Subject 24.242. Logic II. Homework Due Wednesday, February 29

A set of natural numbers is *effectively coenumerable* iff the complement of the set (the set of numbers that aren't in the set) is effectively enumerable.

1. Show that the union of two effectively coenumerable sets is coenumerable.

2. Show that the intersection of two effectively coenumerable sets is coenumerable.

3. Show that, for S a set of natural numbers, the following are equivalent:
   a) S is effectively enumerable.
   b) There is a decidable binary relation R such that $S = \{x: (\exists y) \langle x,y\rangle \in R\}$.
   c) There is an effectively enumerable binary relation R such that $S = \{x: (\exists y) \langle x,y\rangle \in R\}$.

4. True or false? Explain your answer: A set is decidable iff it is either finite or the range of an increasing calculable total function. (A function f is *increasing* iff, for any x and y in its domain, if $x < y$ then $f(x) < f(y)$.)

5. True or false? Explain your answer: A set is decidable iff it is either empty or the range of an nondecreasing calculable total function. (A function f is *nondecreasing* iff, for any x and y in its domain, if $x \leq y$ then $f(x) \leq f(y)$.)

6. Show that, if A and B are disjoint, effectively coenumerable sets of natural numbers, there is a decidable set C with $A \subseteq C$ and $B \cap C = \emptyset$.

7. Show that it's not the case that, for any two effectively coenumerable sets A and B, there are effectively coenumerable sets C and D with $C \subseteq A$, $D \subseteq B$, $C \cap D = \emptyset$, and $C \cup D = A \cup B$.

8. Show that it's not the case that, for any effectively enumerable binary relation R, the partial function f given by

   $f(x) =$ the least y such that $\langle x,y\rangle \in R$, if there is a y with $\langle x,y\rangle \in R$.
   $f(x)$ is undefined if there isn't any y with $\langle x,y\rangle \in R$.

   is calculable.