Enter all your work and your answers directly in the spaces provided on the printed pages. Make sure that your name is on all sheets. Use the backs of the printed pages as scratch paper, but we will only grade the work that you neatly transfer to the spaces on the printed pages. Answers must be derived or explained, not just simply written down. The quiz is closed book, but calculators are allowed.

This quiz contains 14 pages including the cover sheet. Make sure that your quiz contains all 14 pages and that you hand in all 14 pages.

ALL NUMERICAL ANSWERS SHOULD HAVE UNITS.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Grade</th>
<th>Grader</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1: (20 points) For part (A), consider the circuit shown in Figure 1.

\[ v_x = a \cdot V_A + b \cdot V_B + c \cdot I_0 \]  \hspace{1cm} (1)

(A) The voltage \( v_x \) can be expressed as in Equation 1

Find the values of \( a \), \( b \) and \( c \) in terms of \( R_1 \), \( R_2 \) and \( R_3 \).

\[ V_A = V_B = 0, \quad I_0 = 0 \]

\[ V_B = V_A = 0, \quad I_0 = 0 \]

\[ I_0 = V_A = 0, \quad V_B = 0 \]
\[ \sqrt{\alpha} = \frac{R_3}{R_1 + R_2 + R_3} \quad V_A + \frac{R_1 + R_2}{R_1 + R_2 + R_3} \quad V_B - \frac{R_1 R_3}{R_1 + R_2 + R_3} \quad I_0 \]

*Check units!*

\[ [V] = \frac{[\Omega]}{[\Omega]} [V] + \frac{[\Omega]}{[\Omega]} [V] - \frac{[\Omega]^2}{[\Omega]} [A] \]

\[ a = \frac{R_3}{R_1 + R_2 + R_3}, \quad b = \frac{R_1 + R_2}{R_1 + R_2 + R_3}, \quad c = \frac{-R_1 R_2}{R_1 + R_2 + R_3} \]
(B) For part (B), consider the circuit shown in Figure 2. A current source $I_1$ has been connected to the network terminals as shown. How much power is delivered by the current source $I_1$ into the network as a function of $I_1$?

Figure 2: Circuit for Problem 1(B)

Thevenize original network

\[ V_{th} = aV_A + bV_B + cI_0 \]  
(from (a))

\[ R_{th} = R_3 \frac{R_1(R_1 + R_2)}{R_1 + R_2 + R_3} \]

\[ R_{th} = R_3 \frac{R_1 + R_2}{R_1 + R_2 + R_3} \]

\[ V_{thr} = V_{th} - I_1 R_{th} \]

\[ P_{delivered} = V_x I_x, \quad I_x = -I_1, \quad V_x = V_{th} - I_1 R_{th} \]

\[ P = I_1^2 R_{th} - I_1 V_{th} \]

\[ \text{Power} = \frac{I_1^2 R_{th} - I_1 V_{th}}{I_1} \cdot \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3} - I_1(aV_A + bV_B + cI_0) \]
Another option: superposition

\[ v_x = a V_A + b V_B + c I_o + d I_1 \]

\( a, b, c \) same as in (1-A)

\[ d_1: \quad V_A = V_B = I_o = 0 \]

\[ v_{x,1} = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} I_1 \]

\[ d_1 = - \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} \]

\[ P_{\text{delivered}} = - I_1 v_x \]

\[ P = - I_1 (a V_A + b V_B + c I_o + d I_1) \]
**Problem 2:** (25 points) This question concerns the circuit shown in Figure 3. In Parts (A) through (D), use Node 5 as the reference node.

![Circuit Diagram](image)

Figure 3: Circuit for Problem 2

(A) What is $e_4$, the Node voltage at Node 4?

$$e_4 = \frac{V_B}{R_4}$$

(B) (i) What is the current $i_{R_1}$ into Node 2 through resistor $R_1$?

$$i_{R_1} = \frac{I_A}{R_1} = \frac{e_i - e_2}{R_1}$$

(ii) If $e_2$ is known, what is $e_1$ in terms of $e_2$?

$$\frac{e_i - e_2}{R_1} = I_A \Rightarrow e_1 = e_2 + I_A R_1$$

$$e_1 = e_2 + \frac{I_A}{R_1} R_1$$
(C) Write out two KCL node equations at Nodes 2 and 3, involving only two node voltages, $e_2$ and $e_3$, and use them to complete the matrix equation below.

\[
\begin{align*}
\mathbf{e}_2 : & \quad -I_A + e_2 G_{t2} + (e_2 - e_3) G_{t3} = 0 \\
& \quad \left( G_{t2} - G_{t3} \right) e_2 - G_{t3} e_3 = I_A \\
\mathbf{e}_3 : & \quad (e_3 - e_2) G_{t3} + (e_3 - V_B) G_{t4} + e_3 G_{t5} = 0 \\
& \quad -G_{t3} e_2 + (G_{t3} + G_{t4} + G_{t6}) e_3 = G_{t4} V_B
\end{align*}
\]

\[
\begin{pmatrix}
G_{t2} + G_{t3} & -G_{t3} \\
-G_{t3} & G_{t3} + G_{t4} + G_{t5}
\end{pmatrix}
\begin{pmatrix}
e_2 \\
e_3
\end{pmatrix} =
\begin{pmatrix}
I_A \\
G_{t4} V_B
\end{pmatrix}
\]

(D) You are told that $e_3$, the voltage at Node 3, is 4 V when $I_A = 1$ mA and $V_B = 2$ V. You are also told that $e_3$ is 3 V when $I_A = 1$ mA and $V_B = 0$ V. What is $e_3$ when $I_A = 3$ mA and $V_B = 10$ V?

**Superposition:** $e_3 = \alpha I_A + \beta V_B$

\[
\begin{align*}
4 V &= \alpha \cdot 1 mA + \beta \cdot 2 V \\
3 V &= \alpha \cdot 1 mA + \beta \cdot 0 V
\end{align*}
\]

\[
\Rightarrow \begin{cases}
\alpha = 3 k \Omega \\
\beta = 0.5
\end{cases}
\]

\[
I_A = 3 mA, \quad V_B = 10 V
\]

\[
e_3 = 3 k \Omega \cdot 3 mA + 0.5 \cdot 10 V = 9 V + 5 V = 14 V
\]
(E) If the element values are such that, referenced to Node 5, the node voltages \( e_1, e_2, e_3 \) and \( e_4 \) are given in the table below, what are the node voltages if Node 2 is taken as the ground (reference) node?

<table>
<thead>
<tr>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
<th>( e_4 )</th>
<th>( e_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 V</td>
<td>3 V</td>
<td>2 V</td>
<td>1 V</td>
<td>0 V</td>
</tr>
<tr>
<td>( \downarrow ) 3</td>
<td>( \downarrow ) 3</td>
<td>( \downarrow ) 3</td>
<td>( \downarrow ) 3</td>
<td>( \downarrow ) 3</td>
</tr>
<tr>
<td>2 V</td>
<td>0 V</td>
<td>-1 V</td>
<td>-2 V</td>
<td>-3 V</td>
</tr>
</tbody>
</table>

\[ e_3 = 14 \text{ V} \]

\[ e_1' = \frac{2 \text{ V}}{} \]

\[ e_2' = \frac{0 \text{ V}}{} \]

\[ e_3' = \frac{-1 \text{ V}}{} \]

\[ e_4' = \frac{-2 \text{ V}}{} \]

\[ e_5' = \frac{-3 \text{ V}}{} \]
Problem 3: (30 points)

![Figure 4: Circuits and Figures for Problem 3(A) and (B)](image)

The graph from Figure 4 shows the measured behavior of Box A at the terminals a,b. The circuit from Figure 4 is the Norton equivalent circuit of Box B at the terminals c,d.

(A) Determine the short-circuit current of Box A; that is, what is $i_A$ when $v_A=0$?

$$i_A = -3 \, A$$

(B) Determine the open-circuit voltage of Box B; that is, what is $v_B$ when $i_B=0$?

$$v_B = 2 \, A \cdot 2 \, \Omega$$

$$v_B = 4 \, V$$
(C) The two boxes are connected as shown in Figure 5. Determine $i_A$, $i_B$, $v_A$ and $v_B$ with the two boxes connected.

![Figure 5: Circuit for Problem 3(C)](image)

$$i_A = -i_B, \quad v_A = v_B$$

**Method 1 - Analytical**

$$i_A = -3A + \frac{1}{1\Omega} \cdot v_A \quad \text{* from graph}$$

$$i_B + 2A - \frac{v_B}{2\Omega} = 0 \Rightarrow i_B = -2A + \frac{1}{2\Omega} \cdot v_B \quad \text{* from Norton circuit}$$

$$i_A = -i_B, \quad v_A = v_B$$

$$-3A + \frac{1}{1\Omega} \cdot v_A = 2A - \frac{1}{2\Omega} \cdot v_A \Rightarrow v_A = (\frac{1}{1\Omega} + \frac{1}{2\Omega})^{-1} \cdot 5A$$

$$v_A = \frac{2}{3} \Omega \cdot 5A = \frac{10}{3} \text{ V}$$

$$v_B = \frac{10}{3} \text{ V}$$

$$i_A = -3A + \frac{1}{1\Omega} \cdot \frac{10}{3} \text{ V}$$

$$i_A = \frac{1}{3} A$$

$$i_B = -\frac{1}{3} A$$

$$v_A = \frac{10}{3} \text{ V}$$

$$v_B = \frac{10}{3} \text{ V}$$
Method 2 - Combine circuits

Norton of A:

\[ \begin{align*}
\text{Combine sources, resistors} & \\
\text{Reexamine B:} & \\
\text{Method 3 - Graphical} & \\
n_B = i_A, \quad V_A = V_B \rightarrow \text{draw both on } \text{i}_A, V_A
\end{align*} \]
(D) Consider the terminal pair p,q. They are defined below with the boxes connected as in Figure 6.

![Diagram](image)

Figure 6: Circuit for Problem 3(D)

Draw the Thévenin or Norton equivalent circuit seen at the terminals p and q. Also, write an expression for $i$ in terms of $v$.

**Easiest as Norton**

$$v^- = \frac{2}{3} \Omega \cdot (5A + i)$$

$$i = \frac{v^-}{\frac{2}{3} \Omega} - 5A = \frac{3}{2} v^- - 5$$

$$i = \frac{3}{2} v - 5$$
Problem 4: (25 points)

(A) Write the Boolean expression that describes the function of the circuit shown in Figure 7.

![Circuit for Problem 4(A)](image)

\[ \text{OUT} = \overline{(A+B) \cdot C} = C + \overline{A+B} = C + \overline{A} \cdot \overline{B} \]

(B) A new transistor, the BFET, is characterized by the piece-wise model shown in Figure 8 and the following relations:

\[0 < v_{GS} < V_T: \quad i_G = \frac{v_{GS}}{R_G} \quad \text{and} \quad i_D = 0\]

\[v_{GS} > V_T: \quad i_G = \frac{v_{GS}}{R_G} \quad \text{and} \quad i_D = \frac{v_{DS}}{R_{DS}}\]

![Circuit for Problem 4(B)](image)
Draw the equivalent (linear) circuits for the BFET in the two regions of operation.

\[ 0 < V_{GS} < V_T \]

\[ V_T < V_{GS} \]

(C) A digital buffer is constructed as shown in Figure 9, with the following values:
\[ V_S = 5 \, \text{V}, \, V_T = 2.5 \, \text{V}, \, R_G = 100 \, \text{k}\Omega, \, R_{ON} = 2 \, \text{k}\Omega \text{ and } R_1 = 8 \, \text{k}\Omega. \]

![Figure 9: Circuit for Problem 4(C)](image)

The desired static discipline specifies \( V_{IH} = 3 \, \text{V} \) and \( V_{IL} = 2 \, \text{V} \).

Draw the transfer function \( v_z = f(v_x) \) for the case of no loading of the output; that is when \( i_z = 0 \).

What is the output noise margin \( \text{NM}_L \) for this design? Your calculations should justify your answer.
\[ V_y = \frac{R_G}{R_1 + R_G} \cdot V_s \]

\[ = \frac{100k\Omega}{108k\Omega} \cdot 6V = 4.6V > V_T \]

\[ V_Z = \frac{R_{ON}}{R_1 + R_{ON}} \cdot V_S \]

\[ = \frac{2k\Omega}{10k\Omega} \cdot 5V = 1V \]

\[ V_x > V_T \Rightarrow V_y = \frac{R_{ON}}{R_1 + R_{ON}} \cdot V_S = 1V \Rightarrow V_Z = V_S \]

\[ V_{OL} = 1V \]

\[ N_{M_L} = V_{IL} - V_{OL} = 2V - 1V = 1V \]

\[ N_{M_L} = 1V \]
(D) The static discipline specifies that $V_{OH} = 4 \text{ V}$. How many gate inputs can Output Z drive in parallel while still meeting the static discipline? Your calculations should justify your answer.

$$V_z = \frac{R_G}{n} \cdot V_S \geq V_{OH} \quad \frac{100 \text{k}\Omega}{n} \cdot 5\text{ V} > 4\text{ V}$$

$$\frac{100 \text{k}\Omega}{100 \text{k}\Omega + 8 \text{k}\Omega \cdot n} \cdot 5\text{ V} > 4\text{ V}$$

$$500 > 400 + 32n$$
$$32n < 100$$
$$n < 3.1$$

Number of gate inputs = 3