# MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Electrical Engineering and Computer Science 

### 6.002 - Electronic Circuits <br> Fall 2002

## Problem Set 1

Issued: September 4, 2002
Due: September 11, 2002

Reading Assignment:
Readings will be assigned from Agarwal and Lang.

- Chapters 1, 2 and Appendix A (for this week).
- 3.1-3.3 for Tuesday, September 10
- 3.5-3.7 for Thursday, September 12.

Review Exercise 1.1: Let $x(t)$ be the complex function $x(t)=\frac{a+b j}{c+d j} e^{j \omega t}$ where $a, b, c, d$ and $\omega$ are constants, and $j=\sqrt{-1}$. Determine $M$ and $\phi$ such that $\Re\{x(t)\}=M \cos (\omega t+\phi)$, where the symbol $\Re\{z\}$ represents the real part of the complex number $z$.

Review Exercise 1.2: Determine $x(t)$ for $t \geq 0$ given that $\frac{d x(t)}{d t}+a x(t)=b$ and $x(0)=c$ where $a, b$ and $c$ are constants.

Problem 1.1: Figure 1 shows a circuit with four elements: two resistors, a voltage source, and a current source. The resistances of the resistors and the strengths of the sources are given. Branch voltage circuit variables $\left(v_{k}\right)$ and branch current circuit variables $\left(i_{k}\right)$ are defined for each element, in associated reference directions.


Figure 1: Circuit for Problem 1
(A) How many nodes are there in the circuit? Write a KCL equation for each node. How many of the KCL equations are independent? (Independent means, "Cannot be derived from the other equations.")
(B) How many loops are there in the circuit? Write a KVL equation for each loop. How many of these equations are independent?
(C) Write an equation expressing the v-i constraint for each element.
(D) You should now have a set of linear equations in the branch voltage and current variables. If you count only the independent equations, you should have one equation for each unknown branch voltage or current. Solve the equations and fill in the table of results. Compute the power $\left(v_{k} i_{k}\right)$ leaving the circuit by each branch and enter it into your table.

| $v_{1}=$ | $i_{1}=$ | $v_{1} i_{1}=$ |
| :--- | :--- | :--- |
| $v_{2}=$ | $i_{2}=$ | $v_{2} i_{2}=$ |
| $v_{3}=$ | $i_{3}=$ | $v_{3} i_{3}=$ |
| $v_{4}=$ | $i_{4}=$ | $v_{4} i_{4}=$ |

(E) Now compute the sum of the four powers you computed. It should be exactly zero, or you made an error! Which source supplies power and which absorbs power?

Problem 1.2: Repeat the process you did in Problem 1 for the circuit of Figure 2 below. Notice that the sum of the powers is zero here too.


Figure 2: Circuit for Problem 2

Problem 1.3: Make a table of the voltages you computed from Problem 1 paired with the corresponding currents you got in Problem 2 (e.g., get $v_{1}$ from the first table and $i_{1}$ from the second table). What is the sum of the products of these voltages and the currents? More precisely, compute $v_{1} i_{1}+v_{2} i_{2}+v_{3} i_{3}+v_{4} i_{4}$. The result that you get (assuming that you have done the work correctly) is not an accident but the result of a network theorem called Tellegen's Theorem.

