3.091 Fall Term 2009

Homework #2

solution outlines

46. From Equation 6.2 the broadcast frequency is

$$\nu = \frac{2.998 \times 10^8 \text{ m/s}}{3.21 \text{ m}}\left(\frac{1 \text{ MHz}}{1 \times 10^6 \text{ Hz}}\right) = 93.4 \text{ MHz}$$

From Equation 6.2, the wavelength at $\nu = 82$ MHz is

$$\lambda = \frac{2.998 \times 10^8 \text{ m/s}}{82 \text{ MHz}\left(\frac{1 \times 10^6 \text{ Hz}}{1 \text{ MHz}}\right)} = 3.7 \text{ m}$$

Likewise, the wavelength at $\nu = 112$ MHz is 2.68 m, giving a range of 2.68 m to 3.7 m.

*__51.__ We use Equations 6.5 and 6.2 to determine the wavelength of this photon. The energy conversion in the denominator accounts for the fact that we are given 1 mol of photons. Thus,

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})}{\frac{225 \text{ kJ}}{\text{mol}}\left(\frac{1000 \text{ J}}{1 \text{ kJ}}\right)\left(\frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ photons}}\right)} = 5.32 \times 10^{-7} \text{ m} = 532 \text{ nm}$$

54. To begin, we convert the energy output (official name, *power*) to J/s as follows:

$$\text{energy output} = 50 \text{ kW}\left(\frac{1000 \text{ W}}{1 \text{ kW}}\right)\left(\frac{1 \text{ J/s}}{1 \text{ W}}\right) = \frac{5 \times 10^4 \text{ J}}{\text{s}}$$

Now, each of the photons has an energy given by Equation 6.5:

$$E = h\nu = (6.626 \times 10^{-34} \text{ J}\cdot\text{s})\left(\frac{100.7 \text{ MHz}}{1 \text{ MHz}/10^6\text{Hz}}\right) = 6.672 \times 10^{-26} \text{ J/photon}$$

The number N of photons given off each second, then, is

$$N = \frac{5 \times 10^4 \text{ J}}{\text{s}}\left(\frac{1 \text{ photon}}{6.672 \times 10^{-26} \text{ J}}\right) = 7 \times 10^{29} \text{ photons/second}$$

***55.** Equation 6.11 is

$$\Delta E = -\mathcal{R}hc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

A transition from $n = 2$ to $n = 3$ requires absorption of a photon, and an absorption line results. A transition from $n = 3$ to $n = 2$ results in the emission of a photon, and an emission line results. The energy change for both has the same magnitude, but the negative sign in the above Equation makes the emission ΔE negative and the absorption ΔE positive. The energy change for the absorption is

$$\begin{aligned}\Delta E &= -\mathcal{R}ch\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\\ &= -(1.0974 \times 10^7/\text{m})(2.998 \times 10^8\ \text{m/s})(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{3^2} - \frac{1}{2^2}\right)\\ &= 3.028 \times 10^{-19}\ \text{J}\end{aligned}$$

We again use the "final – initial" strategy for placement of the n's. As expected ΔE is positive for absorption. There is no "range" of light; the transition occurs at a single wavelength, determined from

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{3.028 \times 10^{-19}\ \text{J}} = 6.56 \times 10^{-7}\ \text{m} = 656\ \text{nm}$$

***57.** In this case we do not know n_1, the n level where the transition terminates. We can solve Equation 6.11 for n_1 to get

$$n_1 = \sqrt{\left(-\frac{\Delta E}{\mathcal{R}hc} + \frac{1}{n_2^2}\right)^{-1}}$$

The *magnitude* of the energy difference between levels is given by Equations 6.5 and 6.2. Thus,

$$\Delta E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{486\ \text{nm}\left(\dfrac{1\ \text{m}}{1 \times 10^9\ \text{nm}}\right)} = 4.09 \times 10^{-19}\ \text{J}$$

Since this is *emission* of a photon, the ΔE value that we enter in Equation 6.11, as solved above, must be negative. This convention for the signs must always be used since we use the "final – initial" procedure when substituting in n values in Equation 6.11. Thus, we solve

$$\begin{aligned}n_1 &= \sqrt{\left(\frac{-(-4.09 \times 10^{-19}\ \text{J})}{(1.0974 \times 10^7/\text{m})(2.998 \times 10^8\ \text{m/s})(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})} + \frac{1}{4^2}\right)^{-1}}\\ &= \sqrt{4} = 2\end{aligned}$$

Light at 486 nm is blue-green. See Figure 6.9 (b).

*__67.__ Wave effects (also called quantum effects) are not seen in objects with large mass. We expect, therefore, a very small de Broglie wavelength and a very small uncertainty in position. The actual de Broglie wavelength is

$$\lambda_{\text{deB}} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{39\ \text{g}\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\frac{1020\ \text{m}}{\text{s}}\right)} = 1.7 \times 10^{-35}\ \text{m}$$

The uncertainty in position is given by solving Heisenberg's uncertainty principle, Equation 6.19, for Δx. See Example 6.5. We are given Δv as 10 m/s. Thus,

$$\Delta x \geq \frac{h}{4\pi m \Delta v} \geq \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi(0.039\ \text{kg})\left(\frac{10\ \text{m}}{\text{s}}\right)} \geq 1.4 \times 10^{-34}\ \text{m}$$

*__71.__ The number of subshells for a given n is equal to n. Thus, there are three subshells in the $n = 3$ level. They are s, p, and d for which $l = 0$, 1, and 2, respectively.

*__73.__ d orbitals have $l = 2$. There are 5 orbitals in an d subshell.

78. The first quantum number n gives the principal shell directly. The second quantum number l gives the subshell, according to $l = 0$ for s, $l = 1$ for p, $l = 2$ for d, and so on. Thus,

(a) 1, s

(b) 2, p

(c) 3, d

(d) 4, f

86. (a) C has four valence electrons. The valence configuration is $2s^2\ 2p^2$. The electrons in the 2s orbital have the quantum numbers $[2, 0, 0, \frac{1}{2}]$ and $[2, 0, 0, -\frac{1}{2}]$. The electrons in the 2p orbitals have the quantum numbers $[2, 1, 1, \frac{1}{2}]$ and $[2, 1, 0, \frac{1}{2}]$.

(b) Mg has two valence electrons. The valence configuration is $3s^2$. The electrons in the 3s orbital have the quantum numbers $[3, 0, 0, \frac{1}{2}]$ and $[3, 0, 0, -\frac{1}{2}]$.

(c) Br has seven valence electrons. The valence configuration is $4s^2\ 4p^5$. The electrons in the 4s orbital have the quantum numbers $[4, 0, 0, \frac{1}{2}]$ and $[4, 0, 0, -\frac{1}{2}]$. The electrons in the 4p orbitals have the quantum numbers $[4, 1, 1, \frac{1}{2}]$, $[4, 1, 1, -\frac{1}{2}]$, $[4, 1, 0, \frac{1}{2}]$, $[4, 1, 0, -\frac{1}{2}]$, and $[4, 1, -1, \frac{1}{2}]$. (We exclude any d or f orbitals.)

*__93.__ You are urged to obtain these configurations without the use of Table 6.4. Use the periodic table, and check your results against Table 6.4 after you have finished. Only (f) is anomalous.

(a) B: $[He]2s^2\ 2p^1$
(b) Rb: $[Kr]5s^1$
(c) Br: $[Ar]4s^2\ 3d^{10}\ 4p^5$
(d) Ge: $[Ar]4s^2\ 3d^{10}\ 4p^2$
(e) V: $[Ar]4s^2\ 3d^3$
(f) Pd: $[Kr]4d^{10}$
(g) Bi: $[Xe]6s^2\ 4f^{14}\ 5d^{10}\ 6p^3$
(h) Eu: $[Xe]6s^2\ 4f^7$

98. The valence orbital diagram for chlorine is:

Cl: 3*p* ↑↓ ↑↓ ↑
3*s* ↑↓

The valence orbital diagram for Si is

Si: 3*p* ↑ ↑ _
3*s* ↑↓

The valence orbital diagram for Sc is

Sc: 3*d* ↑ _ _ _ _
4*s* ↑↓

*__101.__ Construct orbital occupation diagrams in each case and count the number of unpaired electrons. Below we give the portion of the orbital occupation diagram that is relevant to the number of unpaired electrons.

(a) 2 unpaired electrons

Pb: 6*p* ↑ ↑ _

(b) 1 unpaired electron

Cs: 6*s* ↑

(c) 1 unpaired electron; Cu has an anomalous configuration.

Cu: 4*s* ↑

(d) 2 unpaired electrons

Si: 3*p* ↑ ↑ _

(e) 2 unpaired electrons

Se: 4*p* ↑↓ ↑ ↑

Additional questions:

1. From quantum condition we have

$$\mathrm{mvr} = \frac{\mathrm{nh}}{2\pi}$$

$$\therefore \mathrm{v} = \frac{\mathrm{nh}}{2\pi \mathrm{mr}}$$ get r from the Bohr model, r = 0.529 Å when n=1.

$$= \frac{1 \times 6.6 \times 10^{-34}}{2\pi 9.11 \times 10^{-31} \times 0.529 \times 10^{-10}}$$

$$= 2.18 \times 10^{6} \text{ m/s.}$$

2. $c = \nu\lambda, \quad \therefore \lambda_{\min} = \frac{c}{\nu_{\max}}; \quad \lambda_{\max} = \frac{c}{\nu_{\min}}$

AM $\lambda_{\min} = \frac{3 \times 10^{8}\,\mathrm{m/s}}{1600 \times 10^{3}\,\mathrm{Hz}} = 188\ \mathrm{m}$ $\lambda_{\max} = \frac{3 \times 10^{8}}{530 \times 10^{3}} = 566\ \mathrm{m}$

FM $\lambda_{\min} = \frac{3 \times 10^{8}}{108 \times 10^{6}} = 2.78\ \mathrm{m}$ $\lambda_{\max} = \frac{3 \times 10^{8}}{88 \times 10^{6}} = 3.41\ \mathrm{m}$

3. $E_{\text{incident photon}} = E_{\text{binding}} + E_{\text{scattered e-}}$

$E_{\text{binding}} = -K\left(\frac{1}{3^{2}}\right)$ $E_{\text{scattered}} = \frac{1}{2}\mathrm{mv}^{2}$

$E_{\text{incident photon}} = \frac{\mathrm{hc}}{\lambda}$ $\therefore \frac{\mathrm{hc}}{\lambda} = \frac{K}{9} + \frac{1}{2}\mathrm{mv}^{2}$ $\therefore \left[\left(\frac{\mathrm{hc}}{\lambda} - \frac{K}{9}\right)\frac{2}{\mathrm{m}}\right]^{\frac{1}{2}} = \mathrm{v}$

$$\left[\left(\frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3.091 \times 10^{-7}} - \frac{2.18 \times 10^{-18}}{9}\right)\frac{2}{9.11 \times 10^{-31}}\right]^{\frac{1}{2}} = \mathrm{v}$$

$\therefore$ v = 9.35×10^{5} m/s.

4. $\Delta E_{1\to 6} = \text{q}V \qquad \therefore\ V = \dfrac{\Delta E_{1\to 6}}{\text{q}}$

$$\Delta E_{1\to 6} = -K\left(\frac{1}{1^2} - \frac{1}{6^2}\right) = \frac{35}{36}K$$

q = +2e

$$\therefore V = \frac{35}{36} \times \frac{2.18 \times 10^{18}}{2 \times 1.6 \times 10^{-19}} = 6.62V$$

5. $\bar{v} = Z^2 R\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = 3^2 \times 1.1 \times 10^7\left(\dfrac{3}{4}\right)$

$$= 7.43 \times 10^7 \text{ m}^{-1}$$

6. $\Delta E_{1\to 4} = -Z^2 K\left(\dfrac{1}{4^2} - \dfrac{1}{1^2}\right) = \dfrac{15}{16}KZ^2 = \dfrac{\text{hc}}{\lambda}$

for He, $Z = 2$

$$\therefore \lambda = \frac{\text{hc}}{KZ^2} \bullet \frac{16}{15} = 2.42 \times 10^{-8} \text{m}$$