MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Quiz 1 Solutions

Name: ______________________________ **Recitation Section:** ______________

Recitation Instructor: ______________ **Teaching Assistant:** ______________

Enter all your work and your answers directly in the spaces provided on the printed pages. **Make sure that your name is on all sheets**. Use the backs of the printed pages as scratch paper, but we will only grade the work that you neatly transfer to the spaces on the printed pages. Answers must be derived or explained, not just simply written down. The quiz is closed book, but **calculators are allowed**.

This quiz contains 9 pages including the cover sheet. Make sure that your quiz contains all 9 pages and that you hand in all 9 pages.

Problem	Points	Grade	Grader
1	30		
2	40		
3	30		
Total	100		

Problem 1: (*30 points*) For parts (A)-(C), use the associated branch variables as defined in Figure 1.

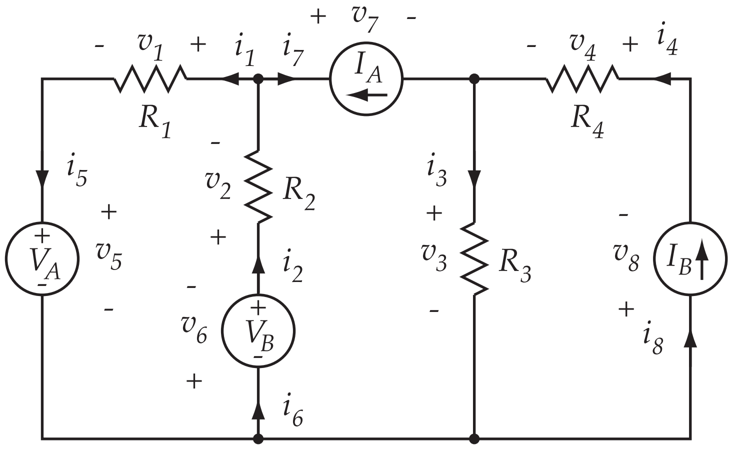


Figure 1: Circuit for Problem 1(A), 1(B) and 1(C)

(A) Write element laws for the resistor R_1, the current source I_A, and the voltage source V_B.

$$\begin{aligned} v_1 &= R_1 i_1 \\ i_7 &= -I_A \\ v_6 &= -V_B \end{aligned}$$

(B) Write a complete set of independent KVL equations expressed only in terms of the branch voltages v_1, v_2, ..., and v_8.

There are several possible answers, one set of independent KVL equations is:

$$\begin{aligned} v_1 + v_2 + v_6 + v_5 &= 0 \\ v_3 + v_7 + v_2 + v_6 &= 0 \\ v_3 + v_4 + v_8 &= 0 \end{aligned}$$

(C) Write a complete set of independent KCL equations expressed only in terms of the branch currents i_1, i_2, ..., and i_8.

Any five of the following is a complete set of independent KCL equations:

$$\begin{aligned} i_1 - i_5 &= 0 \\ i_6 - i_2 &= 0 \\ i_2 - i_1 - i_7 &= 0 \\ i_4 + i_7 - i_3 &= 0 \\ i_8 - i_4 &= 0 \\ i_5 - i_6 + i_3 - i_8 &= 0 \end{aligned}$$

(D) The same circuit is shown in Figure 2, labeled for node analysis. Write out the node equations necessary to solve for the three unknown node voltages e_1, e_2 and e_3. **DO NOT SOLVE** these equations.

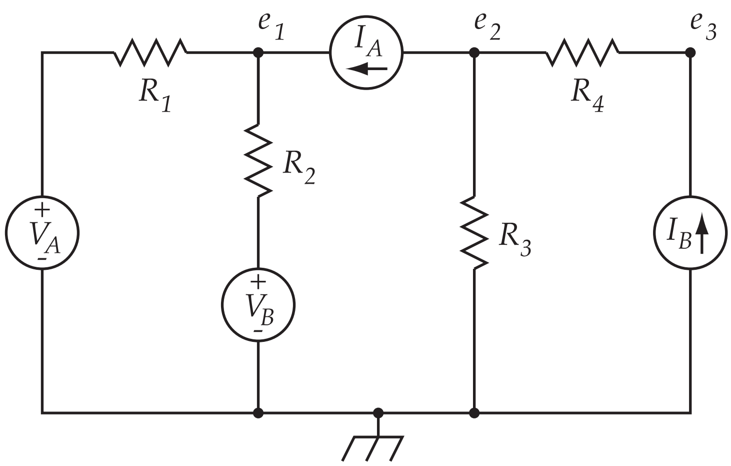


Figure 2: Circuit for Problem 1(D)

The node equations are:

$$\frac{e_1 - V_A}{R_1} + \frac{e_1 - V_B}{R_2} - I_A = 0$$
$$\frac{e_2}{R_3} + \frac{e_2 - e_3}{R_4} + I_A = 0$$
$$\frac{e_3 - e_2}{R_4} - I_B = 0$$

In matrix form, we have:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \begin{bmatrix} \frac{V_A}{R_1} + \frac{V_B}{R_2} + I_A \\ -I_A \\ I_B \end{bmatrix}$$

(E) The current i_1 in Figure 3 can be written in the form:

$$i_1 = aV_A + bV_B + cI_A + dI_B$$

Determine the coefficients a, b, c and d in terms of the resistor variables in the circuit.

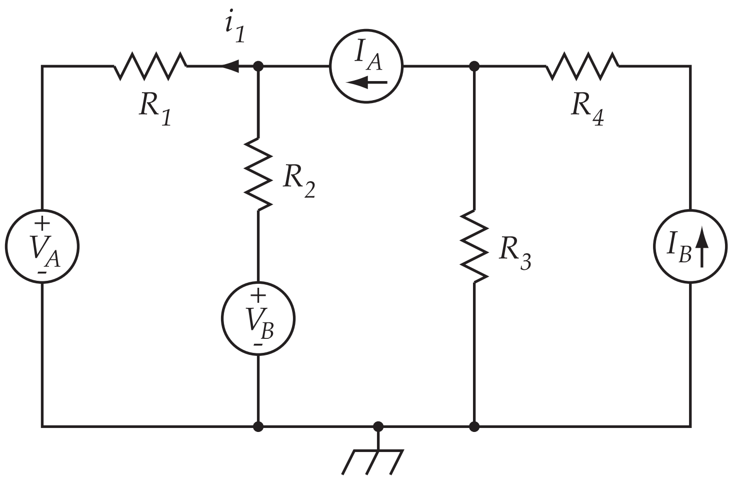
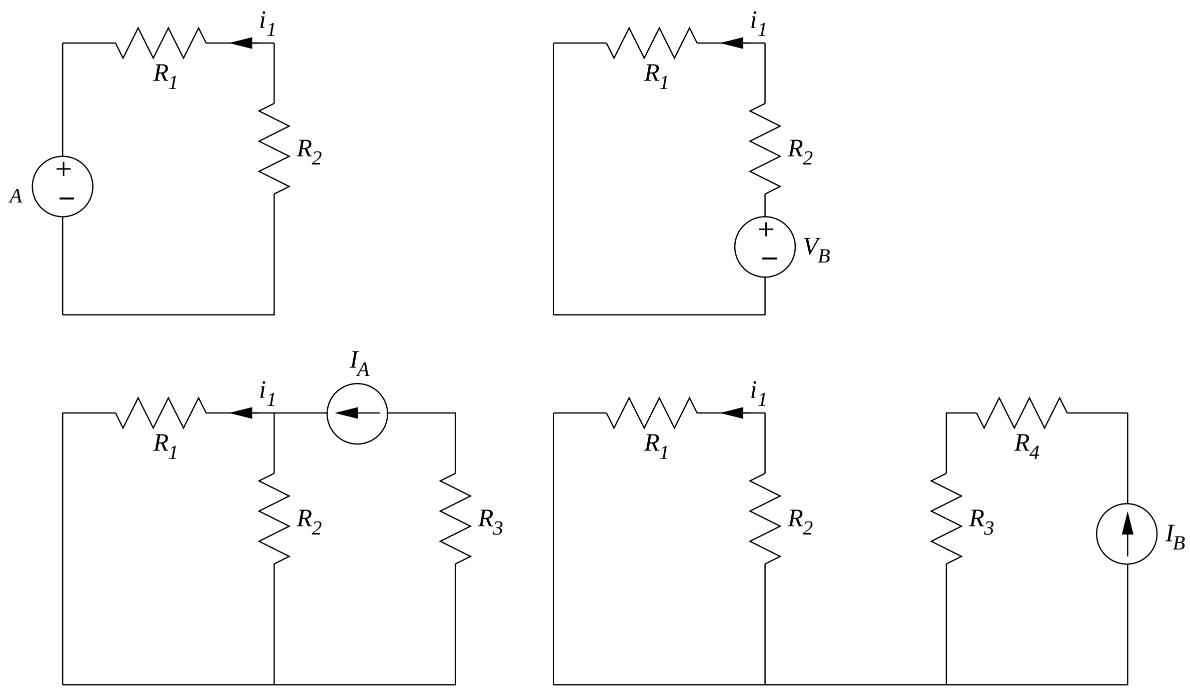


Figure 3: Circuit for Problem 1(E)

Here, superposition is used to determine each constant. The four figures below depict the circuit when only one source is active.



When V_A is the only source active

$$i_1 = -\frac{V_A}{R_1 + R_2}$$

When only V_B is active

$$i_1 = \frac{V_B}{R_1 + R_2}.$$

When only I_A is active

$$i_1 = \frac{R_2}{R_1 + R_2} I_A.$$

From the figures above, I_B does not contribute to i_1.

$a = \underline{-\frac{1}{R_1 + R_2}}$

$b = \underline{\frac{1}{R_1 + R_2}}$

$c = \underline{\frac{R_2}{R_1 + R_2}}$

$d = \underline{0}$

Problem 2: (*40 points*) Network 1, shown in Figure 4, is described by its v-i relationship measured at the terminals. Network 2, shown in Figure 5, is described by a schematic diagram of its components.

(A) Find the Thévenin and Norton equivalent circuits that have the same v-i relationship as Network 1.

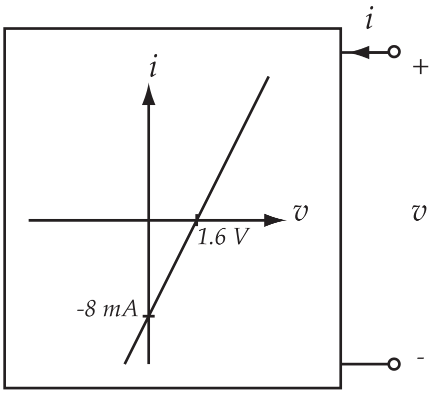
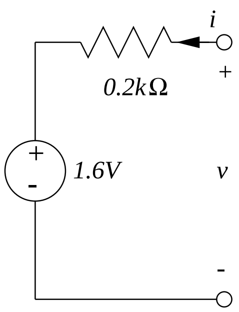
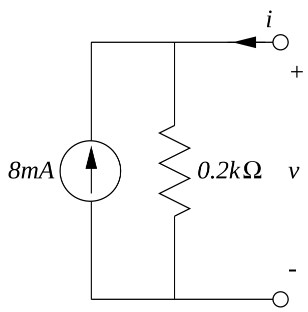


Figure 4: Network for Problem 2(A)

$$v_{OC} = 1.6V$$
$$i_{SC} = -8mA$$
$$R_{TH} = -\frac{v_{OC}}{i_{SC}} = 0.2k\Omega$$



Thevenin



Norton

(B) Find the Thévenin and Norton equivalent circuits that have the same v-i relationship as Network 2.

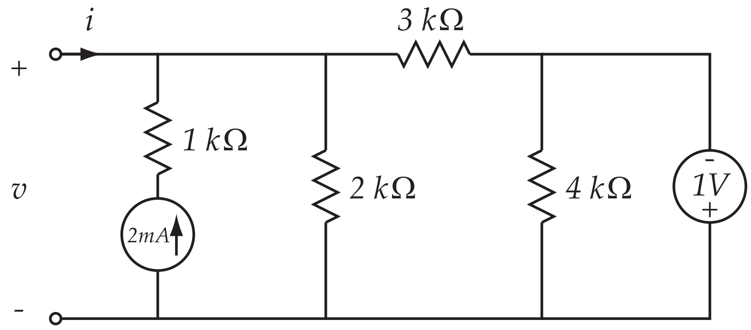


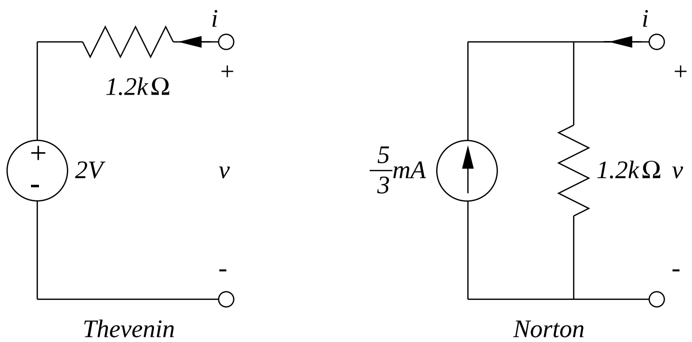
Figure 5: Network for Problem 2(B)

Suppressing all the sources and looking at the resistance into the port gives R_{TH}:

$$R_{TH} = 2k\Omega||3k\Omega = \frac{6}{5}k\Omega = 1.2k\Omega$$

Using superposition, v_{OC} and i_{SC} can be determined:

$$v_{OC} = -\frac{2}{5}V + \frac{2 \cdot 3}{2+3} \cdot 2V = 2V$$
$$i_{SC} = -2mA + \frac{1}{3}mA = -\frac{5}{3}mA$$



(C) Suppose that the two networks are connected together through a resistor as shown in Figure 6. Find the current i_1 and the voltage v_1.

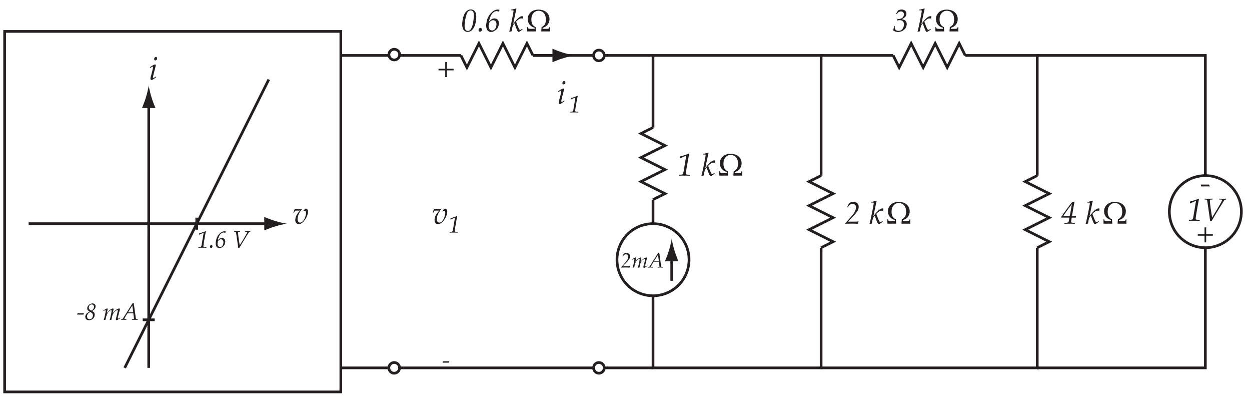
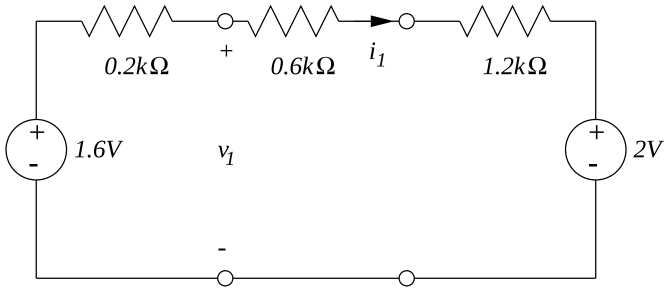


Figure 6: Network for Problem 2(C)

First, redraw the circuit using the Thévenin equivalents of each network:



$$i_1 = \frac{2V - 1.6V}{2k\Omega} = -0.2mA$$

Using superposition and voltage dividers:

$$v_1 = \frac{1.8}{2}1.6 + \frac{0.2}{2}2$$
$$= 1.44 + 0.2 = 1.64V$$

$i_1 =$ $\underline{\quad -0.2mA \quad}$

$v_1 =$ $\underline{\quad 1.64V \quad}$

(D) Suppose that the two networks are connected together through a resistor as shown in Figure 7. Find the current i_2 and the voltage v_2.

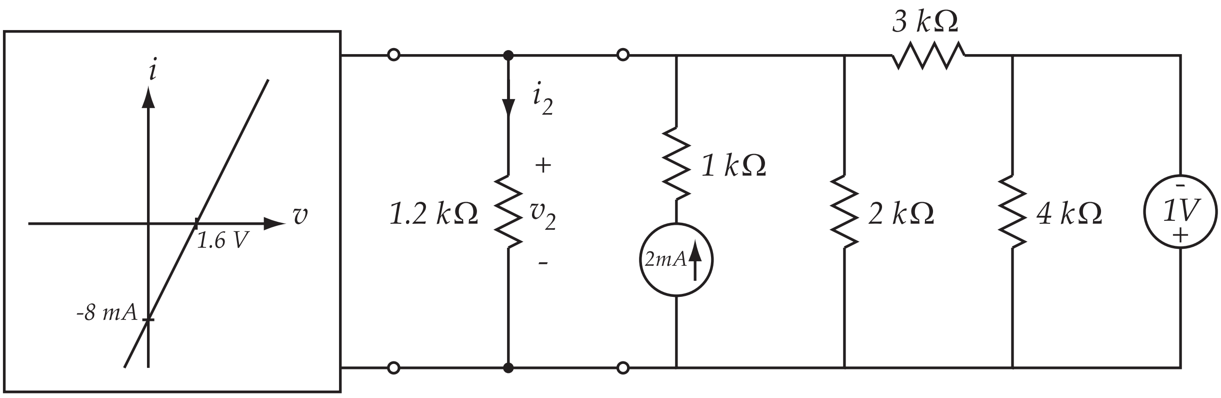
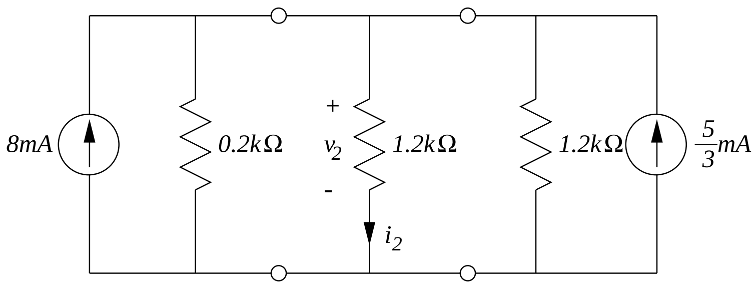


Figure 7: Network for Problem 2(D)

First, redraw the circuit using the Norton equivalents of each network:



$$
\begin{aligned}
v_2 &= \left(\frac{5}{3}mA + 8mA\right)(0.2k\Omega||1.2k\Omega||1.2k\Omega) \\
&= \left(\frac{29}{3}mA\right)(0.15k\Omega) = 1.45V \\
i_2 &= \frac{1.45V}{1.2k\Omega} = \frac{29}{24}mA \approx 1.208mA
\end{aligned}
$$

$i_2 =$ $\underline{\quad 1.208mA \quad}$

$v_2 =$ $\underline{\quad 1.45V \quad}$

Problem 3: (*30 points*) Determine all node potentials in the network shown in Figure 8 in terms of the conductances of the resistors (G_A, G_B, G_C, G_D, G_E, and G_F), the current sources (I_C and I_F), and the voltage sources (V_A, V_B, V_D and V_E).

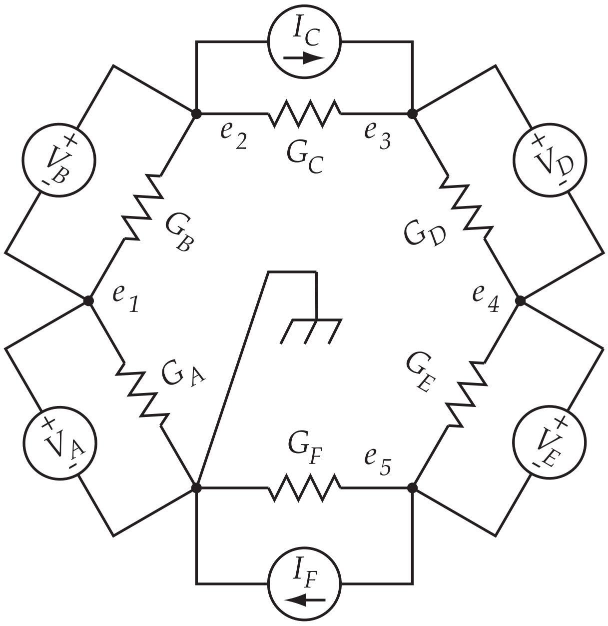


Figure 8: Circuit for Problem 3

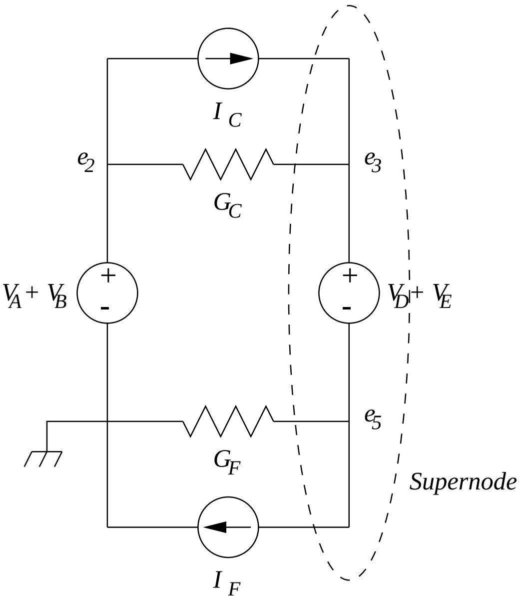
The potentials e_1 and e_2 can be determined immediately from the circuit.

$$
\begin{aligned}
e_1 &= V_A \\
e_2 &= V_A + V_B
\end{aligned}
$$

Noting that e_3, e_4, and e_5 are related in the following way,

$$
\begin{aligned}
e_5 &= e_3 - V_D - V_E \\
e_4 &= e_3 - V_D \\
e_5 &= e_4 - V_E,
\end{aligned}
$$

the circuit can be simplified as shown below.



Now, we write a node equation for the supernode encompassing nodes 3, 4, and 5:

$$(e_3 - V_A - V_B)G_C + e_5 G_F = I_C - I_F$$

Using $e_5 = e_3 - V_D - V_E$ in the equation above gives:

$$e_3 G_C + e_3 G_F = G_C(V_A + V_B) + G_F(V_D + V_E) + I_C - I_F$$
$$e_3 = \frac{G_C(V_A + V_B) + G_F(V_D + V_E) + I_C - I_F}{G_C + G_F}$$

Using the relationship $e_4 = e_3 - V_D$ gives an expression for e_4:

$$e_4 = \frac{G_C(V_A + V_B) + G_F(V_D + V_E) + I_C - I_F}{G_C + G_F} - V_D$$
$$e_4 = \frac{G_C(V_A + V_B - V_D) + G_F V_E + I_C - I_F}{G_C + G_F}$$

Using the relationship $e_5 = e_4 - V_E$ gives an expression for e_5:

$$e_5 = \frac{G_C(V_A + V_B - V_D) + G_F V_E + I_C - I_F}{G_C + G_F} - V_E$$
$$e_5 = \frac{G_C(V_A + V_B - V_D - V_E) + I_C - I_F}{G_C + G_F}$$