Exercise 6.1:

(a) Set the small-signal source to zero and replace the MOSFET with the SCS model:

\[ V_O = V_S - \text{(voltage drop across } R_L) \]
\[ V_O = V_S - \text{(current through } R_L) \cdot R_L \]
\[ V_O = V_S - \frac{K R_L}{2} \left( V_{GS} - V_T \right)^2 \]

(b) Draw the small-signal model:
Solve for \( \frac{v_o}{v_i} \):

\[
v_o = 0V - \text{(voltage drop across } R_L)\]

\[
v_o = (\text{current through } R_L) \cdot R_L
\]

\[
v_o = -R_L \cdot K(V_{GS} - V_T) v_i
\]

\[
\frac{v_o}{v_i} = -K(V_{GS} - V_T) \cdot R_L
\]

(c)

Exercise 6.2:

Replace the MOSFET with its small-signal equivalent:

\[
\begin{align*}
G & \quad D \\
S & \quad S \\
\rightarrow & \quad \downarrow \\
G & \quad K(V_{GS} - V_T) v_{gs}
\end{align*}
\]
\( v_{gs} = v_{ds} \), so current through the device is proportional to the voltage across the device, so it looks like a resistor:

\[
\begin{align*}
K(V_{DS} - V_T) \\
(\text{conductance})
\end{align*}
\]

Exercise 6.3:

(a) TYPO IN BOOK: “...operating-point voltage of \( v_I \)” should read “...operating-point voltage of \( V_I \)”

Replace the MOSFET with the SCS model:

\[
i_{DS} = \alpha(v_{GS} - V_T)^3 = \alpha(v_I - V_T)^3
\]

Solve for \( v_O \):

\[
v_O = V_S - (\text{voltage drop across resistor})
\]

\[
v_O = V_S - (\text{current through resistor}) \cdot R_L
\]

\[
v_O = V_S - \alpha(v_I - V_T)^3 R_L
\]

Operating point:

\[
\begin{align*}
V_O &= V_S - \alpha(V_I - V_T)^3 R_L \\
I_{DS} &= \alpha(V_I - V_T)^3
\end{align*}
\]

(b) \( v_o \) = \[\frac{dv_O}{dv_I}\] \( v_i \)

\[
v_o = -3\alpha(V_I - V_T)^2 R_L v_i,
\]

Gain = \( \frac{v_o}{v_i} = -3\alpha(V_I - V_T)^2 R_L \)
(c) Let $g_m$ be the small-signal transconductance of the MOSFET. The small-signal circuit is:

\[ \text{where } g_m v_i = \frac{d}{d \vgs} \left| \begin{array}{c} \vgs = \vi \\ i = 3 \alpha (V_I - V_T)^2 \end{array} \right. \]

(d) Small-signal gain = $\frac{v_o}{v_i}$

- $v_o = 0V -$ (voltage drop across $R_L$)
- $v_o = -(\text{current through } R_L) \cdot R_L$
- $v_o = -g_m v_i \cdot R_L$

\[ \frac{v_o}{v_i} = - g_m R_L, \text{ where } g_m = 3 \alpha (V_I - V_T)^2 \]

(e) $R_L$ is proportional to the small-signal gain, so doubling $R_L$ will double the gain.

Change in output bias voltage:

\[ \frac{V_S - \alpha (V_I - V_T)^2 (2R_L)}{V_S - \alpha (V_I - V_T)^3 R_L} \]

\text{new output bias} \quad \text{old output bias}

\[ \text{Change in output bias} = - \alpha (V_I - V_T)^3 R_L \]

(f) Since gain is proportional to $(V_I - V_T)^2$, increasing $(V_I - V_T)$ by a factor of $\sqrt{2}$ will double the gain. The required change in $V_I$ depends on its relationship with $V_T$:

\[ \frac{N V_I - V_T}{V_I - V_T} = \sqrt{2} \]

\[ N V_I = \sqrt{2} V_I + (1 - \sqrt{2}) V_T \]

\[ N = \frac{\text{factor change in } V_I}{\frac{V_T}{V_I}} = \sqrt{2} + (1 - \sqrt{2}) \frac{V_T}{V_I} \]
Change in output bias:

\[ V_S - \alpha(\sqrt{2}(V_I - V_T))R_L - [V_S - \alpha(V_I - V_T)R_L] \]

new output bias \hspace{1cm} old output bias

\[ = -(\sqrt{2})^3\alpha(V_I - V_T)R_L + \alpha(V_I - V_T)R_L \]

Change in output bias \( = (1 - 2\sqrt{2})\alpha(V_I - V_T)R_L \approx 1.8 \) times the change in part (e)

**Problem 6.1:**

(a) \( i_P = 10^{-5}\left(\frac{\text{MHOs}}{V^2}\right)(v_{PC} + 5v_{GC})^3 \)

\( i_P = 10^{-5}\left(\frac{\text{MHOs}}{V^2}\right)(15\text{V} - 7.5\text{V})^3 \)

\( i_P = 10^{-5}\left(\frac{\text{MHOs}}{V^2}\right)(7.5^3)\text{V}^3 \)

\( i_P = 10^{-5}(7.5^3)\text{Amps} \)

\( i_P \approx 4.2 \text{ mA} \)

(b) We know from part (a) that the current through \( R \) is 4.2 mA. The voltage across \( R \) is \( 30\text{V} - v_{PC} \). Thus, \( R \) must be \( \frac{30\text{V} - v_{PC}}{4.2 \text{ mA}} \).

\[ R = \frac{30\text{V} - 15\text{V}}{4.2 \text{ mA}} = \frac{15\text{V}}{4.2 \text{ mA}} \]

\( R \approx 3.56 \text{ k\Omega} \)

(c) From the problem definition:

\( i_P = 10^{-5}\left(\frac{\text{MHOs}}{V^2}\right)(v_{PC} + 5v_{GC})^3 \)

The change in \( i_P \) when \( v_{GC} \) varies around an operating point \( V_{GC} \), \( V_{PC} \) is described by the partial derivative of \( i_P \) with respect to \( v_{GC} \) at the operating point:

\[ \left. \frac{\partial i_P}{\partial v_{GC}} \right|_{v_{GC} = V_{GC}, v_{PC} = V_{PC}} = \frac{15 \cdot 10^{-5}\left(\frac{\text{MHOs}}{V^2}\right)(V_{PC} + 5V_{GC})^2v_{GC}}{v_{GC} = V_{GC}, v_{PC} = V_{PC}} \]
The change in $i_p$ when $v_{PC}$ varies around an operating point $V_{GC}$, $V_{PC}$ is described by the partial derivative of $i_p$ with respect to $v_{PC}$ at the operating point:

$$
\frac{i_p}{v_{pc}} = \frac{\partial i_p}{\partial v_{pc}}_{v_{gc} = V_{GC}, v_{pc} = V_{PC}}
$$

$$
i_p = 3 \cdot 10^{-5} \left( \frac{\text{MHOs}}{V^2} \right) (V_{PC} + 5 V_{GC})^2 v_{pc}
$$

The total current $i_p$ is therefore the sum of the two partial currents:

- **left current source value:**
  $$15 \cdot 10^{-5} \left( \frac{\text{MHOs}}{V^2} \right) (V_{PC} + 5 V_{GC})^2 v_{gc}$$

- **right current source value:**
  $$3 \cdot 10^{-5} \left( \frac{\text{MHOs}}{V^2} \right) (V_{PC} + 5 V_{GC})^2 v_{pc}$$

Notice that the right-hand current source’s value is directly proportional to the voltage across the source. Therefore, that dependent current source can be modeled by a resistor:

- **current source value:**
  $$15 \cdot 10^{-5} \left( \frac{\text{MHOs}}{V^2} \right) (V_{PC} + 5 V_{GC})^2 v_{gc}$$

- **resistor value:**
  $$\frac{1}{3 \cdot 10^{-5} \left( \frac{\text{MHOs}}{V^2} \right) (V_{PC} + 5 V_{GC})^2} = \frac{10^5 \cdot V^2 \cdot \Omega}{3(V_{PC} + 5 V_{GC})^2}$$

Substituting in the numerical values for $V_{PC}$ and $V_{GC}$:
Visually, the conductance value of the resistor can be determined by looking at the slope of the $i_P-v_{PC}$ curve (selected by $V_{GC}$) at the operating point ($V_{PC}, I_P$). The conductance value of the dependent current source can be determined by visualizing how much $i_P$ bounces up and down to neighboring $i_P-v_{PC}$ curves as $v_{GC}$ changes (with $v_{PC}$ held constant).

Note that the device could alternately be modeled by a dependent voltage source in series with a resistor (derivation omitted):

\[
G \quad + \quad \frac{4}{3} \frac{k\Omega}{v_{PC}} \quad v_{PC} \quad - \quad v_{GC} \quad - \quad v_{GC} \quad + \quad P \quad C
\]

(d) Substitute in the model from part (c), replace the constant voltage supply with a short (because constant voltage supplies belong in the large-signal model of the circuit), and you get:

\[
8k\Omega \quad + \quad \frac{4}{3} \frac{k\Omega}{15} \quad v_{GC} \quad \frac{4}{15} \frac{k\Omega}{3} \quad v_{PC} \quad v_{GC}
\]
(e) Note that the two resistors in the small-signal model form a current divider.

\[
v_{pc} = 0V - \text{(voltage drop across the 8k}\Omega\text{ resistor)}
\]

\[
v_{pc} = - (\text{current through 8k}\Omega\text{ resistor}) \cdot 8k\Omega
\]

\[
v_{pc} = - \left( \frac{4}{3}k\Omega \cdot \frac{v_{gc}}{24k\Omega + \frac{4}{3}k\Omega + \frac{4}{15}k\Omega} \right) \cdot 8k\Omega
\]

\[
v_{pc} = -\frac{8 \cdot 15}{28} \cdot v_{gc} = -\frac{30}{7} \cdot v_{gc}
\]

\[
A_v = \frac{v_{pc}}{v_{gc}} = -\frac{30}{7}
\]

Problem 6.2:

(a) Alternately, the dependent current source on the right could be replaced by a resistor and dependent current source in parallel:

The resistor and dependent current source in parallel could alternately be modeled by a Thévenin equivalent (a voltage source dependent on \(v_m\) in series with a resistor—not shown).
To find $R_{TH}$, turn off all independent sources and measure the resistance. Because $v_{in} = 0$, $v_m = 0$. Thus, the small-signal current through MOSFET \( \text{②} \) (which corresponds to the right-hand dependent current source in the small-signal model) becomes $-g_{m2}v_{out}$. Because this current is proportional to the voltage across it, that dependent current source can be modeled by a resistor with conductance $g_{m2}$:

The resistance seen from a, a’ is therefore:

$$R_{TH} = \frac{R_2 \cdot \frac{1}{g_{m2}}}{\left( R_2 + \frac{1}{g_{m2}} \right)} = \frac{R_2}{g_{m2}}$$

$$R_{TH} = \frac{R_2}{R_2g_{m2} + 1}$$
(d) \[ \frac{v_{out}}{v_{in}} = A_1 \cdot A_2 \]

\[ \frac{v_{out}}{v_{in}} = \frac{-40000}{201} \approx -200 \]

Small-signal output resistance = \( R_{TH} \) from part (c).

\[ R_{TH} = \frac{R_2}{R_2 g_m + 1} \]

small-signal output resistance = \( \frac{5}{201} \) k\( \Omega \) \approx 25\( \Omega \)