

**Fall Term 2003**  
**Plasma Transport Theory, 22.616**  
**LITTLE Problem Set #3**  
**Prof. Molvig**

Passed Out: Sept. 25, 2003

DUE: Oct. 2, 2003

1. **Moment Equation Structure:** Develop the structure of moment equations (for a single species) that follow from the kinetic equation,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial}{\partial \mathbf{v}} f = 0$$

assuming the distribution is close to Maxwellian,

$$f \simeq f^M = \frac{n}{(2\pi T/m)^{3/2}} \exp\left(-\frac{m(\mathbf{v} - \mathbf{V})^2}{2T}\right)$$

In particular show that the density moment gives,

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{V} = 0$$

that the momentum moment gives,

$$mn \left( \frac{\partial}{\partial t} \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla P + qn \left( \mathbf{E} + \frac{1}{c} \mathbf{V} \times \mathbf{B} \right) - \nabla \cdot \boldsymbol{\pi}$$

where the tensor,  $\boldsymbol{\pi}$ , is the stress moment resulting from deviations from Maxwellian,

$$\boldsymbol{\pi} \equiv \int d^3v m \mathbf{v} \mathbf{v} (f - f^M)$$

and the energy moment can be expressed as,

$$\frac{\partial}{\partial t} \left( \frac{3}{2} nT + \frac{1}{2} mnV^2 \right) + \nabla \cdot \mathbf{Q} = qn\mathbf{V} \cdot \mathbf{E}$$

with,  $\mathbf{Q}$ , the total energy flux,

$$\mathbf{Q} = \frac{1}{2} mnV^2 \mathbf{V} + \frac{5}{2} nT\mathbf{V} + \mathbf{q} + \boldsymbol{\pi} \cdot \mathbf{V}$$

This energy equation contains both internal and hydrodynamic flow energies. Eliminate the hydro flow energy by subtracting,  $\mathbf{V} \cdot$  Momentum Equation, from energy equation to yield,

$$\frac{\partial}{\partial t} \left( \frac{3}{2} nT \right) + \nabla \cdot \left( \frac{5}{2} nT\mathbf{V} + \mathbf{q} \right) - \mathbf{V} \cdot \nabla P = -\boldsymbol{\pi} : \nabla \mathbf{V}$$

which can also be manipulated into a form for the temperature alone,

$$\frac{3}{2} n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T + P \nabla \cdot \mathbf{V} = -\nabla \cdot \mathbf{q} - \boldsymbol{\pi} : \nabla \mathbf{V}$$

Finally show that with zero viscous stress,  $\boldsymbol{\pi} = 0$ , and zero heat flux,  $\mathbf{q} = 0$ , that this is equivalent to the adiabatic equation of state,

$$\frac{D}{Dt} \left( P n^{-5/3} \right) = 0$$

implying that  $P n^{-5/3}$  is constant following the fluid.