Name: __________________________________________

Recitation Instructor (circle one):
Baldo    Hutchinson    Kolodziejski    Schindall    Wilson

Recitation Hour (circle one):
9   10   11   12   1   2

NOTE THAT PROBLEMS HAVE WEIGHTS RANGING FROM 10 TO 16

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PROBLEM 1

Joe Nerdle, who took 6.002 some time ago, wants to assemble a three-input AND gate. He finds sufficient identical MOSFETS of unknown specifications.

The several parts below are independent.

(A) In fifty words or less, describe measurements which Joe can make to determine the threshold voltage $V_T$ and the on resistance $R_{ON}$ of the FETs. Include a circuit diagram if you wish.

Joe establishes a static discipline in which a logical one corresponds to $v > 4$ volts and a logical zero corresponds to $v < 1$ volt. A three input NAND gate and the switch-resistor model for the FET are shown below:

(B) Write a Boolean expression which describes the logical function of this gate.
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For the remaining parts of this problem, use these values:

\( R_{ON} = 1\,\text{k}\Omega \), \( V_T = 1\,\text{V} \)

(C) When \( v_A = v_B = v_C = 5\,\text{V} \), determine \( v_D \).

(D) Because Joe wants an AND gate, he connects the output of the gate above to a FET inverter as shown below:

When he builds this circuit he finds that \( v_D \) is low for all input conditions. Explain.

(E) In an effort to understand the reason the AND gate does not work, Joe isolates the inverter and connects an oscilloscope to its output as shown below.

He can’t find a scope probe so he uses a long piece of coaxial cable. This unfortunate arrangement provides a capacitive load of \( C = 20\,\text{pF} = 2 \times 10^{-11}\,\text{F} \) to the inverter. The resulting circuit is the one shown above right.

Sketch and label the waveform \( v_O(t) \) which Joe sees on the scope when the switch changes the input voltage from \(+5\,\text{V}\) to \(0\,\text{V}\) at \( t = 0 \). Also sketch and label the waveform \( v_O(t) \) which Joe sees on the scope much later \((t > 5\tau)\) when the switch changes the input voltage from \(0\,\text{V}\) to \(+5\,\text{V}\) at \( t' = 0 \). Show asymptotes and determine the time constant. Use the coordinates provided on page 4.
Waveform when switch moves from +5 volts to 0 volts at $t=0$

$V_o$(volts)

$+5$

$0$

$-5$

$t$(seconds)

Waveform when switch moves from 0 volts to +5 volts at $t'=0$

$V_o$(volts)

$+5$

$0$

$-5$

$t'$(seconds)
PROBLEM 2

The circuit below contains both dependent and independent sources.

Determine a Thevenin equivalent circuit at terminals $a-a'$. Specify polarities and element values.
PROBLEM 3

A FET used as an amplifier is shown in the circuit below:

The capacitor $C_1$ couples the incremental input voltage $v_i$ to the gate. Because it is an open circuit for DC, $v_i$ does not affect $V_{GS}$, the operating-point value of the gate-to-source voltage.

The FET is described by:

$$i_D = \begin{cases} 0 & \text{for } v_{GS} < V_T \\ \frac{K}{2}(v_{GS} - V_T)^2 & \text{for } v_{GS} > V_T \text{ and } v_{DS} > (v_{GS} - V_T) \end{cases}$$

$$K = 0.4 \frac{mA}{V^2} \quad V_T = 2V$$

(A) Determine $V_{GS}$ and $R_L$ such that the operating point of the amplifier is:

$$V_O = 7V \quad I_D = 0.2mA$$

(B) Determine the incremental transconductance $g_m$ of the FET at the operating point defined in part (A).
(C) Given that $R_1 + R_2 = 500\,\Omega$, determine $R_1$ and $R_2$ such that the desired value of $V_{GS}$ is established.

(D) Employing the usual incremental model for the FET, sketch and label a complete incremental model for the circuit. Label elements of the model by their symbols, *NOT* by their numerical values.
PROBLEM 4

A quartz piezoelectric crystal exhibits a strong (high Q) resonance, which is why it is used as time or frequency standards in clocks, tuners, etc.

\[ R \quad S \quad L \quad C \]

equivalent circuit for a quartz crystal

(A) A quartz crystal is used in the filter below. Determine the transfer function of this filter. That is, find \( \mathcal{A}(s) = \frac{V_o}{V_I} \) where \( s = j\omega \). Express your result as a ratio of polynomials in \( s \).

\[ R \quad S \quad L \quad C \]
circuit symbol for a quartz crystal

\[ + \quad V_I \quad - \]

Let \( R = R_S \)
For a specific set of element values in this circuit, the transfer function is:

\[ \mathcal{A}(s) = 2.5 \times 10^3 \frac{s}{s^2 + 5 \times 10^3 s + 0.25 \times 10^14} \quad \text{(for } s \text{ in sec}^{-1}) \]

(B) Sketch the frequency dependence of the magnitude and phase of \( \mathcal{A}(j\omega) \) on the coordinates provided on page 10. Determine and label the resonant frequency \( \omega_0 \), the magnitude \( A_0 \) and phase of \( \mathcal{A}(j\omega_0) \) at that frequency, and the high and low frequency asymptotes of magnitude and phase. Determine the bandwidth of the resonance defined by the frequencies where \( |\mathcal{A}| = A_0/\sqrt{2} \).
PROBLEM 5

The circuit below is driven by $v_I(t) = 20 \cos(\omega t)$:

Using the indicated element values, the transfer function is:

$$\frac{V_o}{V_I} = 0.5 \frac{j\omega + 2 \times 10^4}{j\omega + 10^4}$$

Determine $v_O(t)$ for $\omega = 10^4 \text{ sec}^{-1}$. 

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PROBLEM 6

The circuits below function as filters in the sinusoidal steady state.

Label those which are Low-Pass Filters with \( \text{LP} \)
Label those which are High-Pass Filters with \( \text{HP} \)
Label those which are Band-Pass Filters with \( \text{BP} \)
Label those which are Band-Stop Filters with \( \text{BS} \)

Grading will be based on \((\# \text{ correct}\times3-\# \text{ incorrect})\)
PROBLEM 7

The op-amp circuit below employs a diode, which may be assumed ideal, and an n-channel FET. The op-amp has very high gain. The characteristics of both are shown.

Sketch the nonlinear transfer characteristic \( v_O \) vs. \( v_I \) on the coordinates below for \(-20 \text{ V} < v_I < 20 \text{ V}\).

**Hint:** Start from \( v_I = 0 \) and let \( v_I \) be of either polarity.
PROBLEM 8

The operational amplifier shown below has very high gain. The supply voltages (at which the amplifier saturates) are ±10 volts. The circuit is an oscillator. Assume that the switch is open for all \( t < 0 \).

At \( t = 0 \) the switch closes and stays closed.

(A) Determine and sketch on the axes on the next page the waveforms of \( v_+ \), \( v_- \) and \( v_O \) for two full cycles of the oscillation.
(B) What is the period of the steady-state oscillation in seconds?

\[ R = 10 \, \text{k}\Omega = 10^4 \, \Omega \quad \text{and} \quad L = 10 \, \text{mH} = 10^{-2} \, \text{H} \]