MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Problem Set 10

Issued: November 6, 2002 **Due: November 13, 2002**

Reading Assignment:

- A&L Chapter 14 and Appendices B and C for Thursday, November 7.
- A&L Chapter 15 for Tuesday, November 12.

Problem 10.1:

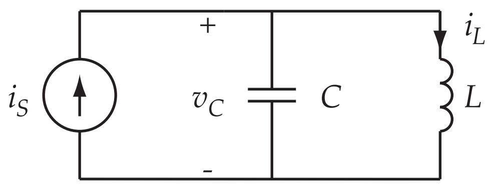

Figure 1: Circuit for Problem 10.1

(A) In the network shown in Figure 1, $i_S(t) = I_S u(t)$ and $v_C(0^-) = i_L(0^-) = 0$. Find $i_L(t)$.

(B) Repeat part (A) for $i_S(t) = 0$, $v_C(0^-) = 0$ and $i_L(0^-) = I_L$.

(C) Repeat part (A) for $i_S(t) = I_S u(t)$, $v_C(0^-) = 0$ and $i_L(0^-) = I_L$.

If you have done parts (A)-(C) correctly, you will have found that the solution to part (C) is the sum of the solutions to parts (A) and (B). This is not a coincidence, but instead a general property of the solutions to LCCODE's.

The response in a network driven by sources but with zero initial conditions is known as a *zero-state response*. The response of the same network with the sources set to zero but with non-zero initial conditions is known as the *zero-input response*. This problem illustrates the general principle that the response of a network driven by sources with non-zero initial conditions is the superposition of the zero-state response and the zero-input response.

Problem 10.2:

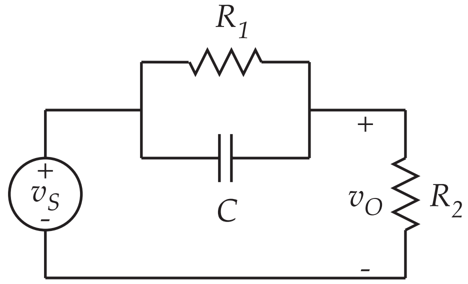

Figure 2: Circuit for Problem 10.2

(A) Find a differential equation relating $v_O(t)$ to $v_S(t)$ in the circuit of Figure 2.

(B) Determine a forced solution to the complex voltage source $v_{Sf}(t) = V_s e^{j\omega t}$. Denote your answer by $v_{Of}(t)$.

(C) In part (B), you have determined a forced response to the complex source $v_{Sf}(t)$. Symbolically, $v_{Sf}(t) \rightarrow v_{Of}(t)$. For LCCODE's with real coefficients, $Re\{v_{Sf}(t)\} \rightarrow Re\{v_{Of}(t)\}$, i.e., a forced response to $Re\{v_{Sf}(t)\}$ is $Re\{v_{Of}(t)\}$. Use this result to find the forced response to $Re\{v_{Sf}(t)\} = V_s \cos \omega t$. Put your results in the form $v_O(t) = V_o \cos(\omega t + \theta)$.

Problem 10.3:

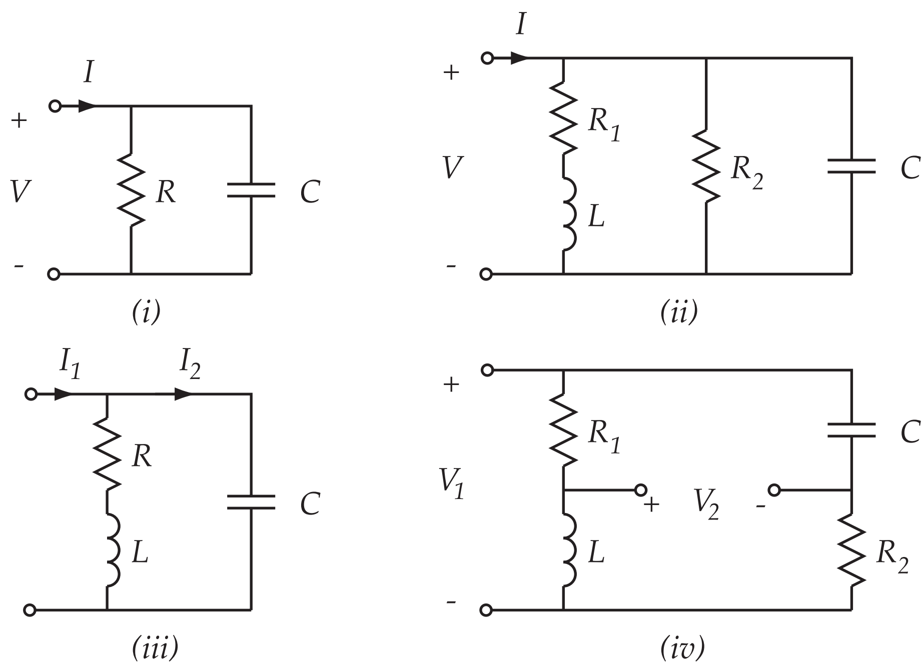

Figure 3: Circuits for Problem 10.3

(A) (i) Find the complex impedance $Z = V/I$ in Figure 3(i).

(ii) Find the complex admittance $Y = I/V$ in Figure 3(ii).

(iii) Find the complex transfer function $H(j\omega) = I_2/I_1$ in Figure 3(iii).

(iv) Find the complex transfer function $H(j\omega) = V_2/V_1$ in Figure 3(iv).

(B) In the circuit of Figure 3(iii), the current $i_1(t) = I \cos \omega t$. Find the steady-state current $i_2(t)$.

(C) In the circuit of Figure 3(iv), the voltage $v_1(t) = V \sin \omega t$. Determine the steady-state voltage $v_2(t)$. Evaluate your result for the *special* case $L/R_1 = R_2 C$

Problem 10.4:

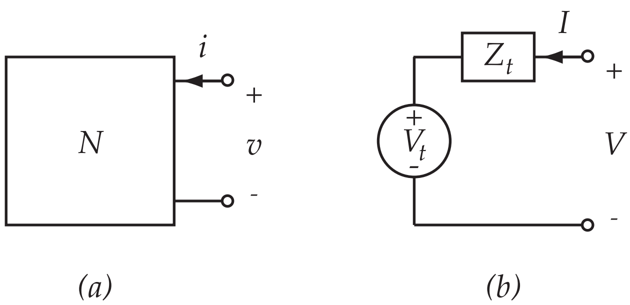

Figure 4: Networks for Problem 10.4

The network N shown schematically in Figure 4(a) is composed of resistors, inductors, capacitors, and sinusoidal sources. With the terminal pair open-circuited, the voltage $v = 10V \sin(10^4 t)$ is measured at the indicated terminal pair. When the terminal pair is short-circuited, the current $i = 5mA \sin(10^4 t - \pi/6)$ is measured. (For both open and short-circuit measurements, the same reference signal at frequency $\omega = 10^4$ was used to define the phase.)

(A) The circuit of Figure 4(b) is proposed as a Thévenin equivalent to represent the circuit behavior at the indicated terminal pair. Note that this representation is valid only for the complex amplitudes of v and i at the frequency $\omega = 10^4$. Determine V_t and Z_t.

(B) A $0.05\mu F$ capacitor is placed across the terminals of the network in Figure 4(a). Find $i(t)$.

Problem 10.5: In communication and RF systems, a technique called *matching* is used in order to extract the maximum power from a source with a given source resistance (or impedance). In this problem, we'll explore the motivation for matching a load to a source and then illustrate a method by which this can be done, at least for a narrow range of frequencies.

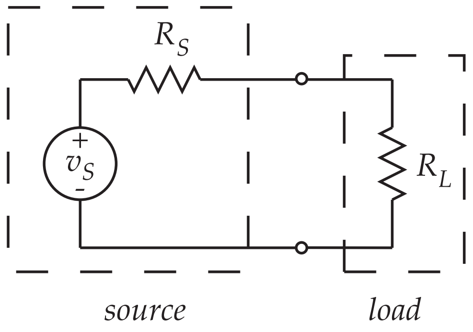

Figure 5: Circuit for Problem 10.5(A) and (B)

(A) Figure 5 shows a source with a given source resistance connected to a load. Calculate the power delivered to the load R_L and show that this is maximized when R_L is chosen to be equal to R_S. What is the maximum power that is then available to be delivered to the load?

(B) Show that the time-averaged power dissipated in a resistor R in the sinusoidal steady-state is $\frac{1}{2}|I|^2 R$ where I is the complex amplitude of the current in the resistor.

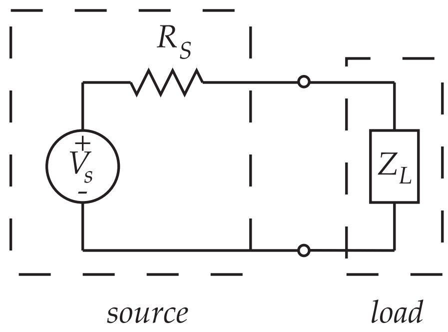

Figure 6: Circuit for Problem 10.5(C)

(C) Figure 6 shows a source with a source resistance R_S connected to a load $Z_L = R_L + jX_L$. The circuit is operating in the sinusoidal steady-state and is therefore drawn in a form appropriate for determining the complex amplitudes of the current and voltage. Show that the maximum power delivered to the load occurs for $R_L = R_S$ and $X_L = 0$.

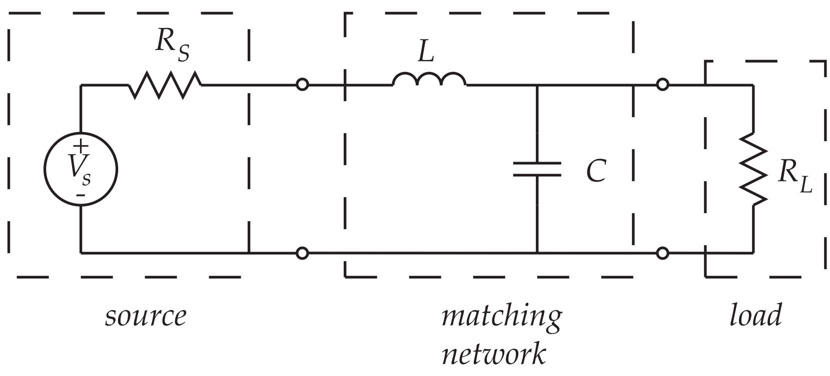

Figure 7: Circuit for Problem 10.5(D)

(D) Figure 7 shows a load R_L connected to a source with a source resistance R_S. It is proposed to insert the L-C section between load and source in order to match the load to the source, i.e., to cause the impedance seen by the source to be R_S.

(i) Calculate the impedance formed by the parallel combination of the resistor and capacitor. Put your result in the form $Z = R + jX$.

(ii) Assume that $R_L > R_S$. Calculate a value for the capacitor C (actually for ωC) for which the impedance seen by the source will have a real part equal to R_S.

(iii) Calculate the value of the inductance L for which the impedance seen by the source will be $R_S + j0$.

From this problem, you have found that a load R_L that is greater than a source resistance R_S can always be transformed to the source resistance R_S by inserting an L-C network between source and load. What change in the L-C network should be made to match a load whose resistance is less than a source resistance?