

Subject 24.242. Logic II. Assignment due Wednesday, April 10.

Show the following sentences are consequences of PA. You may use any of the theorems proved on the handout:

1. $(\forall x)(x \bullet [1]) = x$
2. $(\forall x)(\forall y)(\forall z)(x E (y + z)) = ((x E y) \bullet (x E z))$
3. $(\forall x)(\forall y)(\forall z)(x E (y \bullet z)) = ((x E y) E z)$
4. $(\forall x)(\forall y)(\forall z)((x \bullet y)Ez) = ((xEz) \bullet (yEz))$
5. $(\forall x)(\forall y)(x < y \leftrightarrow (\exists z)(x + sz) = y)$
6. $(\forall x)(\forall y)(x < y \leftrightarrow sx < sy)$
7. $(\forall x)(\forall y)(x < y \leftrightarrow sx \leq sy)$, where $\tau \leq \rho$ is an abbreviation for $(\tau < \rho \vee \tau = \rho)$
8. $(\forall x) \neg x < x$
9. $(\forall x)(\forall y)(\forall z)((x < y) \wedge (y < z)) \rightarrow x < z)$
10. Show that, taking (Q11) to be one of the axioms of PA is redundant; that is, “ $(\forall x)(\forall y)(x < y \vee (x = y \vee y < x))$ ” is derivable from the other axioms of PA.