

1. Ambipolar Potential in a Magnetized Plasma Column

The radial current accounts for viscosity and inertial rotation

$$J_r = \frac{c}{B} \left(-\frac{1}{r} \frac{\partial}{\partial r} r n_i m_i \eta_i \frac{\partial}{\partial r} V_{i\theta} + n_i m_i \frac{\partial}{\partial t} V_{i\theta} \right)$$

By ambipolarity, intrinsically, $J_r = 0$, i.e.

$$\frac{\partial}{\partial t} V_{i\theta} = \eta_i \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} V_{i\theta}$$

it is a diffusion equation in cylindrical system.

A general solution is

$$V_{i\theta} = \sum_n A_n e^{-k_n^2 t} J_0 \left(\frac{k_n}{\sqrt{\eta_i}} r \right)$$

When $t \rightarrow \infty$, asymptotically, $V_{i\theta} = 0$.

Usually (main term)

$$V_{i\theta} = \frac{c \mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mathbf{b} \times \nabla P_i}{n_i m_i \Omega_i}$$

$$= -\frac{c E_r}{B} + \frac{1}{n_i m_i \Omega_i} \frac{dP_i}{dr} = 0$$

$$\text{So } E_r = \frac{B}{n_i m_i \Omega_i c} \frac{dP_i}{dr} \quad \text{i.e.}$$

$$-\frac{d\phi}{dr} = \frac{1}{n_i e_i} \frac{dP_i}{dr} = \frac{1}{e_i} \frac{dT_i}{dr}$$

(2)

$$\Rightarrow e_i \phi(r) = \bar{T}_i T_i(r) \quad , \quad (\text{Assume at edge } e_i \phi(a) = T_i(a) = 0)$$

Therefore in this steady state $V_{i0} = 0$, the net rotation ~~is only from electrons~~ will die away. The electron ~~rotation~~ ^{velocity} will contribute to current Not rotation.

$$V_{e0} = \frac{c \underline{E} \times \underline{B}}{B} + \frac{\underline{b} \times \nabla \rho_e}{n_e m_e c} \quad \text{will contribute to current Not rotation.}$$

Note the friction induced radial electron and ion flow cancel each other. ~~to~~ to keep the ambipolarity.

2. self-Adjoint property of Collision operator

$$C_e(\hat{f}) = \frac{\Gamma}{2} \frac{\partial}{\partial \underline{v}} \cdot \int d^3v' f_0 f_0' \underline{U} \cdot \left(\frac{\partial \hat{f}}{\partial \underline{v}} - \frac{\partial \hat{f}'}{\partial \underline{v}'} \right)$$

$$\int d^3v \hat{g} C_e(\hat{f}) = \frac{\Gamma}{2} \int d^3v \hat{g} \frac{\partial}{\partial \underline{v}} \cdot \int d^3v' f_0 f_0' \underline{U} \cdot \left(\frac{\partial \hat{f}}{\partial \underline{v}} - \frac{\partial \hat{f}'}{\partial \underline{v}'} \right)$$

Integrate by parts (Assume \hat{f} & \hat{g} is reasonably behaved at ∞)

$$\int d^3v \hat{g} C_e(\hat{f}) = -\frac{\Gamma}{2} \int d^3v d^3v' f_0 f_0' \frac{\partial \hat{g}}{\partial \underline{v}} \cdot \underline{U} \cdot \left(\frac{\partial \hat{f}}{\partial \underline{v}} - \frac{\partial \hat{f}'}{\partial \underline{v}'} \right) \quad \text{--- (1)}$$

make change $\underline{v} \rightleftharpoons \underline{v}'$, \underline{U} is symmetric about \underline{v} and \underline{v}'

$$\int d^3v \hat{g} C_e(\hat{f}) = -\frac{\Gamma}{2} \int d^3v d^3v' f_0 f_0' \left(-\frac{\partial \hat{g}}{\partial \underline{v}'} \right) \cdot \underline{U} \cdot \left(\frac{\partial \hat{f}}{\partial \underline{v}} - \frac{\partial \hat{f}'}{\partial \underline{v}'} \right) \quad \text{--- (2)}$$

$$\frac{\textcircled{1} + \textcircled{2}}{2} \Rightarrow$$

③

$$\int d^3v \hat{g} C(\hat{f}) = -\frac{\Gamma}{4} \int d^3v d^3v' f_0 f_0' \left(\frac{\partial \hat{g}}{\partial v} - \frac{\partial \hat{g}'}{\partial v'} \right) \cdot \underline{U} \cdot \left(\frac{\partial \hat{f}}{\partial v} - \frac{\partial \hat{f}'}{\partial v'} \right)$$

this is a symmetric form about \hat{g} & \hat{f} , so.

$$\int d^3v \hat{g} C(\hat{f}) = \int d^3v \hat{f} C(\hat{g}), \quad (\text{self-adjoint!})$$

3. Conservation of laws for linearized collision operator

Use the self-adjointness of C .

$$\int d^3v \begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} C(\hat{f}) = \int d^3v \hat{f} C \left(\begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} \right) \neq$$

$$C \left(\begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} \right) = \frac{\Gamma}{2} \frac{\partial}{\partial \underline{v}} \cdot \int d^3v' f_0 f_0' \underline{U} \cdot \left(\frac{\partial}{\partial \underline{v}} \begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} - \frac{\partial}{\partial \underline{v}'} \begin{bmatrix} 1 \\ m\underline{v}' \\ \frac{1}{2}m v'^2 \end{bmatrix} \right)$$

$$\text{ex. } \frac{\partial}{\partial \underline{v}} 1 - \frac{\partial}{\partial \underline{v}'} 1 = 0$$

$$\frac{\partial}{\partial \underline{v}} \underline{v} - \frac{\partial}{\partial \underline{v}'} \underline{v}' = \underline{I} - \underline{I} = 0$$

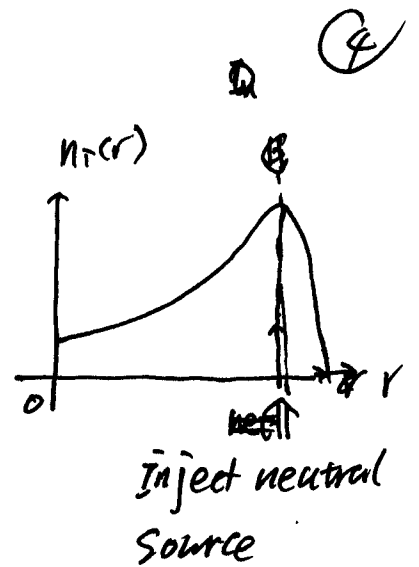
$$\frac{\partial}{\partial \underline{v}} \frac{1}{2}m v^2 - \frac{\partial}{\partial \underline{v}'} \frac{1}{2}m v'^2 = m(\underline{v} - \underline{v}'), \quad \text{but } \underline{U} \cdot (\underline{v} - \underline{v}') = 0$$

$$\text{So } \int d^3v \hat{f} C \left(\begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} \right) = 0 \Rightarrow \int d^3v \begin{bmatrix} 1 \\ m\underline{v} \\ \frac{1}{2}m v^2 \end{bmatrix} C(\hat{f}) = 0$$

4. Solution:

The radial ion flux is

$$\Gamma_i = -n_i \frac{T_i}{m_i \Omega_i^2 T_{i2}} \left(\frac{p_i'}{p_i} - \frac{T_2}{2T_i} \frac{p_2'}{p_2} - \frac{3}{2} \frac{T_i'}{T_i} \right)$$



From the ambipolarity condition

$$\sum_a e_a \Gamma_a = e(\Gamma_i + 2\Gamma_e - \Gamma_e)$$

But $\Gamma_e \propto n_e \frac{T_e}{m_e \Omega_e T_e} \ll \Gamma_i$, so $\Gamma_e = -\frac{1}{2} \Gamma_i$.

This is to say, if we can make ions flow in, the impurities would flow out.

If we take ~~some~~ $\frac{p_i'}{p_i} - \frac{T_2}{2T_i} \frac{p_2'}{p_2} - \frac{3}{2} \frac{T_i'}{T_i} > 0$, ions will flow inward.

Assume $T_2 \sim T_i$, then $(\ln n_i - \frac{1}{2} \ln n_2)' > (\frac{1}{2} \ln T_i (\frac{1}{2} + \frac{1}{2}))'$

~~i.e. if $\frac{p_i'}{p_i} > \frac{3}{2} \frac{T_i'}{T_i}$~~ $\frac{n_i(r)}{n_i(0)} > \left(\frac{n_2(r)}{n_2(0)} \right)^{\frac{1}{2}} \left(\frac{T_i(r)}{T_i(0)} \right)^{\frac{1}{2} - \frac{1}{2}}$ --- (1)

So if we can add some ~~net~~ neutral source at the tokamak edge to make a density (pressure) profile to satisfy Condition (1).

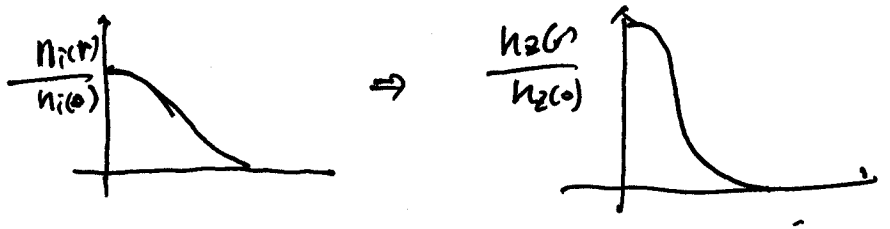
The ~~particle~~ ions will flow in until a steady state is reached.

$$\Gamma_i = 0, \quad \frac{n_i(r)}{n_i(0)} = \left(\frac{n_2(r)}{n_2(0)} \right)^{\frac{1}{2}} \left(\frac{T_i(r)}{T_i(0)} \right)^{\frac{1}{2} - \frac{1}{2}}$$

So For Helium ash, $z=2$

$$\frac{n_i(r)}{n_i(0)} = \left(\frac{n_z(r)}{n_z(0)} \right)^{\frac{1}{z}} \Rightarrow \left(\frac{n_z(r)}{n_z(0)} \right) = \left(\frac{n_i(r)}{n_i(0)} \right)^z$$

Assume the steady state ion distribution is



So there is a big accumulation of ions in the center. impurities (He)

However, for high z impurities, e.g. Carbon, $z=6$

$$\frac{n_z(r)}{n_z(0)} = \left(\frac{n_i(r)}{n_i(0)} \right)^6 \left(\frac{T_i(r)}{T_i(0)} \right)^{-2}$$

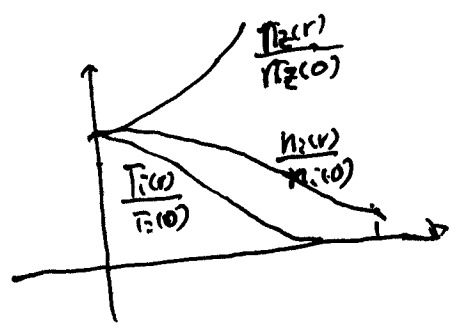
There is a good chance that we can make $n_z(a) > n_z(0)$

For example, we can take $\frac{n_z(r)}{n_z(0)} = \frac{1}{1 + 4 \frac{r^2}{a^2}}$

But if we ~~control~~ make up a temperature profile, s.t. $\frac{T_i(r)}{T_i(0)} = \frac{1}{(1 + 4r^2/a^2)^4}$

Then $\frac{n_z(r)}{n_z(0)} = (1 + \frac{4r^2}{a^2})^2$, there would be no accumulation

of impurities in the center.



5. Diamagnetic Fluxes.

$$n m v_x y = n \int d^3v m v_y f_1 = \frac{n m}{\Omega} \int d^3v v_y^2 \left(A_1 + A_2 \left(\frac{m v^2}{2T} - \frac{5}{2} \right) \right) f_m$$

$$1^{\circ} \int d^3v v_y^2 f_m = \frac{1}{3} \int d^3v v^2 \frac{n}{(2\pi T/m)^{3/2}} e^{-\frac{m v^2}{2T}}$$

$$= \frac{n}{3} \left(\frac{m}{2\pi T} \right)^{3/2} \times 4\pi \int_0^{\infty} dv v^4 e^{-\frac{m v^2}{2T}}$$

let $x = \sqrt{\frac{m}{2T}} v$, then

$$\int d^3v v_y^2 f_m = \frac{4\pi n}{3} \left(\frac{m}{2\pi T} \right)^{3/2} \left(\frac{2T}{m} \right)^{5/2} \int_0^{\infty} dx x^4 e^{-x^2}$$

$\frac{3\sqrt{\pi}}{8}$

$$= \frac{n T}{m}$$

$$2^{\circ} A \cdot \int d^3v v_y^2 \left(\frac{m v^2}{2T} - \frac{5}{2} \right) f_m$$

$$= \frac{1}{3} \int d^3v v^2 \left(\frac{m v^2}{2T} - \frac{5}{2} \right) f_m$$

$$= \frac{4\pi n}{3} \left(\frac{m}{2\pi T} \right)^{3/2} \int_0^{\infty} dv v^4 \left(\frac{m v^2}{2T} - \frac{5}{2} \right) e^{-\frac{m v^2}{2T}}$$

(let $x = \sqrt{\frac{m}{2T}} v$, then)

$$= \frac{4\pi n}{3} \left(\frac{m}{2\pi T} \right)^{3/2} \left(\frac{2T}{m} \right)^2 \int_0^{\infty} dx x^4 \left(x^2 - \frac{5}{2} \right) e^{-x^2}$$

But let $t = x^2$, then

$$\int_0^{\infty} dx x^4 (x^2 - \frac{5}{2}) e^{-x^2}$$

$$= \frac{1}{2} \int_0^{\infty} dt t^{3/2} (t - \frac{5}{2}) e^{-t}$$

$$= \frac{1}{2} \left(\Gamma(\frac{7}{2}) - \frac{5}{2} \Gamma(\frac{5}{2}) \right)$$

$$= 0$$

So, we have $\int d^3v v_y^2 \left(\frac{mv^2}{2T} - \frac{5}{2} \right) f_m = 0$

Then $n m v_{ry} = \frac{m}{\Omega} \frac{n T}{m} A_1 = \frac{\cancel{m} T}{\Omega} A_1 \propto A_1$

6. Generalized Flux-Friction Relations

Take the energy ^{flux} moment of the leading order kinetic equation

$$\Omega \underline{v} \times \underline{b} \cdot \frac{\partial}{\partial \underline{v}} f \approx C(f, f)$$

$$\Rightarrow \int d^3v \frac{1}{2} m v^2 \underline{v} \cdot \Omega \underline{v} \times \underline{b} \cdot \frac{\partial}{\partial \underline{v}} f = \int d^3v \frac{1}{2} m v^2 \underline{v} \cdot C(f, f)$$

$$\text{But } \left(\int d^3v \frac{1}{2} m v^2 \underline{v} \cdot \Omega \underline{v} \times \underline{b} \cdot \frac{\partial}{\partial \underline{v}} f \right)_i$$

$$= \frac{\Omega m}{2} \int d^3v \underline{v}^2 v_i \epsilon_{jke} v_k b_e \frac{\partial f}{\partial v_j}$$

Integrate by parts

$$= -\frac{\Omega m}{2} \int d^3v f \Sigma_{jke} b_e \frac{\partial}{\partial v_j} (v^2 v_i v_k)$$

$$= -\frac{\Omega m}{2} \int d^3v f \Sigma_{jke} b_e (2v_j v_k' + v^2 \delta_{ij} v_k + v^2 \delta_{jk}' v_i)$$

$$(\Sigma_{jke} \delta_{jk} = 0, \Sigma_{jke} b_e v_j v_k = \underline{v} \cdot (\underline{v} \times \underline{b}) = 0)$$

$$= -\frac{\Omega m}{2} \int d^3v f \Sigma_{jke} b_e v^2 v_k$$

$$\left(-\frac{\Omega m}{2} \right)$$

Therefore we can write above result in vector form

$$\int d^3v \frac{1}{2} m v^2 \underline{v} \Omega \underline{v} \times \underline{b} \cdot \frac{\partial}{\partial \underline{v}} f = \Omega \underline{b} \times \int d^3v \frac{1}{2} m v^2 \underline{v}$$

$$= -\Omega \underline{Q} \times \underline{b} = \Omega Q_x \underline{e}_y - \Omega Q_y \underline{e}_x$$

$$\Rightarrow \Omega Q_x = + \int d^3v \frac{1}{2} m v^2 v_y G(f, f)$$

$$= + \int d^3v \frac{1}{2} m v^2 v_y G(\hat{f})$$

$$(f \equiv (\hat{f} + 1) f_0)$$

7. like-particle (Ion) Collision Fluxes

Assume the kinetic equation for the guiding center distribution

$$\frac{\partial f}{\partial t} = C_0(f, f) + C_2(f, f)$$

where C_0 is the "velocity" operator

$$C_0(f, f) \equiv \frac{\Gamma^{ii}}{2} \frac{\partial}{\partial v} \cdot \int d^3v' \underline{U} \cdot \left(\frac{\partial}{\partial \underline{v}} - \frac{\partial}{\partial \underline{v}'} \right) f f'$$

$$C_2(f, f) = \frac{\partial}{\partial \underline{X}} \int d^3v' D(\underline{v}, \underline{v}') \left(\frac{\partial f}{\partial \underline{X}} f' - f \frac{\partial f'}{\partial \underline{X}} \right)$$

$$D(\underline{v}, \underline{v}') \equiv \Gamma^{ii} \underline{e}_y \cdot \underline{U} \cdot \underline{e}_y \frac{1}{\Omega^2}$$

Zeroth order

$$C_0(f_0, f_0) = 0 \Rightarrow f_0 = f_M \text{ (sufficient)}$$

second order:

$$\begin{aligned} \frac{\partial f_0}{\partial t} &\approx C_2(f_0, f_0) + C_0(f_0, f_2) + C_0(f_2, f_0) \\ &= C_0(f_{00}, f_2) + C_0(f_{00}, f_2) + C_2(f_M, f_M) \\ &= C_0(\hat{f}_2) + C_2(f_{00}, f_{00}) \end{aligned} \quad \dots (1)$$

~~Take the~~ Notice $\int d^3v \left(\frac{1}{2} m v^2 \right) C_0(\hat{f}_2) = 0$, proved in prob 3.

1° Take the particle moment of Eq(1).

$$\begin{aligned} \frac{\partial n}{\partial t} &= \int d^3v G(\hat{v}, \hat{f}_2) + \int d^3v G(f_m, f_m) \\ &= \int d^3v G(f_m, f_m) \end{aligned}$$

Write as $\frac{\partial n}{\partial t} + \frac{\partial}{\partial X} \Pi_i = 0$, then Π_i is defined as

$$\Pi_i = - \int d^3v d^3v' D(v, v') \left(\frac{\partial f_m}{\partial X} f_m' - f_m \frac{\partial f_m'}{\partial X} \right)$$

2°. Take the energy moment of Eq(1).

$$\frac{\partial}{\partial t} \int \frac{1}{2} m v^2 f_m d^3v \rightarrow \int d^3v \frac{1}{2} m v^2 G(f_m, f_m) \quad \text{---}$$

i.e.

$$\frac{\partial}{\partial t} \frac{3}{2} n T_i \leftarrow \frac{\partial}{\partial X} \int d^3v d^3v' \frac{1}{2} m v^2 D(v, v') \left(\frac{\partial f_m}{\partial X} f_m' - f_m \frac{\partial f_m'}{\partial X} \right)$$

$$= \frac{\partial}{\partial X} \frac{5}{2} T_i \int d^3v d^3v' D(v, v') \left(\frac{\partial f_m}{\partial X} f_m' - f_m \frac{\partial f_m'}{\partial X} \right)$$

$$+ \frac{\partial}{\partial X} T_i \int d^3v d^3v' \left(\frac{m v^2}{2 T_i} - \frac{5}{2} \right) D(v, v') \left(\frac{\partial f_m}{\partial X} f_m' - f_m \frac{\partial f_m'}{\partial X} \right)$$

$$= - \frac{\partial}{\partial X} \frac{5}{2} T_i \Pi_i \quad \text{---} \quad - \frac{\partial}{\partial X} Q_i$$

with

$$q_i \equiv - \bar{\Gamma}_i \int d^3v d^3v' \left(\frac{mv^2}{2T_i} - \frac{\xi}{2} \right) D(\underline{v}, \underline{v}') \left(\frac{\partial f_0}{\partial \underline{X}} f_0' - f_0 \frac{\partial f_0'}{\partial \underline{X}} \right)$$

$$\frac{q_i}{\bar{\Gamma}_i} = -A_2$$

From above

$$\bar{\Gamma}_i = - \int d^3v d^3v' D(\underline{v}, \underline{v}') \left(\frac{\partial f_0}{\partial \underline{X}} f_0' - f_0 \frac{\partial f_0'}{\partial \underline{X}} \right)$$

change of variables $\underline{v} \Leftrightarrow \underline{v}'$, and $D(\underline{v}, \underline{v}') = D(\underline{v}', \underline{v})$

$$\text{So } \bar{\Gamma}_i = - \int d^3v d^3v' D(\underline{v}, \underline{v}') \left(\frac{\partial f_0'}{\partial \underline{X}} f_0 - f_0' \frac{\partial f_0}{\partial \underline{X}} \right)$$

$$\text{So } \bar{\Gamma}_i = \bar{\Gamma}_i \Rightarrow \boxed{\bar{\Gamma}_i = 0}$$

$$f_0 = \frac{n}{(2\pi T_i/m)^3} e^{-\frac{mv^2}{2T_i}} \Rightarrow \frac{\partial f_0}{\partial \underline{X}} = \left(A_1 + \left(\frac{mv^2}{2T_i} - \frac{\xi}{2} \right) A_2 \right) f_0$$

$$\text{So } \frac{q_i}{\bar{\Gamma}_i} = - \int d^3v d^3v' \left(\frac{mv^2}{2T_i} - \frac{\xi}{2} \right) D(\underline{v}, \underline{v}') f_0 f_0' \left(\frac{mv'^2}{2T_i} - \frac{mv^2}{2T_i} \right) A_2$$

$$\stackrel{1)}{=} -A_2 \int d^3v d^3v' \left(\frac{mv^2}{2T_i} - \frac{\xi}{2} \right) D(\underline{v}, \underline{v}') f_0 f_0' \left(\frac{\frac{1}{2}mv^2 - \frac{1}{2}mv'^2}{T_i^2} \right)$$

$$\stackrel{2)}{=} -A_2 \int d^3v d^3v' \left(\frac{mv'^2}{2T_i} - \frac{\xi}{2} \right) D(\underline{v}, \underline{v}') f_0 f_0' \frac{\frac{1}{2}mv'^2 - \frac{1}{2}mv^2}{T_i^2}$$

$$\frac{q_1 + q_2}{2} = -A_2 \int d^3v d^3v' \frac{\left(\frac{1}{2}m v^2 - \frac{1}{2}m v'^2\right)^2}{2T_i^2} D(v, v') f_0 f_0'$$

In terms of the overall transport matrix

$$\begin{pmatrix} T_i \\ q_i / \Omega T_i \end{pmatrix} = \frac{1}{n} \begin{bmatrix} -D & T_{12} \\ T_{21} & -\chi_i \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

Since A_1 & A_2 are independent to each other $T_i = 0$

Then $D = T_{12} = 0$.

Also we have $T_{21} = 0$ through above calculation or Onsager

Symmetry.

$$\chi_i = \frac{n_i}{2T_i^2} \int d^3v d^3v' \frac{\left(\frac{1}{2}m v^2 - \frac{1}{2}m v'^2\right)^2}{2T_i^2} D(v, v') f_0 f_0'$$

So only χ_i is not zero. with

$$D(v, v') = \frac{e_y \cdot v \cdot e_y}{\Omega^2} \frac{\pi''}{\Omega^2}$$

Calculation of χ_i

change of variables $(\underline{v}, \underline{v}') \rightarrow (\underline{v}_1, \underline{v}_2)$

$\underline{v}_1 = \underline{v} - \underline{v}'$, $\underline{v}_2 = \frac{1}{2}(\underline{v} + \underline{v}')$, therefore

$$\chi_i = \frac{n_i \Gamma_{ii}}{2 \Omega^2 T_i^2} \int d^3v \int d^3v' \frac{m_i^2}{|\underline{v} - \underline{v}'|} \left(\frac{1}{2}v^2 - \frac{1}{2}v'^2 \right)^2 \left(1 - \frac{(v_y - v'_y)^2}{|\underline{v} - \underline{v}'|^2} \right)$$

$$\times \frac{n^2}{(2\pi T_i/m_i)^3} e^{-\frac{m}{2T}(v^2 + v'^2)}$$

$$= \frac{n_i \Gamma_{ii}}{2 \Omega^2 T_i^2} \frac{n_i^2 m_i^5}{(2\pi T_i)^3} \int d^3v_1 \frac{1}{v_1} \int d^3v_2 (v_1 \cdot v_2)^2 \left(1 - \frac{e_y \cdot v_1 v_1 \cdot e_y}{v_1^2} \right)$$

$$\times e^{-\left(\frac{m}{4T}v_1^2 + \frac{m}{T}v_2^2\right)}$$

Note $\frac{1}{(\pi T/m)^{3/2}} \int_{-\infty}^{\infty} dv_x v_x^2 e^{-\frac{m}{T}v_x^2} = \frac{T}{2m}$

So $\frac{1}{(\pi T/m)^{3/2}} \int d^3v_2 v_2 v_2 e^{-\frac{m}{T}v_2^2} = \frac{1}{2} \frac{T}{m} \frac{T}{m}$

$$\text{So } \chi_i = \frac{n_i \Gamma_{ii}}{2 \Omega^2 T_i^2} \frac{n_i^2 m_i^5}{(2\pi T_i)^3} \frac{1}{2} \frac{T}{m} \int d^3v_1 v_1 \left(1 - \frac{v_{1y}^2}{v_1^2} \right) e^{-\frac{m v_1^2}{4T}} \times \left(\pi \frac{T}{m} \right)^{3/2}$$

$$= \frac{\Gamma_{ii}}{2 \Omega^2 T_i^2} \frac{n_i^2 m_i^5}{(2\pi T_i)^3} \frac{\pi^{3/2}}{2} \left(\frac{T}{m_i} \right)^{5/2} \times \frac{64\pi}{3} \left(\frac{T_i}{m_i} \right)^2$$

Note $\int d^3v_i v_i \left(1 - \frac{v_i^2}{v_i^2}\right) e^{-\frac{m v_i^2}{4T_i}}$

$$= 4\pi \int_0^\infty dv_i v_i^3 \left(1 - \frac{1}{3}\right) e^{-\frac{m v_i^2}{4T_i}} = \frac{64\pi}{3} \left(\frac{T_i}{m_i}\right)^2$$

Therefore we get

$$\chi_i = \frac{2N_i}{3\sqrt{\pi}} \frac{\gamma_{ii}}{\Omega_i^2} \left(\frac{m_i}{T_i}\right)^{\frac{1}{2}} N_i^2$$

Notice $\gamma_{ii} = \frac{4\pi z^4 e^4 / m_i}{m_i^2} = \frac{\nu_{ii}}{N_i} \left(\frac{2T_i}{m_i}\right)^{3/2}$

So $\chi_i = \frac{4\sqrt{2}}{3\sqrt{\pi}} \nu_{ii} \frac{N_i^2 T_i}{m_i \Omega_i^2}$

From the book $\frac{1}{\tau_i} = \frac{2\sqrt{2}}{3\sqrt{\pi}} \nu_{ii}$, (Eq. 1.5)

So $\chi_i = 2 \frac{\nu_{ii}^2 T_i}{m_i \Omega_i^2} \frac{1}{\tau_i}$

This is exactly Braginskii's result Eq 4.48