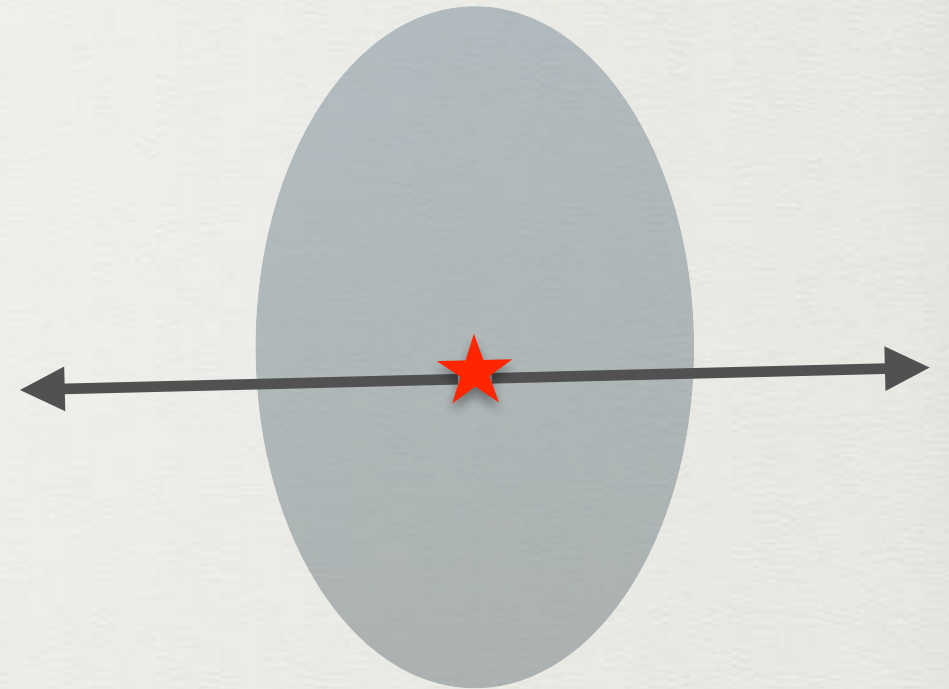
Medium (AA collisions)
Ratios of AA/pp collisions

Large droplet of QGP

Path length dependence



\neq



More energy loss
Longer time in QGP
More interactions

Less energy loss
Shorter time in QGP
Few interactions

Jet energy loss observables

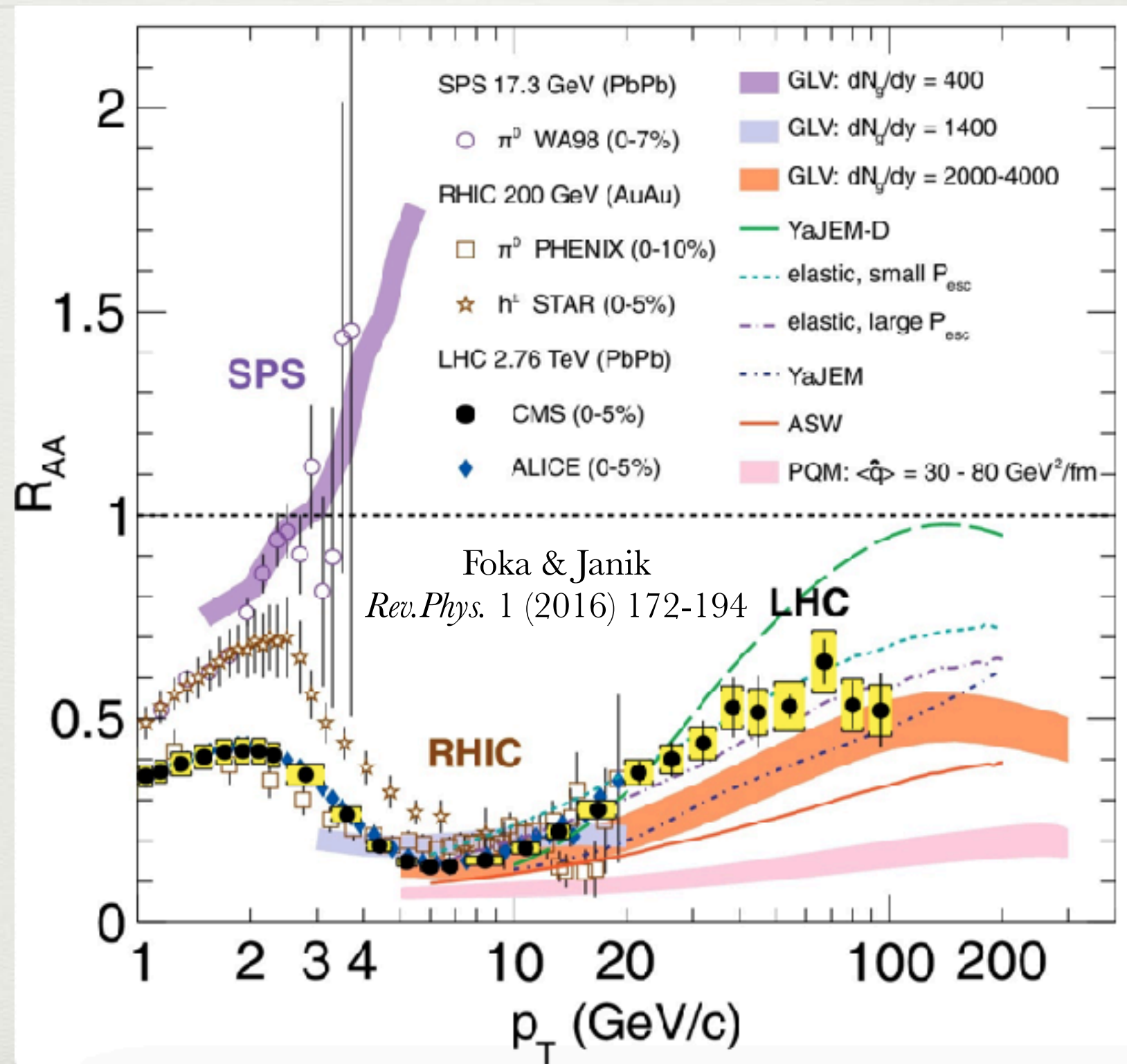
Nuclear modification factor R_{AA}

$$R_{AA} = \frac{1}{N_{coll}} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T} \quad \frac{\text{Medium}}{\text{Vacuum}}$$

We expect $R_{AA} \ll 1$ when large amount of energy loss
 $R_{AA} \sim 1$ when no (or little) energy loss

At low-ish p_T expect small R_{AA} , but expect $R_{AA} \rightarrow 1$ at
very high p_T

Measured R_{AA}



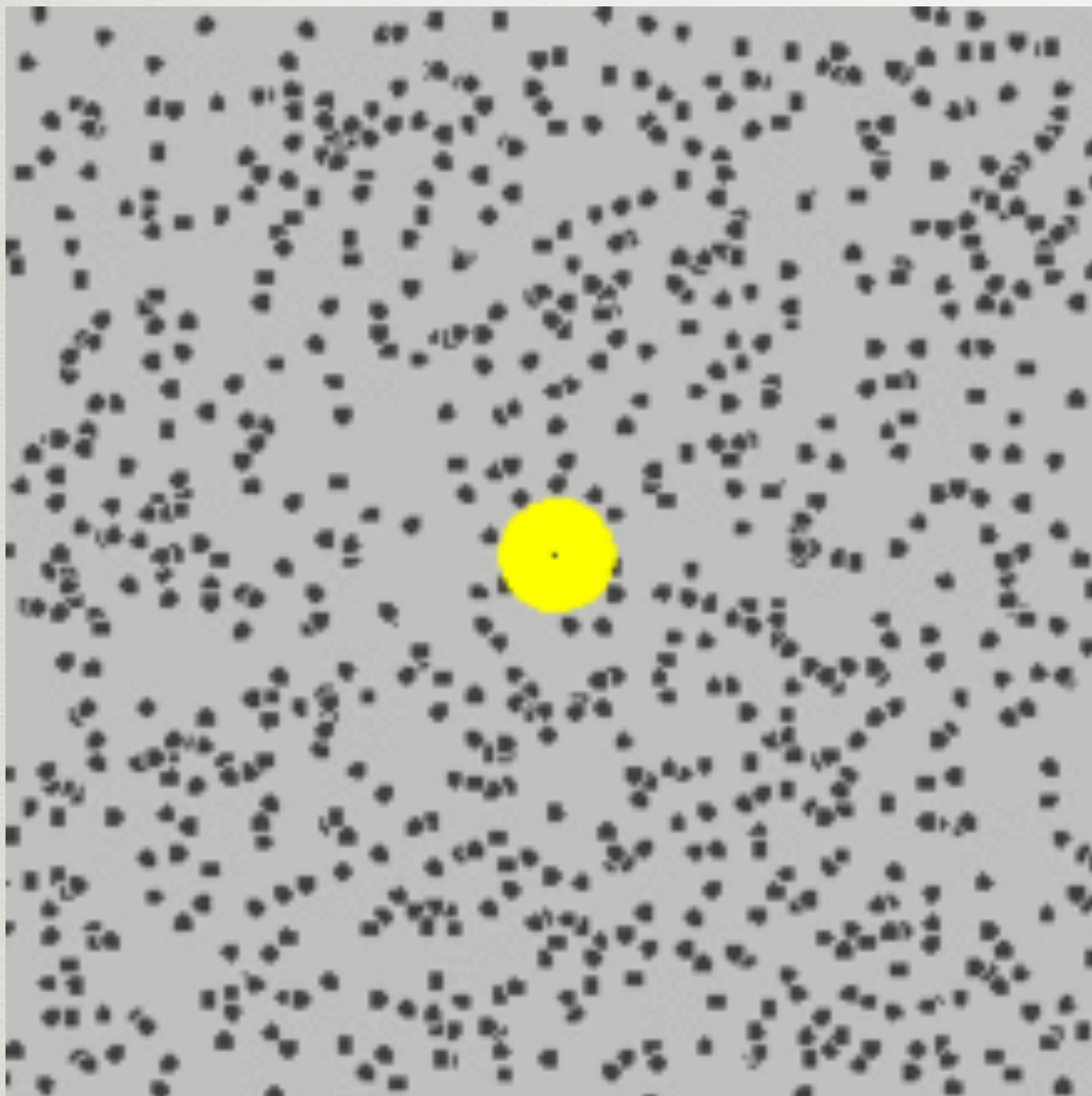
Heavy Flavor Probes

Moore and Teaney *Phys.Rev.C* 71 (2005) 064904

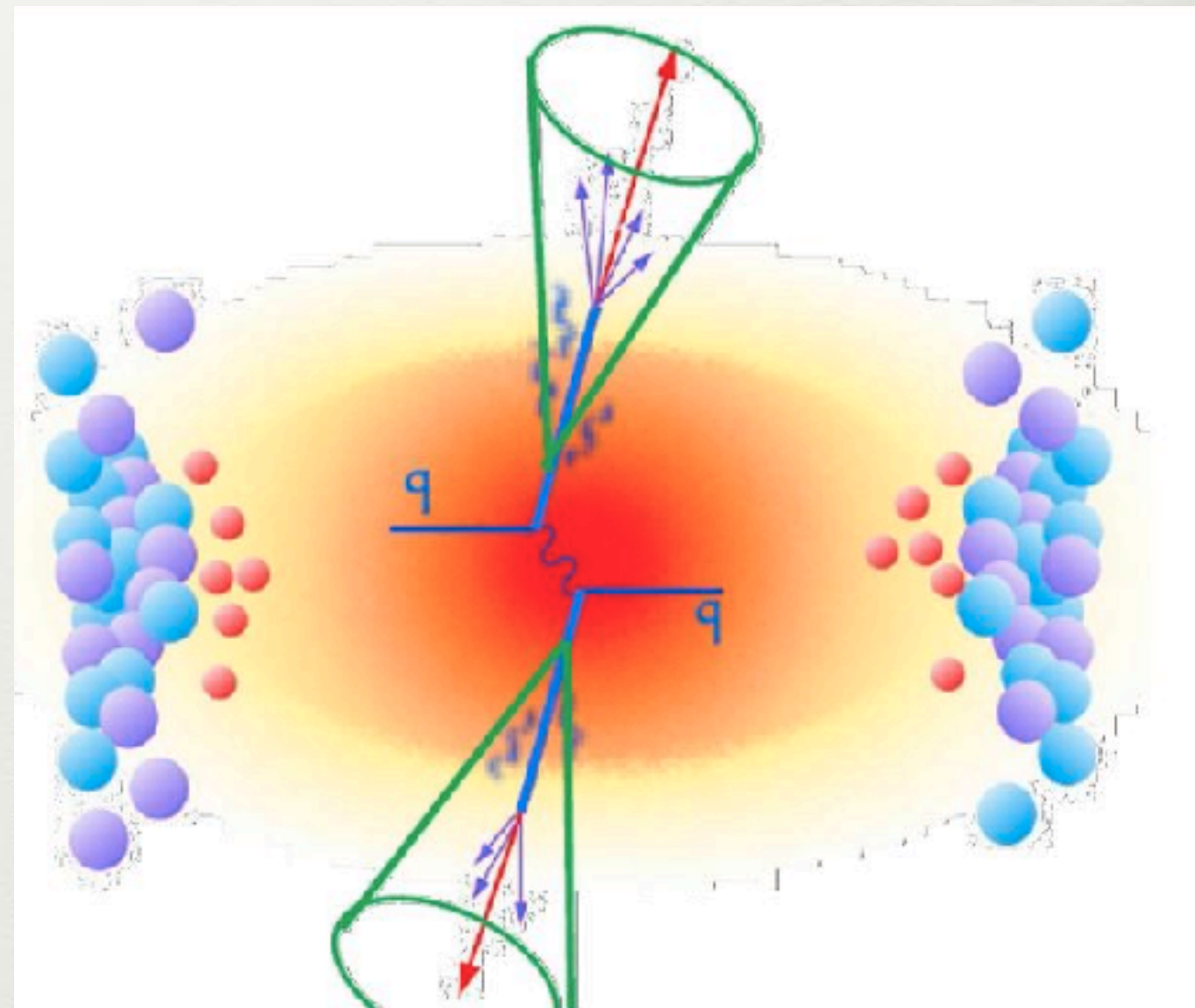
- Light quark time scale $\tau \sim \frac{\eta}{e + p}$
- Charm quark time scale $\tau_c \sim \frac{m_c}{T} \frac{\eta}{e + p}$
- At top LHC with $m_c \sim 1.3$ GeV and $T_{LHC}^{max} \sim 600$ MeV
 $\tau_c \sim 2\tau$
- At RHIC with $m_c \sim 1.3$ GeV and $T_{RHIC} \sim 200$ MeV
 $\tau_c \sim 6\tau$

D mesons to test R_{AA} vs. v_2 in OO

Langevin (low p_T)



Energy loss (high p_T)



Relativistic Langevin

How do you describe a particle randomly getting bumped around by the fluid?

non-relativistic $m \frac{d\vec{v}}{dt} = -a\vec{v} + b(t)$

relativistic $dx_i = \frac{dp_i}{E} dt$

$dp_i = -\Gamma(p)p_i dt + \sqrt{dt}\sqrt{\kappa}\rho_i$

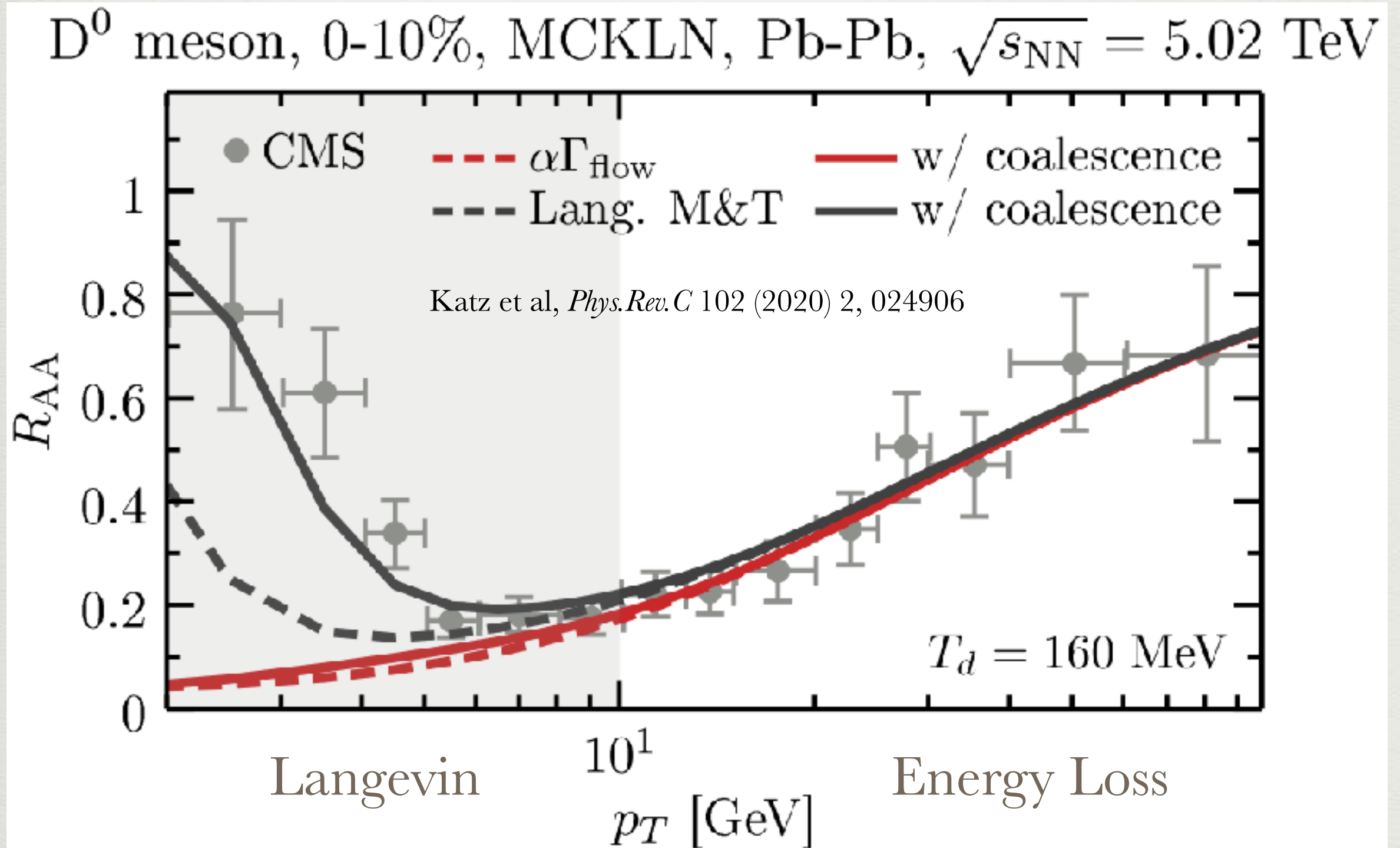
Drag coefficient $\kappa = 2T^2/D$

Noise term

Spatial Diffusion Coefficient

```
graph TD; NR["non-relativistic  
m dv/dt = -a v + b(t)"]; R["relativistic  
dx_i = dp_i/E dt"]; DR["Drag coefficient  
κ = 2T²/D"]; NT["Noise term"]; NR -- "Noise term" --> NT; R -- "Noise term" --> NT; DR -- "Drag coefficient" --> R;
```

D mesons: p_T dependence and theory



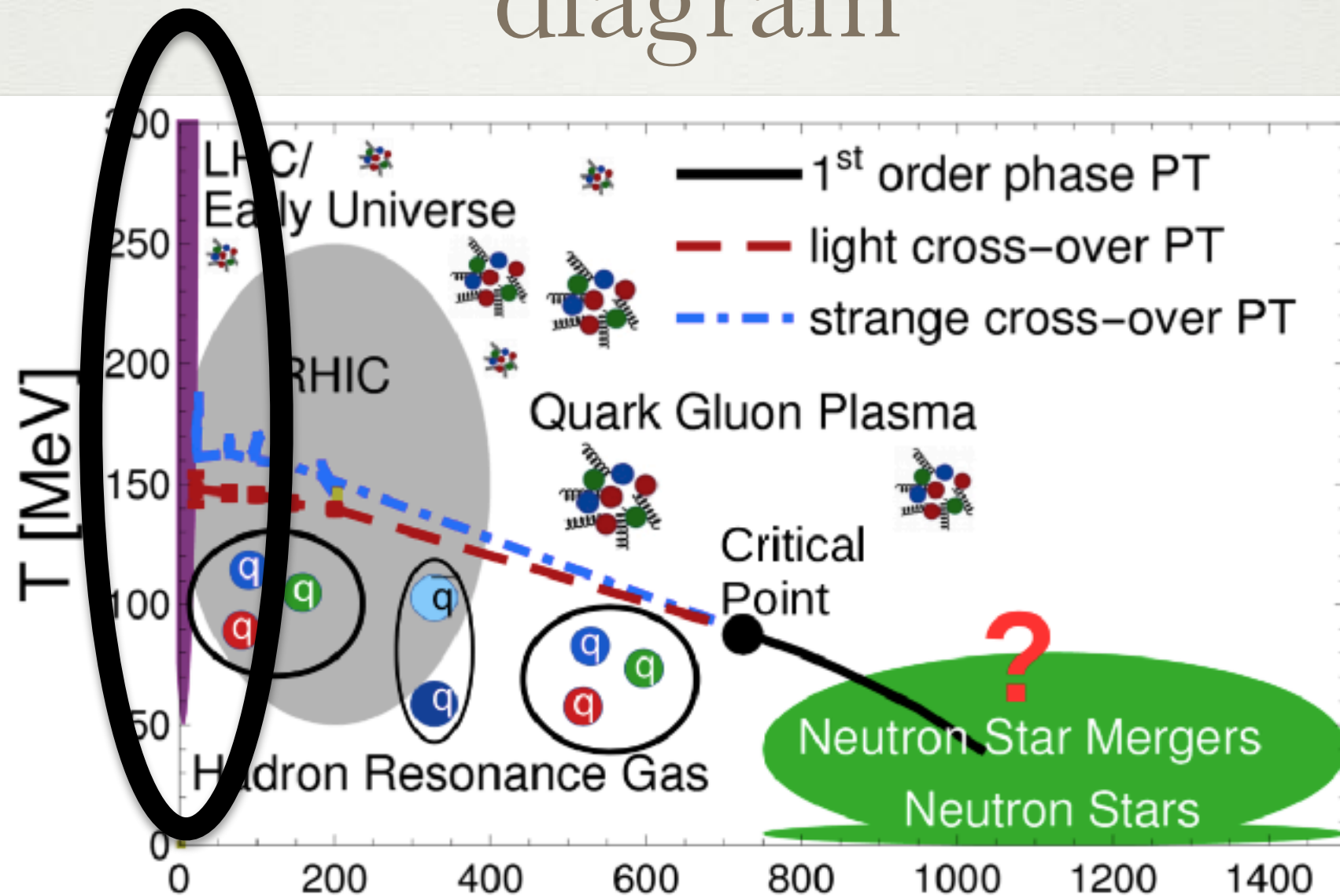
What are people studying today?

Jets & Heavy flavor

- Jet shapes, sub-structure, mass...
- Azimuthal anisotropies of jets and heavy mesons, especially in small systems
- Identifying quark vs gluon jets
- Jet-medium interactions
- Jet size (R) dependence on observables

Future facilities

Current Cartoon of the QCD phase diagram



Baryons = anti-baryons μ_B [MeV]

References

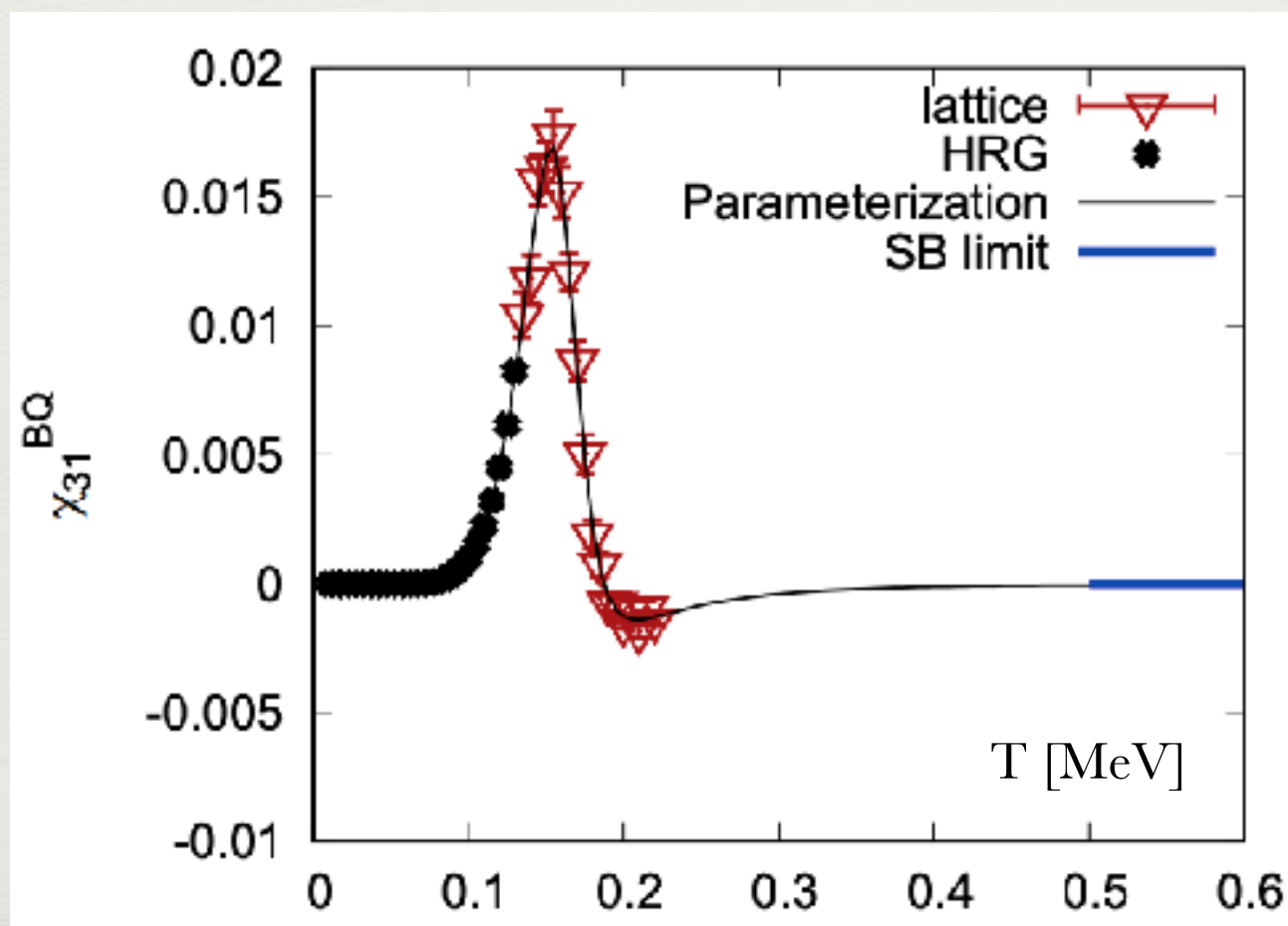
Light transition Phys.Lett. B738 (2014) 305-310; **Strange Transition** JNH and Ratti arxiv 1804.10661 ; **Neutron Star (mergers)** V. Dexheimer arXiv:1708.08342; **Holography** Critelli, JNH et al, Phys.Rev. D96 (2017) no.9, 096026

EoS: Lattice QCD, EOS in 4D

JNH, Paolo Parotto, Claudia Ratti, Jamie Stafford Phys.Rev.C 100 (2019) 6, 064910

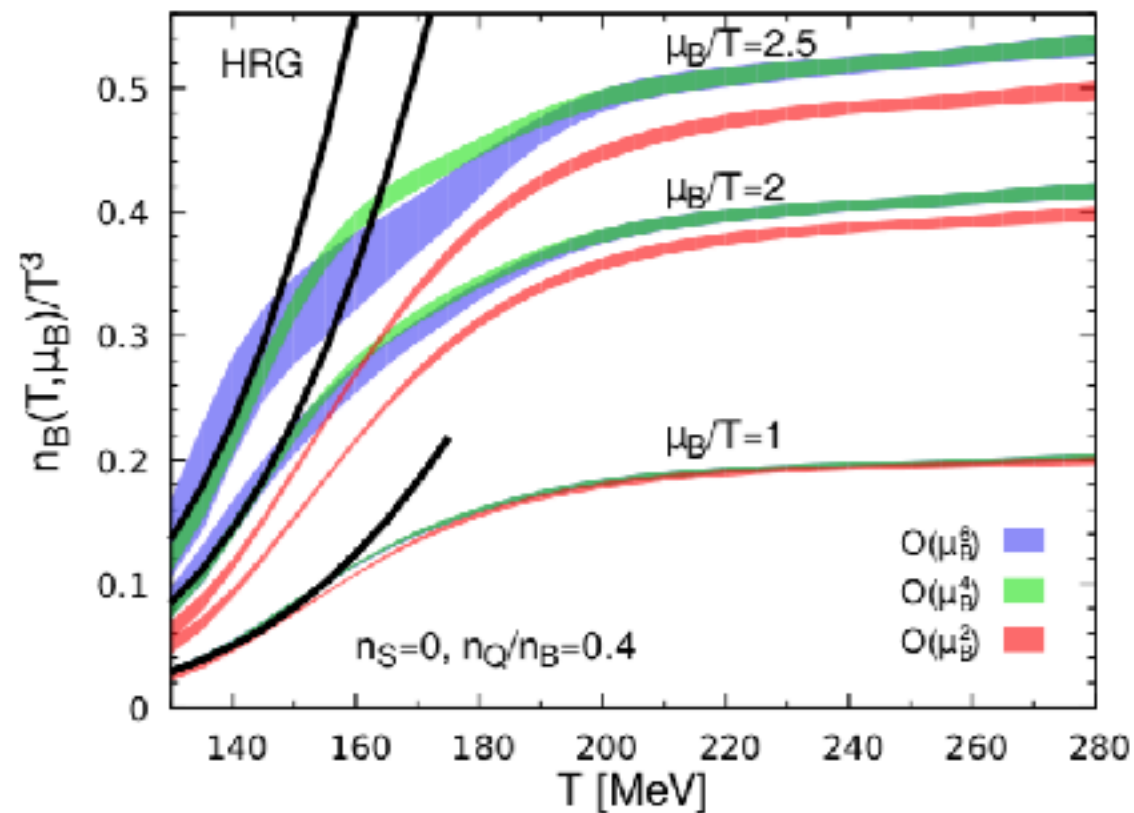
$$\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$$\chi_{ijk}^{B,S,Q} = \frac{\partial^{i+j+k}(p/T^4)}{\partial(\mu_B/T)^i \partial(\mu_S/T)^j \partial(\mu_Q/T)^k} \Big|_{\mu_{BSQ}=0}$$

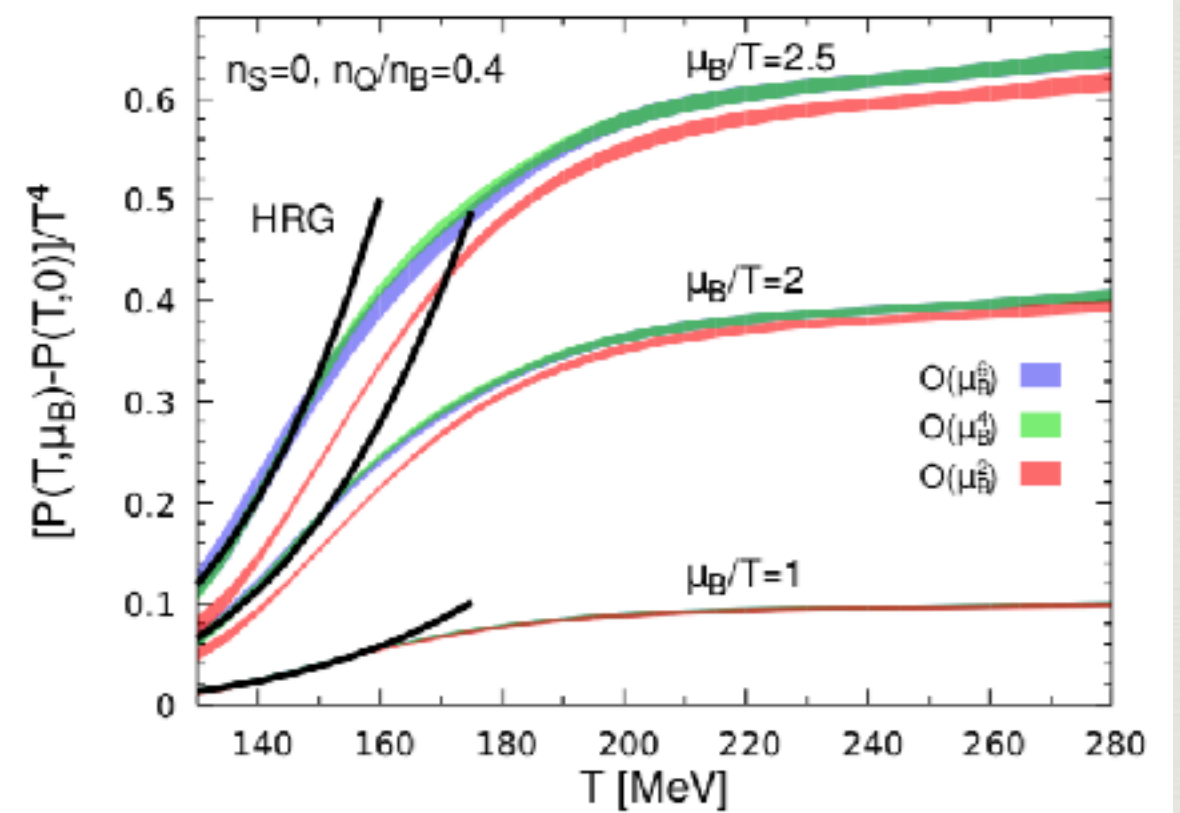


Applicable up to
 $\mu_B \sim 450$ MeV with
 current Lattice
 QCD results

Reconstructing the Equation of State at finite μ_B



HotQCD Phys.Rev. D95 (2017) no.5, 054504



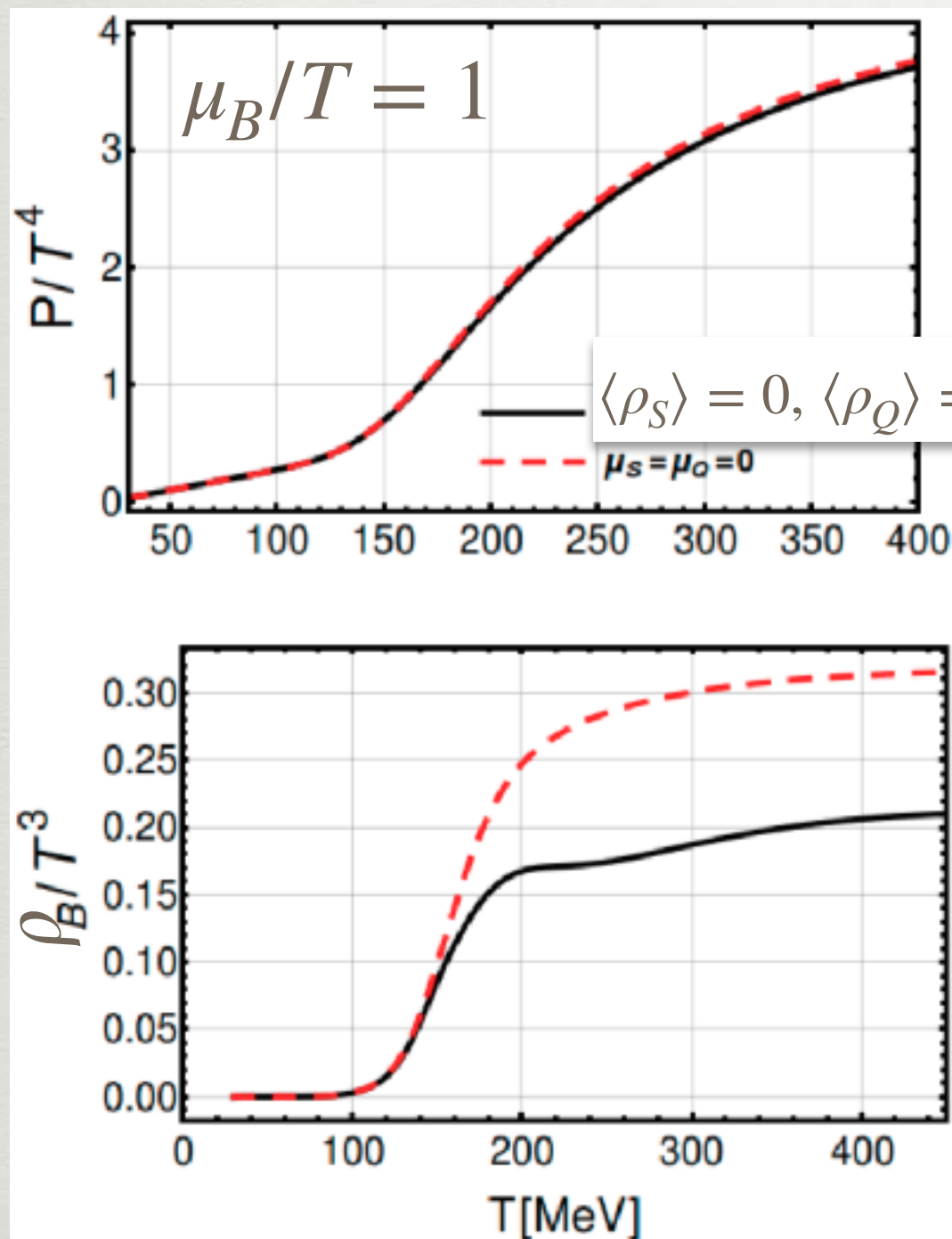
HotQCD Phys.Rev. D95 (2017) no.5, 054504

Taylor series expanded around $\mu_B = 0$

$$\frac{P(T, \mu_B)}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T} \right)^2 + c_4 \left(\frac{\mu_B}{T} \right)^4 + c_6 \left(\frac{\mu_B}{T} \right)^6 + \mathcal{O}(\mu_B^8)$$

BSQ Lattice QCD equation of state

JNH (Paolo Parotto and Jamie Stafford) et al, arXiv:1902.06723



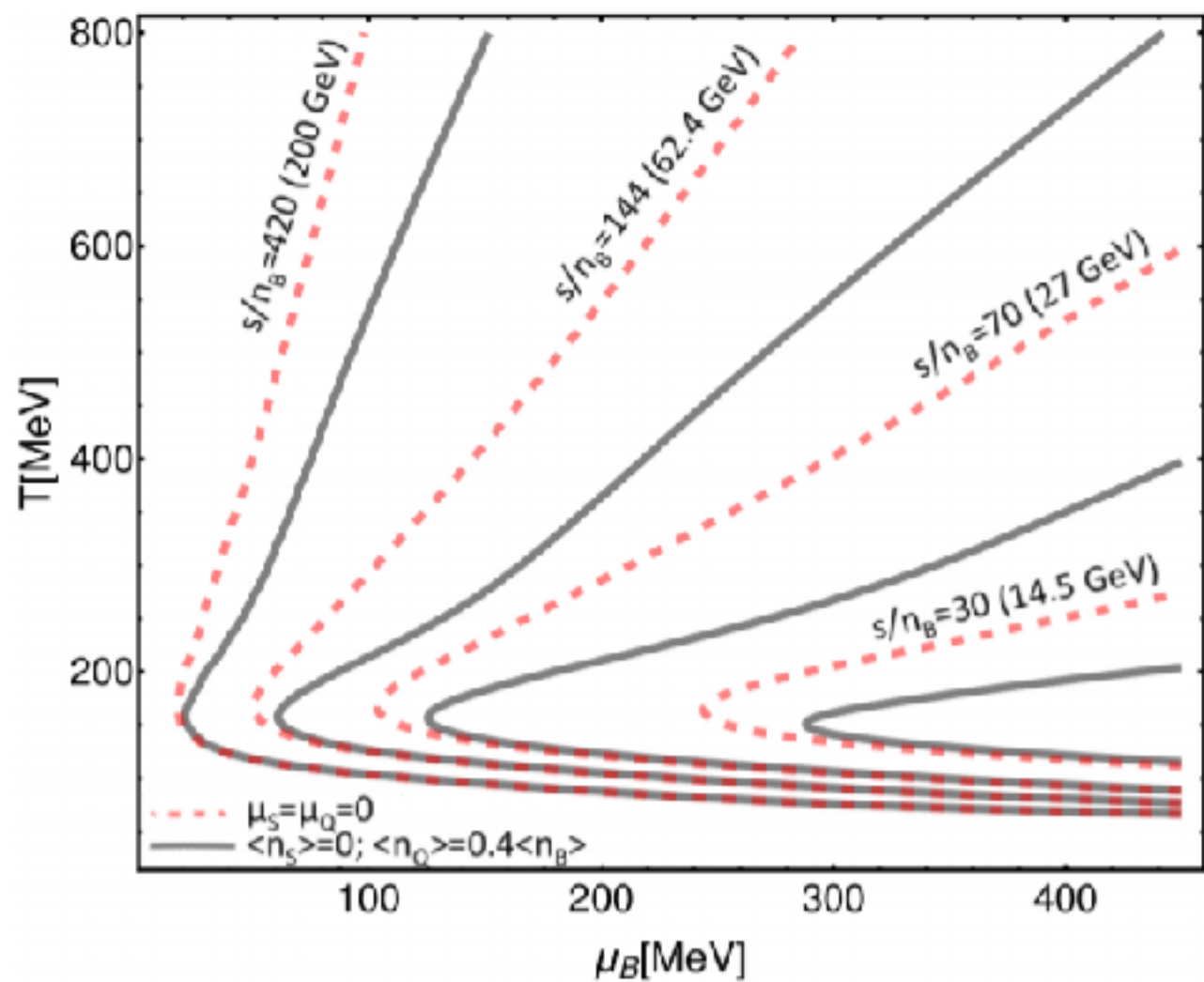
Lattice QCD reconstructed
QCD equation of state up to
fourth order

Strange hadrons make up $\sim 10\%$
of the total number of particles.

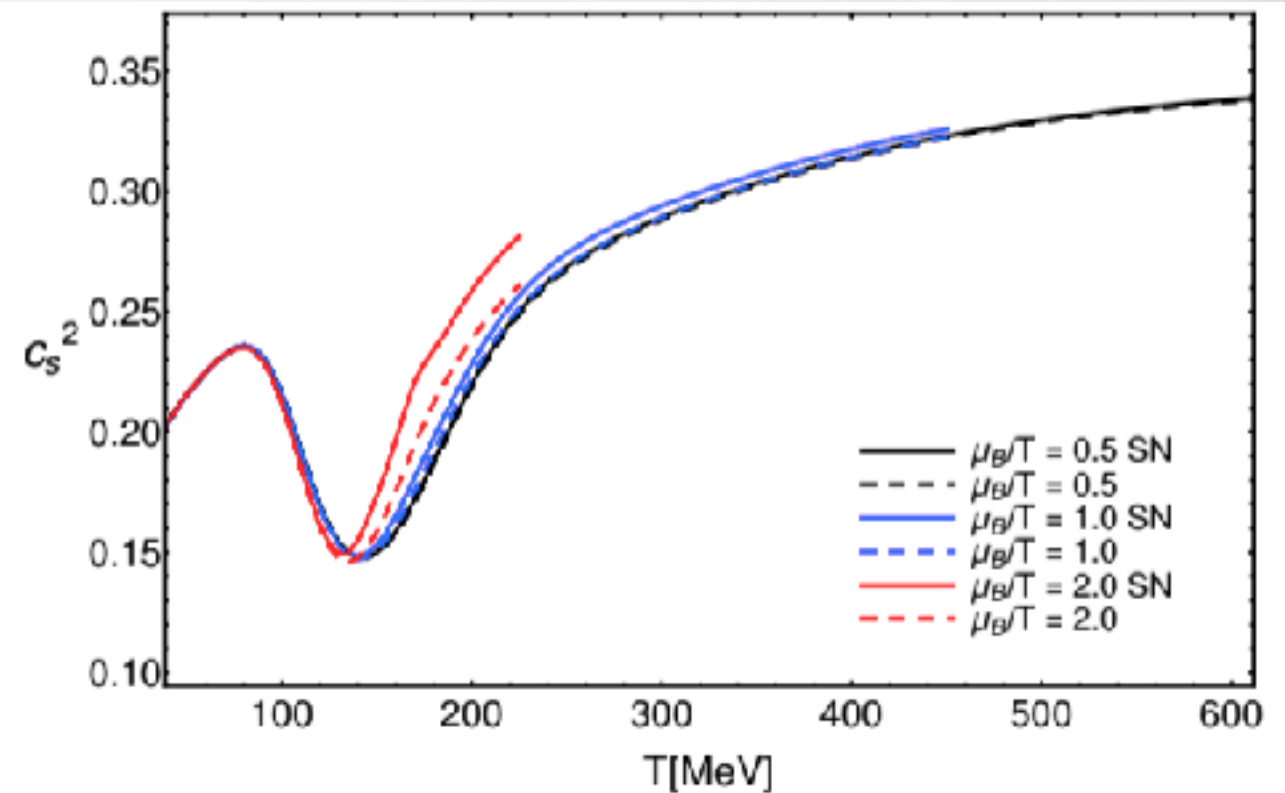
Speed of sound

JNH, Paolo Parotto, Claudia Ratti, Jamie Stafford Phys.Rev.C 100 (2019) 6, 064910

Isentrope trajectories



Steeper speed of sound



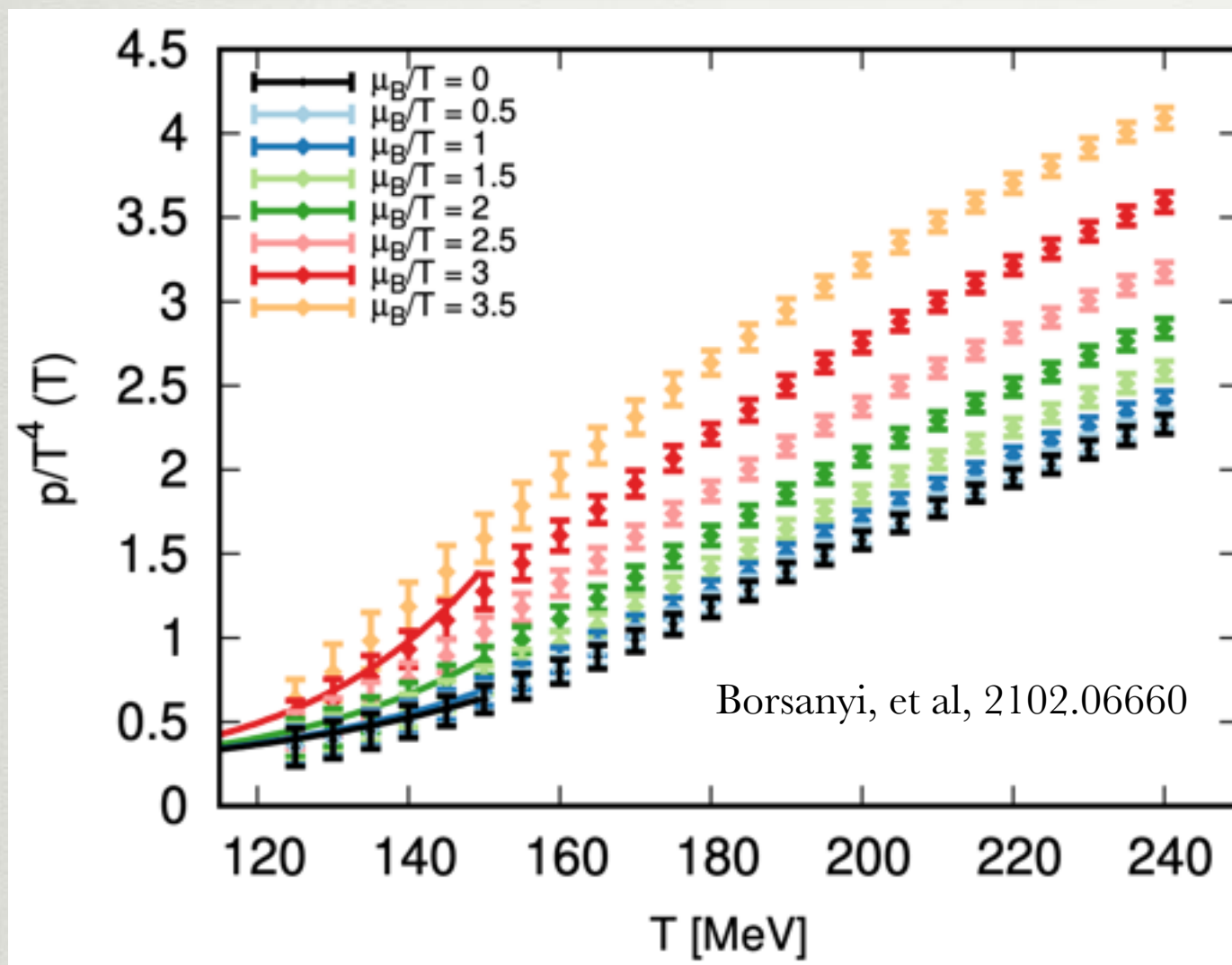
$$c_s^2 = \left. \frac{\partial p}{\partial \epsilon} \right|_{n_i} + \sum_i \frac{n_i}{\epsilon + p} \left. \frac{\partial p}{\partial n_i} \right|_{\epsilon, n_j}$$

Reaches larger chemical potentials

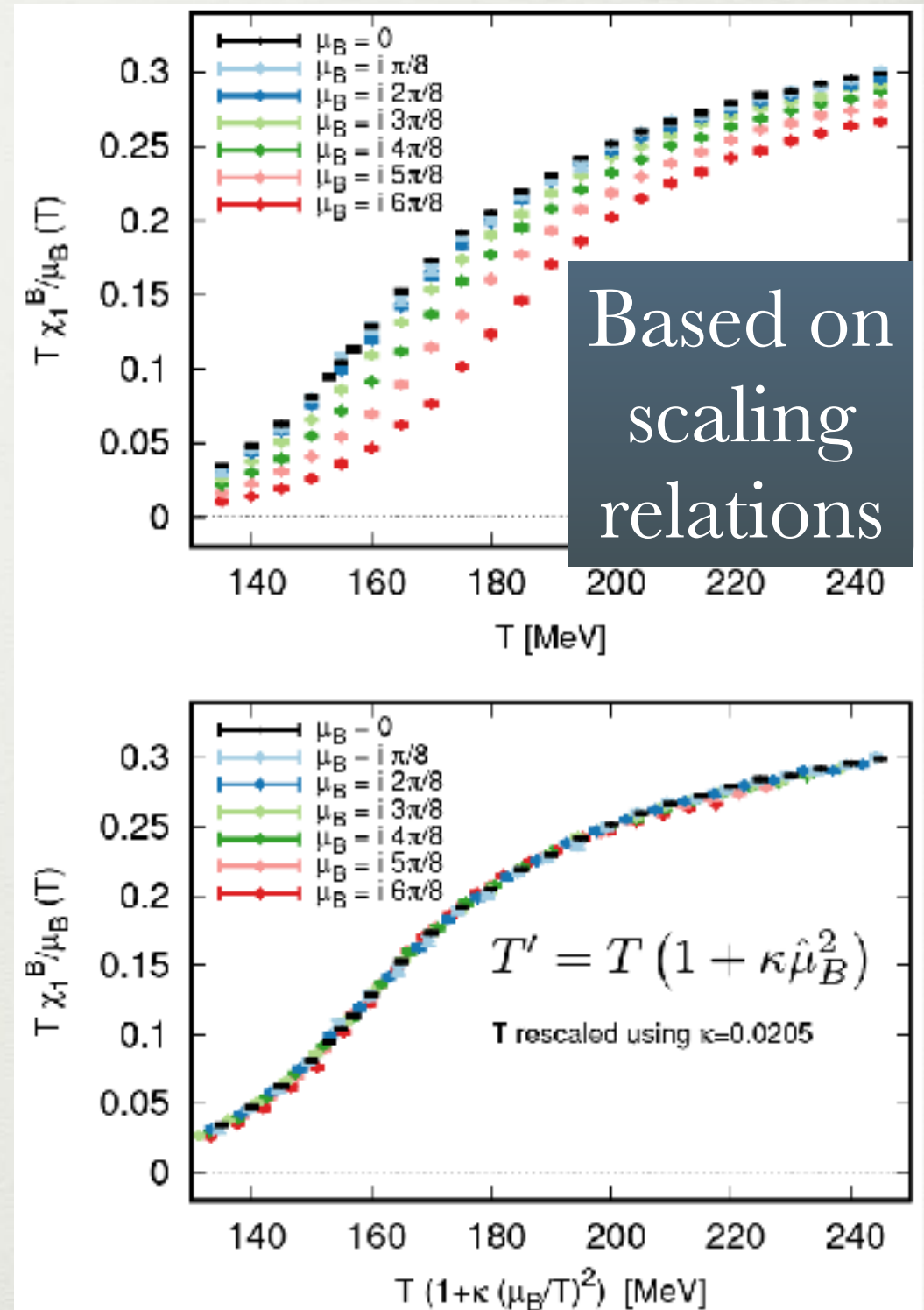
See also Monnai, Schenke, Shen, [arXiv:1902.05095](https://arxiv.org/abs/1902.05095)

Lattice QCD new resummation

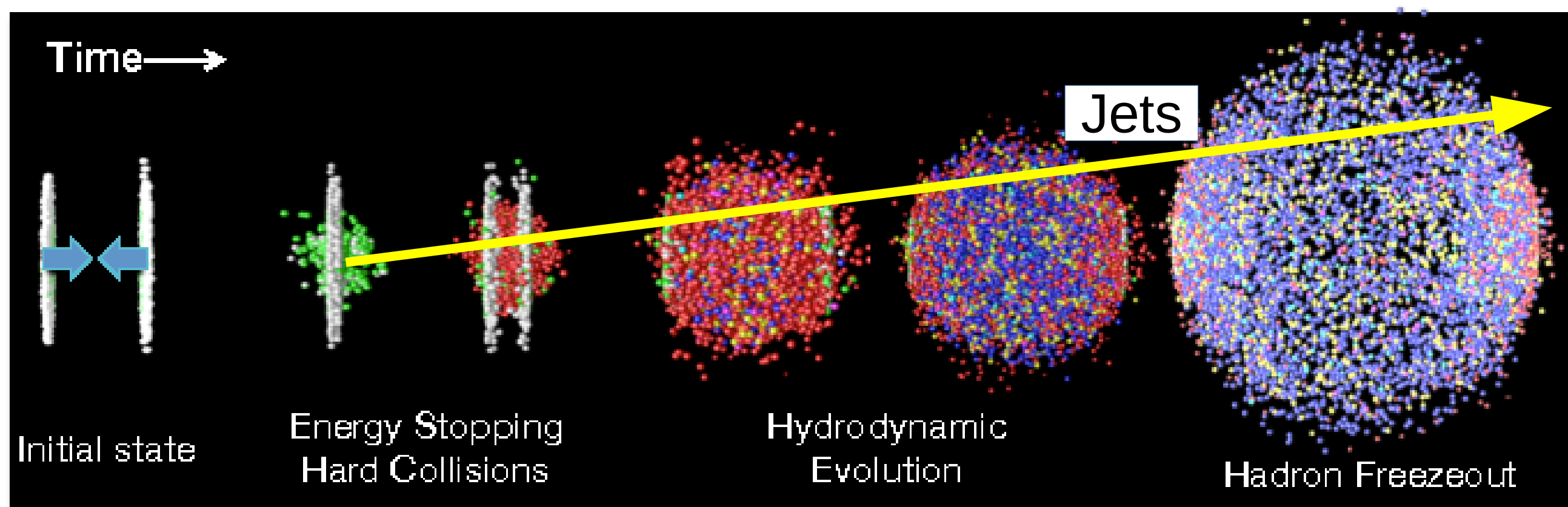
New resummation scheme to reach larger μ_B



For $T \sim 150\text{MeV}$
reaches $\mu_B \sim 525\text{MeV}$



Future Experiments



Electron Ion Collider (EIC) - Nucleon/Nuclei Structure affect the initial state (important for small systems)

Late 2020's

sPHENIX/LHC - Jets probe shorter scales i.e. a QGP microscope

2023+

Beam Energy Scan (RHIC)/FAIR – High baryon densities, hadron gas phase

2018-2020, > 2028

Mapping the QCD phase diagram

