Subject 24.242. Logic II. Homework due Wednesday, May 8.

1. Goldbach's conjecture is the statement that every even number greater than 2 is the sum of two primes. No one knows whether Goldbach's conjecture is true, nor does anyone know whether it's decidable in PA. It is known, however, that if Goldbach's conjecture is undecidable in PA, then it's true. Show this.

2. Take α so that Q $\vdash (\alpha \leftrightarrow \neg \mathrm{Bew}_{PA}([\alpha]))$. Take β so that Q $\vdash (\beta \leftrightarrow \neg \mathrm{Bew}_{PA \cup \{\alpha\}}([\beta]))$. Is β provable in the theory PA $\cup \{\alpha\}$? Is β true? Explain your answers.

3. Take α so that Q $\vdash (\alpha \leftrightarrow \neg \mathrm{Bew}_{PA}([\alpha]))$. Take γ so that Q $\vdash (\gamma \leftrightarrow \neg \mathrm{Bew}_{PA \cup \{\neg \alpha\}}([\gamma]))$. Is γ provable in the theory PA $\cup \{\neg \alpha\}$? Is γ true? Explain your answers.

4. Take δ so that Q $\vdash (\delta \leftrightarrow \mathrm{Bew}_{PA}([\ulcorner \neg\delta \urcorner]))$. Is δ decidable in PA? Is δ true? Explain your answers.

5. Let Γ be a consistent Δ set of axioms that includes Q. The *Rosser sentence* for Γ is the sentence obtained by the self-referential lemma so that Q $\vdash (\rho \leftrightarrow (\forall y)(y\ B_\Gamma[\ulcorner\rho\urcorner] \rightarrow (\exists z < y)\ z\ B_\Gamma[\ulcorner \neg \rho \urcorner]))$. Show that ρ is undecidable in Γ. Is ρ true?