MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Problem Set 7

Issued: October 16, 2002 **Due: October 23, 2002**

Reading Assignment:

- A&L Section 10.6 for Thursday, October 17.
- A&L Chapter 12 for Tuesday, October 22.

Problem 7.1: Consider again the simple MOSFET buffer analyzed in Problem 6.5, whose circuit diagram is reproduced in Figure 1. We have seen in Problem 6.5, and also observed in the lecture demonstration, that the gate-to-source capacitance of $M2$, C_{GS}, causes a delay in the propagation of pulses through the buffer. This delay is characterized by the time constants associated with the circuit charging and discharging C_{GS}.

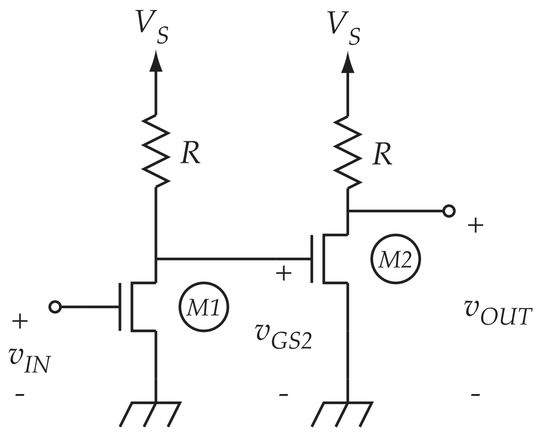

Figure 1: MOSFET buffer circuit for Problem 7.1

Closer examination of the lecture demo reveals the presence of an additional time constant (perhaps some of you noticed the short "shelf" in the gate-to-source voltage of $M2$ in the lecture demo). This time constant is due to the capacitance between the gate and drain of $M2$ and affects the output voltage as $M2$ passes through its saturation region. Somewhat surprising, the effect of C_{GD} is significantly enhanced by the voltage gain of $M2$. In this problem, we will examine this so-called *Miller effect*.

Consider the small-signal equivalent circuit in Figure 2. Here v_t and R_t represent the Thévenin equivalent circuit of $M1$ as seen looking back from the gate of $M2$ and C_{GD} and $g_m = k(v_{GS} - V_T)$

are the gate-to-drain capacitance and incremental current gain of $M2$, respectively. Notice that although the switching problem should be analyzed using a large-signal, nonlinear model, we are here illustrating the Miller effect by restricting ourselves to a small-signal linear analysis. Note too that C_{GS} associated with $M2$ is omitted for simplicity.

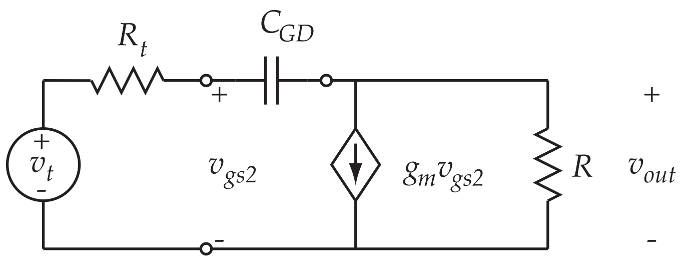

Figure 2: Small-signal equivalent circuit for Problem 7.1

(A) Determine the Thévenin equivalent resistance "seen" by the capacitor C_{GD}.

(B) What is the time constant associated with charging this capacitor? Notice that typically $g_m R >> 1$ so that the time constant is much larger than may have been expected.

(C) Determine and sketch $v_{out}(t)$ when $v_t(t) = V_t u(t)$. Assume that the capacitor C_{GD} is uncharged for $t < 0$.

Problem 7.2:

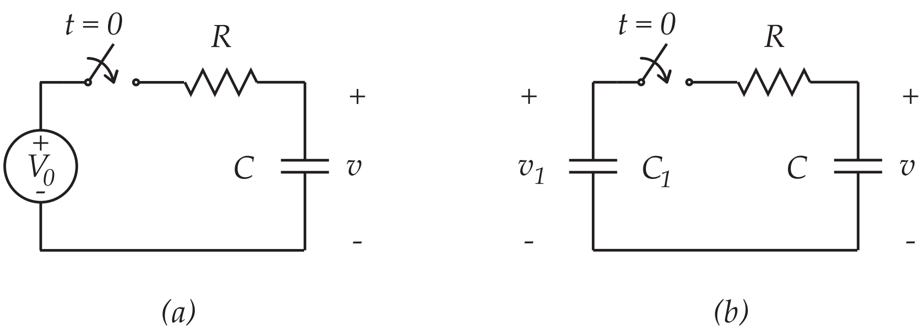

Figure 3: Circuits for Problem 7.2(A)-(D)

(A) In the circuit of Figure 3(a), an initially uncharged capacitor is charged by a constant voltage source V_0 that is switched into the circuit at $t = 0$. In order to limit the peak value of current, the resistor R is inserted between the source and the capacitor. Determine the voltage $v(t)$.

(B) Determine the final energy stored in the capacitor and compare with the energy supplied by the voltage source during the charging process. If there is a difference, where did the missing energy go?

(C) In an effort to find a more efficient charging system, the circuit of Figure 3(b) is proposed. Assume that capacitor C_1 is initially charged to voltage V_1. Sketch the waveform of v for $t > 0$, indicating the charging time constant. What should be the relation of V_1 to the source value V_0 in part (A) in order for the same energy to end up in the capacitor C?

(D) Is the charging process in part (C) more or less efficient than that in part (A)? I.e., which circuit dissipates more energy, assuming that the final energy stored in capacitor C is the same?

(E) Consider now the inductor circuit shown in Figure 4.

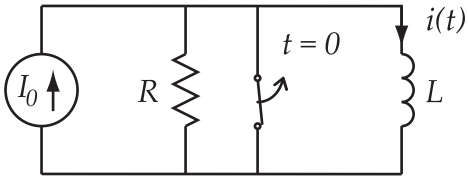

Figure 4: Circuit for Problem 7.2(E)

The inductor is energized by the constant current source that is switched into the circuit at time $t = 0$. Determine the inductor current $i(t)$ assuming that $i(t) = 0$ for $t < 0$. Also, determine the energy stored in the inductor and the total energy which has been supplied by the source and which has been dissipated in the resistor after the circuit has reached steady-state conditions.

Problem 7.3: Figure 5(a) shows a simple RC circuit driven by a voltage source $v_s(t)$. The purpose of this problem is to illustrate several methods of finding $v(t)$ when the source is an impulse, $v_s(t) = \delta(t)$. For simplicity, we will assume that the initial state of the network is zero, i.e., $v(t) = 0$ for $t < 0$.

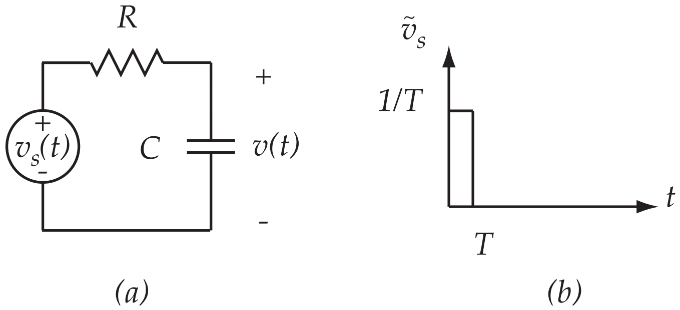

Figure 5: Networks and input for Problem 7.3

The differential equation relating $v(t)$ to $v_s(t)$ is easily found to be:

$$RC\frac{dv}{dt} + v = v_s(t) \qquad (1)$$

(A) *Approximating $\delta(t)$ by a short pulse.* The first method is based on approximating $\delta(t)$ by the short, intense pulse shown in Figure 5(b). Notice that the area of the pulse is $1V{\cdot}s$, independent of T, and is in fact equal to the area of the (unit) impulse.

(i) Find $v(t)$ for $0 < t < T$ due to the waveform shown in Figure 5(b). Do this by constructing the general solution of (1) with $v_s = 1/T$ and determining the constant that multiplies the solution to the homogeneous equation by applying the appropriate initial condition.

(ii) Find $v(t)$ for $t > T$ by matching the general solution valid for this interval at time T^+ with the solution found in (i) at $t = T^-$.

(iii) Now let $T \to 0$ in the solution found in (ii) to determine the impulse response.

The next two methods follow by observing that the impulse response is a solution of the homogeneous equation since $\delta(t) = 0$ for $t > 0$. The problem then reduces simply to finding the appropriate initial condition.

(B) *Getting $v(0^+)$ from the differential equation.* Determine the initial condition $v(0^+)$ by integrating (1) from $t = 0^-$ to $t = 0^+$, noting that $\int_{0^-}^{0^+} v(t')dt' = 0$. Use $v(0^+)$ to determine the arbitrary constant in the solution to the homogeneous equation.

(C) *Getting $v(0^+)$ from the circuit.* Since the capacitor voltage remains finite at $t = 0$, the voltage $v_s = \delta(t)$ must appear across the resistor. Thus,

$$i_c = i_R = \frac{\delta(t)}{R} \qquad 0^- < t < 0^+$$

(It may be helpful here to think of $\delta(t)$ as the short, intense pulse in Figure 5(b). Integrate this equation from $t = 0^-$ to $t = 0^+$ and determine $v(0^+)$. Compare with the result obtained in (B).

(D) *Getting $v(t)$ from the step response.* A reliable way of getting an impulse response is to first find the step response and then differentiate with respect to time. This works because of the following property of solutions to (1) (*with zero state initial conditions!*):

$$\text{If} \quad \mathrm{v_s(t) \rightarrow v(t)} \quad \text{then} \quad \frac{\mathrm{dv_s}}{\mathrm{dt}} \rightarrow \frac{\mathrm{dv}}{\mathrm{dt}}$$

Find the response $v(t)$ due to $v_s(t) = u(t)$ and differentiate it to get the response to $v_s(t) = \delta(t)$. Compare your result with results of parts (A)-(C).

Problem 7.4: The waveform of the current source in the circuit of Figure 6(a) consists of the train of impulses shown in Figure 6(b). Each impulse has area Q (in Coulombs) and the period between impulses is T.

Assume that the circuit operates in the steady-state and that the voltage v(t) is periodic with period T.

(A) Let $v(t) = V_0$, an unknown constant, at $t = 0^-$. Determine $v(t)$ for $0 < t < T$ in terms of V_0. Hint: make use of your results from Problem 7.3.

(B) By equating $v(T^-) = V_0$, determine V_0. Determine and sketch $v(t)$ over one full period for the two cases $RC << T$ and $RC >> T$.

(C) In part (B), you found the voltage for one period of a periodic waveform. Let that periodic waveform be denoted by $v^*(t)$. Suppose now that the impulse train is applied beginning at

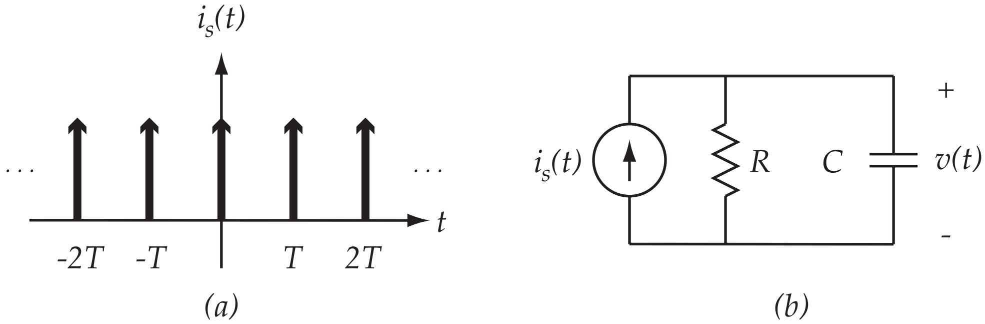

Figure 6: Circuit and input for Problem 7.4

$t = 0$ with $v(0^-) = 0$. The solution for the semi-infinite impulse train for $t > 0^-$ can be written in the form

$$v(t) = v^*(t) + v_1(t) \qquad t > 0^-$$

Determine $v_1(t)$.