Subject 24.242. Logic II. Homework due March 6.

1. Show that the function Pair given by $\mathrm{Pair}(x,y) = \frac{1}{2}(x^2 + 2xy + y^2 + x + 3y)$ is a bijection from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$.

2. Show that the set of prime numbers is a bounded set.

3. Write down a bounded formula that says that $x > 0$ and z is the remainder on dividing y by x.

4. Using the result from problem 3, show that Goldbach's conjecture – "Every even number > 2 is the sum of two primes" – can be formalized as a Π sentence.

5. Show that, for any Σ sets A and B, there exist Σ sets $C \subseteq A$ and $D \subseteq B$ with $C \cap D = \varnothing$ and $C \cup D = A \cup B$.

6. Show that, for any Σ binary relation R, there is a Σ partial function f with $\mathrm{Dom}(f) = \{x: (\exists y) <x,y> \in R\}$ and with $<x,f(x)> \in R$ for each $x \in \mathrm{Dom}(f)$.

7. Show that, for any two nonoverlapping Π sets A and B there is a Δ set C that includes A and is disjoint from B.