

Subject 24.142. Logic II. Homework assignment due Wednesday, May 1

In answering these questions, you may use abbreviation, in particular, “Pair” and “Triple.”

1. What is $\ulcorner ([1] + [1]) \urcorner$?
2. What is $\ulcorner (s([2] \cdot [2])E[2]) \urcorner$?
3. What is the Gödel number for (Q1)?
4. What is the Gödel number for (Q8)?

In the last two problems, you’ll need to replace “x” and “y” with official variables, like “ x_0 ” and “ x_1 .”

Consider the version of the sentential calculus in which the atomic sentences are “ p_0 ,” “ p_1 ,” “ p_2 ,” “ p_3 ,” and so on, which we give Gödel numbers by setting:

$$\begin{aligned}\ulcorner p_i \urcorner &= \text{Pair}(17, i) \\ \ulcorner \neg \phi \urcorner &= \text{Pair}(10, \ulcorner \phi \urcorner) \\ \ulcorner (\phi \vee \psi) \urcorner &= \text{Triple}(11, \ulcorner \phi \urcorner, \ulcorner \psi \urcorner) \\ \ulcorner (\phi \wedge \psi) \urcorner &= \text{Triple}(12, \ulcorner \phi \urcorner, \ulcorner \psi \urcorner) \\ \ulcorner (\phi \rightarrow \psi) \urcorner &= \text{Triple}(13, \ulcorner \phi \urcorner, \ulcorner \psi \urcorner) \\ \ulcorner (\phi \leftrightarrow \psi) \urcorner &= \text{Triple}(14, \ulcorner \phi \urcorner, \ulcorner \psi \urcorner)\end{aligned}$$

5. Show that the set of codes of SC sentences is Σ
6. Show that the set of codes of SC sentences is Π .
7. Show that the set of codes of tautological SC sentences is Π .
8. Show that the set of codes of tautological SC sentences is Σ .

Life is too precious to spend it solving the last four problems in detail. I’ll be content with a plausible sketch.