Principles of Charged Particle Acceleration

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To my parents, Katherine and Stanley Humphries
**Preface to the Digital Edition**

I created this digital version of *Principles of Charged Particle Acceleration* because of the large number of inquiries I received about the book since it went out of print two years ago. I would like to thank John Wiley and Sons for transferring the copyright to me. I am grateful to the members of the Accelerator Technology Division of Los Alamos National Laboratory for their interest in the book over the years. I appreciate the efforts of Daniel Rees to support the digital conversion.

STANLEY HUMPHRIES, JR.

University of New Mexico
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**Preface to the 1986 Edition**

This book evolved from the first term of a two-term course on the physics of charged particle acceleration that I taught at the University of New Mexico and at Los Alamos National Laboratory. The first term covered conventional accelerators in the single particle limit. The second term covered collective effects in charged particle beams, including high current transport and instabilities. The material was selected to make the course accessible to graduate students in physics and electrical engineering with no previous background in accelerator theory. Nonetheless, I sought to make the course relevant to accelerator researchers by including complete derivations and essential formulas.

The organization of the book reflects my outlook as an experimentalist. I followed a building block approach, starting with basic material and adding new techniques and insights in a programmed sequence. I included extensive review material in areas that would not be familiar to the average student and in areas where my own understanding needed reinforcement. I tried to make the derivations as simple as possible by making physical approximations at the beginning of the derivation rather than at the end. Because the text was intended as an introduction to the field of accelerators, I felt that it was important to preserve a close connection with the physical basis of the derivations; therefore, I avoided treatments that required advanced methods of mathematical analysis. Most of the illustrations in the book were generated numerically from a library of demonstration microcomputer programs that I developed for the courses. Accelerator specialists will no doubt find many important areas that are not covered. I apologize in advance for the inevitable consequence of writing a book of finite length.
I want to express my appreciation to my students at Los Alamos and the University of New Mexico for the effort they put into the course and for their help in resolving ambiguities in the material. In particular, I would like to thank Alan Wadlinger, Grenville Boicourt, Steven Wipf, and Jean Berlijn of Los Alamos National Laboratory for lively discussions on problem sets and for many valuable suggestions.

I am grateful to Francis Cole of Fermilab, Wemer Joho of the Swiss Nuclear Institute, William Herrmannsfeldt of the Stanford Linear Accelerator Center, Andris Faltens of Lawrence Berkeley Laboratory, Richard Cooper of Los Alamos National Laboratory, Daniel Prono of Lawrence Livermore Laboratory, Helmut Milde of Ion Physics Corporation, and George Fraser of Physics International Company for contributing material and commenting on the manuscript. I was aided in the preparation of the manuscript by lecture notes developed by James Potter of LANL and by Francis Cole. I would like to take this opportunity to thank David W. Woodall, L. K. Len, David Straw, Robert Jameson, Francis Cole, James Benford, Carl Ekdahl, Brendan Godfrey, William Rienstra, and McAllister Hull for their encouragement of and contributions towards the creation of an accelerator research program at the University of New Mexico. I am grateful for support that I received to attend the 1983 NATO Workshop on Fast Diagnostics.

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