What is a "Point Group"?

## Point Group Definition

- A classification scheme for finite objects (molecules)
- Molecules having the same set of symmetry elements/operations "belong to" the same point group
- Point groups have labels we will learn
- We will use the Schoenflies notation (spectroscopy) rather than the Hermann-Mauguin notation (crystallography)


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Labels are the Schoenflies symbols: here are six limiting symmetry types

- High symmetry, multiple higher-order $(n>2)$ rotation axes. Examples: $T_{d}, I_{h}, O_{h}$
- Low symmetry, only the identity or that plus only a single mirror plane or an inversion center: $C_{1}, C_{s}, C_{i}$
- Linear molecules: $C_{\infty v}, D_{\infty h}$
- C groups: $C_{n v}, C_{n h}, C_{n}$
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- $S$ groups: $S_{4}, S_{6}, S_{8}$, etc.; only operations present are based upon $S_{n}$
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The Platonic Solids: Polyhedra with Regular Polygon Faces Symmetry lowered from the sphere, but still present are multiple higher-order axes


Tetrahedron


Octahedron


Cube


Icosahedron


Dodecahedron

## High Symmetry Groups

These have multiple higher order $(n>2)$ rotation axes. Example: $C_{60}$, icosahedral

$Q \curvearrowright$

## High Symmetry Groups

These have multiple higher order $(n>2)$ rotation axes. Example: $\left[\mathrm{B}_{12} \mathrm{H}_{12}\right]^{2-}$, icosahedral
$\square$ Show All Proper
$\square$ Show All Improper
$\square \mathrm{C}_{5}$ axis Rotate$\square \mathrm{C}_{3}$ axis$\mathrm{C}_{2}$ axis$\mathrm{C}_{2}$ axis
$\square \mathrm{C}_{2}$ axis

Element Operation

| $\square$ Show All | nes |
| :---: | :---: |
| $\square$ inv ctr | Invert |
| $\square$ plane ( $\sigma$ ) | Reflect |
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## High Symmetry Groups

The tetrahedron has four $C_{3}$ axes but lacks inversion center


## High Symmetry Groups

The group $T_{h}$ has four $C_{3}$ axes (through octahedral faces) and adds the inversion center
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High Symmetry Groups
The group $T$ is a pure rotation group with no mirror planes or inversion centers


## High Symmetry Groups

The group $O_{h}$ has three $C_{4}$ axes and an inversion center: $\left[\mathrm{Mo}_{6} \mathrm{Cl}_{14}\right]^{2-}$


## High Symmetry Groups

The group $O_{h}$ has three $C_{4}$ axes and an inversion center: $\mathrm{SF}_{6}$



## Groups of Low Symmetry

The identity alone, or together with one mirror or an inversion center

- The identity only, $C_{1}$
- The identity plus one mirror plane: $C_{s}$
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## Linear Molecules

Distinguish based upon presence or absence of $\perp C_{2}$ axes

- A linear molecule has a $C_{\infty}$ axis of rotation
- Nitrous oxide, $\mathrm{N}_{2} \mathrm{O}, \mathrm{N}=\mathrm{N}=\mathrm{O}$, the two ends are different so no $C_{2} \perp$ to the $C_{\infty}$, the point group assignment is $C_{\infty v}$
- Carbon dioxide, $\mathrm{CO}_{2}, \mathrm{O}=\mathrm{C}=\mathrm{O}$, the two ends are "symmetry related" and exchangeable by $\perp C_{2}$ or by $\sigma_{h}$ so the point group assignment is $D_{\infty h}$
- In general, $D$ groups have $n C_{2}$ axes $\perp$ to the $C_{n}$ (single principal rotation axis)


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## Examples of C groups

A single $C_{n}$ plus $n$ vertical mirror planes


## Examples of C groups

A single $C_{n}$ plus $n$ vertical mirror planes


## Examples of C groups

A single $C_{n}$ plus a horizontal mirror plane


## Examples of C groups

A single $C_{2}$ with no mirror planes: ansa metallocene example of point group $C_{2}$


## Examples of $D$ groups

$D_{n h}$ has $n C_{2} \perp$ to the $C_{n}$, plus a $\sigma_{h}$



## Examples of $D$ groups

$D_{n h}$ has $n C_{2} \perp$ to the $C_{n}$, plus a $\sigma_{h}$


## Examples of $D$ groups

$D_{n d}$ has $n C_{2} \perp$ to the $C_{n}$, plus $n \sigma_{d}$ but no $\sigma_{h}$; example is $\mathrm{S}_{4} \mathrm{~N}_{4}$


## Examples of $D$ groups

$D_{n d}$ has $n C_{2} \perp$ to the $C_{n}$, plus $n \sigma_{d}$ but no $\sigma_{h}$; example is $\mathrm{S}_{8}$


## Examples of $D$ groups

$D_{n}$ has $n C_{2} \perp$ to the $C_{n}$, but no mirror planes; example is $\left[\mathrm{Fe}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}\right]^{3-}$
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## Example of $S_{n}$ groups

$S_{n}$ only for $n=2,4,6, \ldots$ but note, $S_{2}=$ inversion so that is $C_{i}$


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