Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
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Homework #1 Solutions

Exercise 1.1:

Answer: A complex number of the form $a + bj$ can be rewritten as $\sqrt{a^2+b^2}e^{j\tan^{-1}\frac{b}{a}}$. Using this fact, we can rewrite $x(t)$ as

$$\frac{\sqrt{a^2+b^2}e^{j\tan^{-1}\frac{b}{a}}}{\sqrt{c^2+d^2}e^{j\tan^{-1}\frac{d}{c}}}e^{j\omega t}$$

Combining the exponentials together gives

$$\frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}e^{j\left(\omega t+\tan^{-1}\frac{b}{a}-\tan^{-1}\frac{d}{c}\right)}$$

Using Euler's formula we can write out the real part of the above expression such that

$$\Re\{x(t)\} = M\cos(\omega t + \phi)$$

where

$$M = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \quad \text{and} \quad \phi = \tan^{-1}\frac{b}{a} - \tan^{-1}\frac{d}{c}$$

Exercise 1.2:

Answer: The solution to a first-order linear differential equation is the sum of a particular solution, and a homogeneous solution of the form Ae^{st}. The forcing term b is a constant, so the particular solution that solves this differential equation is a constant, M. Substituting this into the differential equation yields

$$0 + aM = b$$

so $M = \frac{b}{a}$.

The exponent-part of the homogeneous solution is determined by the differential equation. Substituting our general solution into the equations gives

$$Ase^{st} + aAe^{st} = 0$$

Canceling the Ae^{st} terms gives $s = -a$. The scalar A is determined by the initial conditions. Setting our solution $\frac{b}{a} + Ae^{-at}\Big|_{t=0} = c$ and solving for A gives the final solution

$$x(t) = \frac{b}{a} + \left(c - \frac{b}{a}\right)e^{-at}$$

Problem 1.1:

Answer:

(A) There are three nodes in this circuit, which means we will have two independent KCL equations. The three dependent equations are are:

$$\begin{aligned} i_1 - i_2 - i_3 &= 0 \\ i_3 - i_1 - i_4 &= 0 \\ i_2 + i_4 &= 0 \end{aligned}$$

(B) There are three loops in the circuit, which means we will have two independent KVL equations. The three dependent equations are are:

$$\begin{aligned} v_1 + v_3 &= 0 \\ v_4 - v_2 - v_1 &= 0 \\ v_3 + v_4 - v_2 &= 0 \end{aligned}$$

(C) The $v - i$ constraints for each element are as follows

$$\begin{aligned} i_3 &= I_S \\ v_4 &= V_S \\ v_1 &= R_1 i_1 \\ v_2 &= R_2 i_2 \end{aligned}$$

(D) Solving the system of linear equations yields:

v_1	=	5 V	i_1	=	2.5 A	$v_1 i_1$	=	12.5 W
v_2	=	-2 V	i_2	=	-.5 A	$v_2 i_2$	=	1 W
v_3	=	-5 V	i_3	=	3 A	$v_3 i_3$	=	-15 W
v_4	=	3 V	i_4	=	.5 A	$v_4 i_4$	=	1.5 W

(E) The sum is zero. I_S supplies power, while V_S is absorbing power.

Problem 1.2:

Answer:

(A) Again, we have three nodes, and two independent equations.

$$\begin{aligned} i_1 - i_2 - i_3 &= 0 \\ i_3 - i_1 - i_4 &= 0 \\ i_2 + i_4 &= 0 \end{aligned}$$

(B) Again, we have three loops, and two independent equations.

$$\begin{aligned} v_1 + v_3 &= 0 \\ v_4 - v_2 - v_1 &= 0 \\ v_3 + v_4 - v_2 &= 0 \end{aligned}$$

(C) The $v - i$ constraints are

$$\begin{aligned} v_1 &= R_1 i_1 \\ v_2 &= V_\text{S} \\ v_3 &= R_3 i_3 \\ v_4 &= R_4 i_4 \end{aligned}$$

(D) Solving the system of equations yields

$$\begin{array}{lllllllll} v_1 & = & \text{-1 V} & i_1 & = & \text{-1 A} & v_1 i_1 & = & \text{1 W} \\ v_2 & = & \text{3 V} & i_2 & = & \text{-2 A} & v_2 i_2 & = & \text{-6 W} \\ v_3 & = & \text{1 V} & i_3 & = & \text{1 A} & v_3 i_3 & = & \text{1 W} \\ v_4 & = & \text{2 V} & i_4 & = & \text{2 A} & v_4 i_4 & = & \text{4 W} \end{array}$$

(E) Again, the individual powers sum to zero, and we find that V_S supplies power.

Problem 1.3:

Answer: Here's the table:

Voltage from Problem 1.1	Current from Problem 1.2	Resulting $v - i$ products
$v_1 = 5$ V	$i_1 = -1$ A	$v_1 i_1 = -5$ W
$v_2 = -2$ V	$i_2 = -2$ A	$v_2 i_2 = 4$ W
$v_3 = -5$ V	$i_3 = 1$ A	$v_3 i_3 = -5$ W
$v_4 = 3$ V	$i_4 = 2$ A	$v_4 i_4 = 6$ W

The resulting sum of the $v - i$ products is 0 W.