

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Homework #3 Solutions

Problem 3.1 Answer:

When the two networks are connected together, $v_1 = v_2$. We also know that $i_1 + i_2 + \frac{v_{1,2}}{1k\Omega} = 0$ from KCL written at the top node.

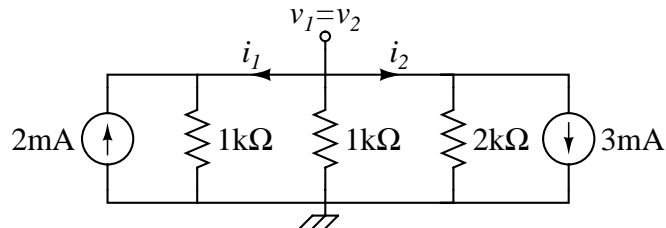
We know that for each network N_i , $v_i = v_i(0) - i_i \cdot \frac{v_i(0)}{i_{\text{intercept}}}$ where $I_{\text{intercept}}$ is the current i_i when $v_i = 0$. Given these facts, we can write:

$$\begin{aligned} v_1 &= 2V + 1k\Omega \cdot i_1 \\ v_2 &= -6V + 2k\Omega \cdot i_2 \\ v_{1,2} &= -1k\Omega(i_1 + i_2) \end{aligned}$$

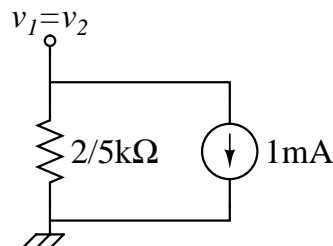
We can solve these equations for the unknowns and find:

$$v_1 = v_2 = -.4V \quad i_1 = -2.4mA \quad i_2 = 2.8mA$$

Another way to solve this problem is by drawing the Norton equivalent circuits. The Norton currents are just the i intercepts on the graphs for each network, and the resistance is just the slope of the v - i graph. The resulting circuit looks like



The resistors combine to form one $\frac{2}{5}k\Omega$ resistor. The current sources can be combined into one current source feeding 1mA into the bottom node. The resulting circuit is



It is clear that $v_1 = v_2 = -.4V$. From the first circuit, above, we know that $i_1 = \frac{v_1}{1k\Omega} + -2mA = -2.4mA$, and $i_2 = \frac{v_1}{2k\Omega} + 3mA = 2.8mA$.

Problem 3.2 Answer:

- (A) When just the 2 mA source is acting alone, it has two 2kΩ and one 1kΩ resistor connected across it. These combine to form a total resistance of $\frac{1}{2}k\Omega$, so

$$v_A = I_S \cdot .5k\Omega = 2mA \cdot .5k\Omega = 1V$$

When just the voltage source acts alone, the two 2kΩ resistors combine to form a 1kΩ resistor. The 4V source gets halved by the resulting voltage divider, which gives

$$v_B = -\frac{1}{2}V_S = -2V$$

Combining these gives the final answer of

$$v = v_A + v_B = -1V$$

- (B) The current through the resistor is $\frac{v}{2k\Omega}$, and the voltage across it is v , so the power dissipated by the resistor is $\frac{v^2}{2k\Omega} = \frac{1}{2}mW$
- (C) No, superposition does not apply to power. Write $v = v_A + v_B$. The power, as found in part (B) above, is $\frac{v^2}{R} = \frac{(v_A+v_B)^2}{R} = \frac{v_A^2+2v_Av_B+v_B^2}{R}$. This is not equal to the sum of the powers from the two sources acting alone, which would be $p = p_A + p_B = \frac{v_A^2}{R} + \frac{v_B^2}{R} = \frac{v_A^2+v_B^2}{R}$.

Problem 3.3 Answer:

- (A) We know that I_S has to be equal to the sum of the current i plus the current through the 1kΩ resistor (write KCL at the top node). The current through the resistor is equal to $\frac{v_O}{1k\Omega}$. We can write

$$I_S = i + \frac{v_O}{1k\Omega}$$

Substituting in the device law for the nonlinear device, and the value of I_S gives us a cubic equation which we can solve for v_O .

$$5mA = v_O^3mA + \frac{v_O}{1k\Omega}A$$

Multiply this equation by 10^3 (to change all the measurements to mA units) to get the polynomial

$$5 = v_O^3 + v_o$$

- (B) This polynomial can be solved in a number of ways. In matlab, type `roots([1 0 1 -5])` and it returns two imaginary and one real root for v_O . The real root is clearly the correct answer, which is

$$v_O \approx 1.5160 \quad i \approx 3.4840$$

- (C) We can write out the Taylor series for the equation in Part (A) above as

$$I_S + i_s = (V_O)^3 + 3V_O^2 \cdot v_o + (V_O + v_o)$$

Note that again that all of these units are in milliamps. Subtract the equation from Part (A) evaluated at the operating point from this to arrive at

$$i_s = 3V_O^2 \cdot v_o + v_o$$

or

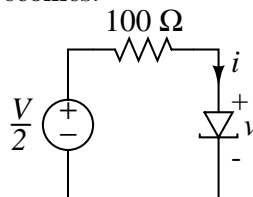
$$i_s = 6.8946 \cdot v_o + v_o = 7.8936v_o$$

We can solve this for $\frac{v_o}{i_s}$ to find

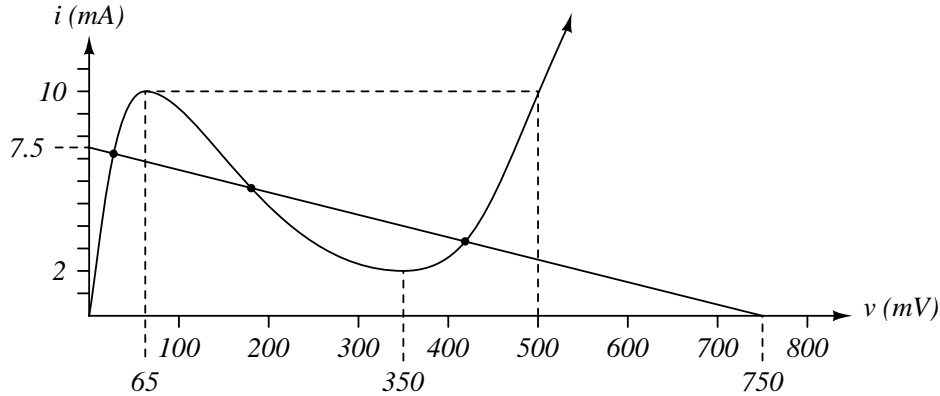
$$\frac{v_o}{i_s} = \frac{1}{7.8946} \frac{\text{V}}{\text{mA}} = 126.7\Omega$$

Problem 3.4 Answer:

- (A) The first step is to simplify the given circuit down to a Thévenin equivalent circuit connected to the tunnel diode. The circuit becomes:



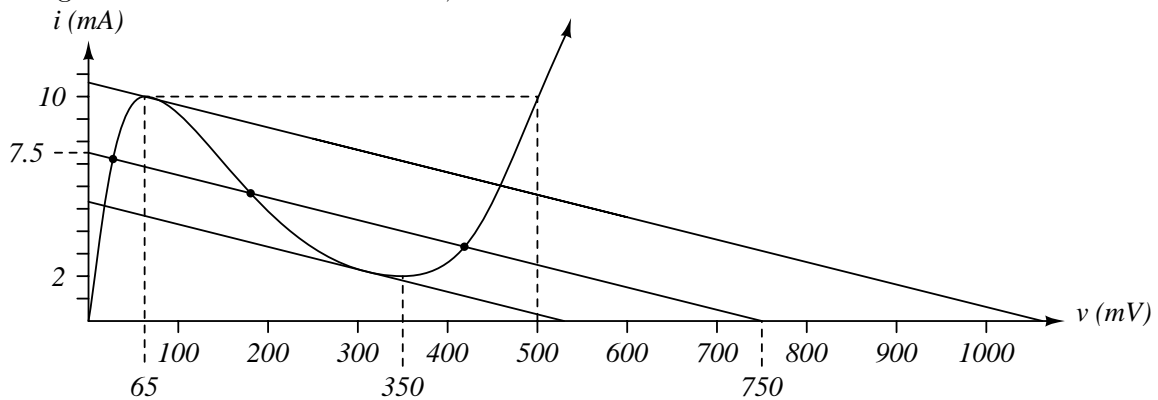
We can then draw the load line $i = 7.5\text{mA} - \frac{v}{100}$ on the tunnel diode's transfer curve to find the operating points, as shown below.



There are three different equilibrium values in the graph above. They are approximately:

$v(\text{mV})$	$i(\text{mA})$
50	7.25
175	5.75
420	3.4

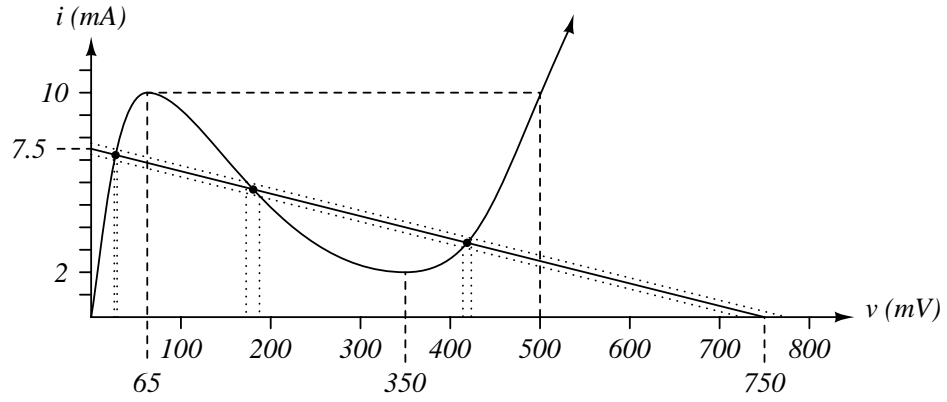
- (B) As V increases or decreases, the Thévenin equivalent circuit's load line moves up and down on the graph. The last multiple solutions (multiple intersections) occur where the load line is tangent to the tunnel diode curve, as shown below.



The values of V that correspond to these lines are $V \approx 1.1\text{V} \rightarrow 2.2\text{V}$.

- (C) For a small change in v_S , given as v_s , we would like to know the corresponding change in v_D , v_d . We can evaluate the small-signal gain of the circuit, then, by dividing v_d by v_s .

If we are to do this graphically, let v_s move 50mV in either direction from the operating point (total 100mV swing). Draw the new load lines on the graph, and find about how much v_D changes, which is v_d . This is illustrated in the figure below. Note that the 100mV swing in v_S corresponds to a total 50mV swing in the x-intercept of the load line.



Now, we can estimate $\frac{v_d}{v_s}$ by finding v_d from the graph above, and dividing it by the 100mV swing we've imposed on v_s . These results are summarized in the table below:

$\Delta v_D = v_d$	$\frac{v_d}{v_s}$
3.1mV	.031
-14.9mV	-.149
9.4mV	.094

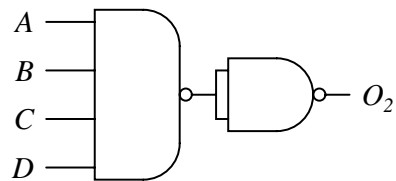
Problem 3.5 Answer:

(A) The truth table is filled in below.

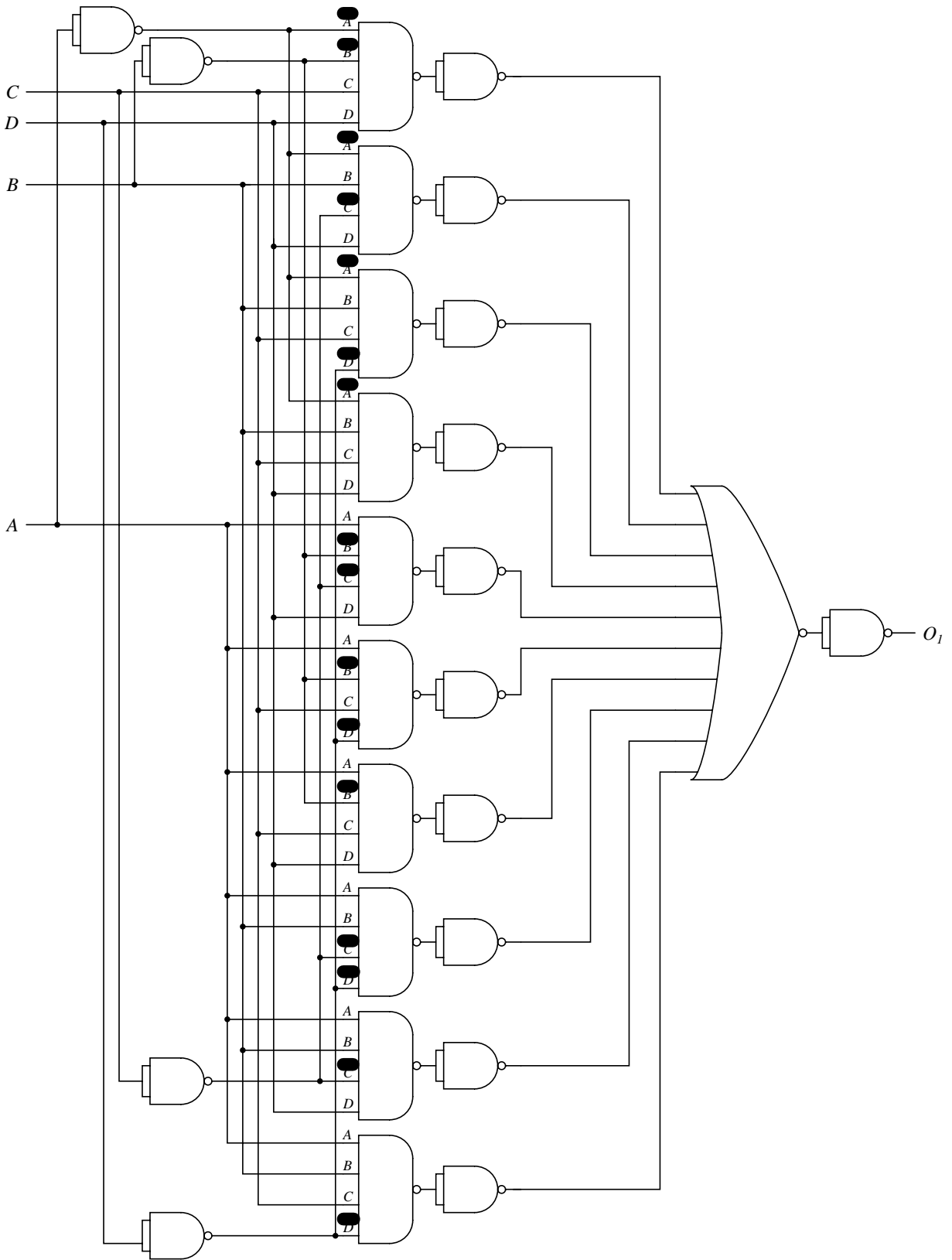
A	B	C	D	O_2	O_1	O_0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	0

(B) There are many answers to this problem. The easiest approach has been taken here. NAND gates are used as inverters, AND and OR gates are made by inverting the outputs of NAND and NOR gates, respectively. Each unique product (AND) of terms that produces a one at the output is ORed together to form each output bit.

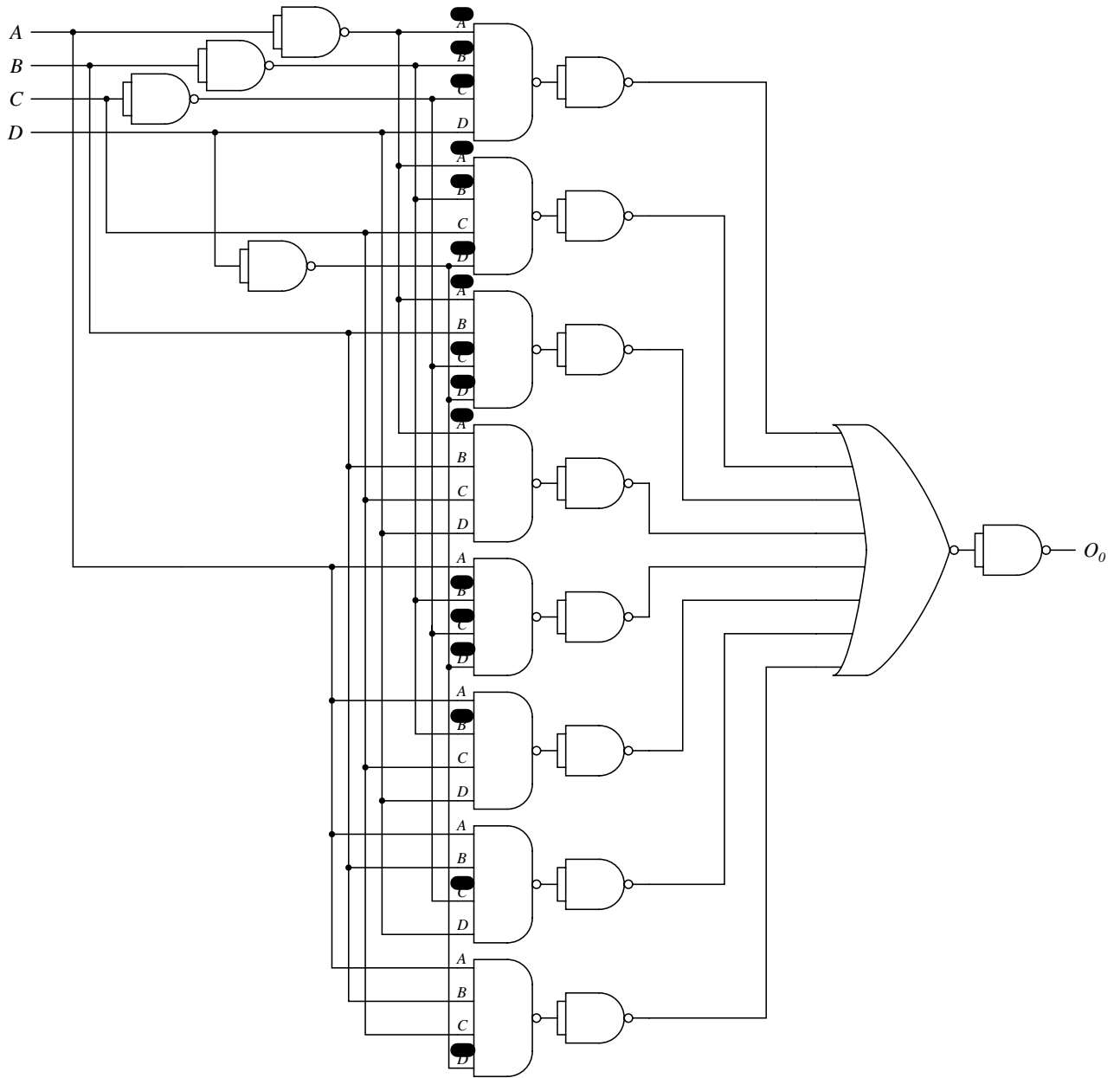
For bit O_2 :



For bit O_1 :



For bit O_0 :



Problem 3.6 Answer:

- (A) Ignoring noise implies that $v_{OL} = v_{IL}$ and $v_{OH} = v_{IH}$. The largest value that we can assign v_{OL} without having the transfer curve enter the forbidden region is .65 V. Likewise, the lowest value we can assign v_{OH} without entering the forbidden region is the same, .65 V. Remember that V_{OL} and V_{IL} must allow a corresponding choice for V_{OH} and V_{IH} that can still satisfy the static discipline ($V_{OL} < V_{OH}$).
- (B) Notice that the noise margin for low voltages $NM_L = V_{IL} - V_{OL}$. Notice that for a given choice of V_{OL} that the “best” value for V_{IL} is the one that sits at the intersection of the buffer’s transfer curve and the V_{OL} line. That is to say that $V_{IL} = f(V_{OL})$. A maximum in the NM_L occurs when $\frac{d}{dV_{OL}}NM_L = 0$. Evaluating this derivative gives

$$\frac{d}{dV_{OL}}NM_L = \frac{d}{dV_{OL}}V_{IL} - 1$$

This expression is zero when $\frac{d}{dV_{OL}}V_{IL} = 1$. A similar argument applies for NM_H .

So, to maximize the noise margins, we find the points on the transfer curve where $\frac{d}{dv_{OUT}}v_{IN} = 1$. If we move any of v_{OL} , v_{IL} , v_{OH} or v_{IH} away from these points, the noise margin will decrease. The two places where $\frac{d}{dv_{OUT}}v_{IN} = 1$ occur at $v_{IN} = .5V$ and $v_{IN} = .7V$.

- (i) The highest value that could be chosen for v_{OL} is .2 V.
- (ii) The highest value that could be chosen for v_{IL} is .5 V.
- (iii) The maximum noise margin for low inputs is $.5 - .2 = .3$ V.
- (iv) The lowest value that could be chosen for v_{OH} is .8 V.
- (v) The lowest value that could be chosen for v_{IH} is .7 V.
- (vi) The maximum noise margin for low inputs is $.8 - .7 = .1$ V.