Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

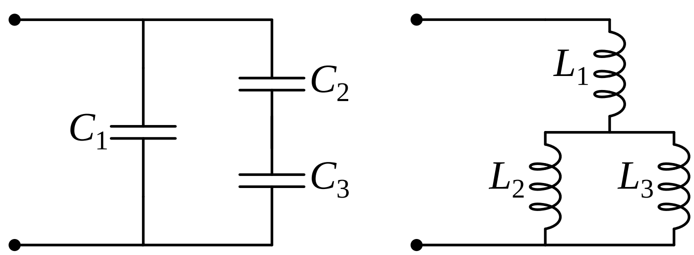
6.002 – Electronic Circuits
Spring 2002

Homework #6

Issued 3/13/02 – Due 3/20/02

Exercise 6.1: Consider an amplifier with an input-output relation that takes the form $v_{\mathrm{OUT}} = V_{\mathrm{A}}(v_{\mathrm{IN}}/V_{\mathrm{B}})^5$, where V_{A} and V_{B} are voltage constants. Determine its output bias voltage V_{OUT} and its small-signal gain $v_{\mathrm{out}}/v_{\mathrm{in}}$ for a given input bias voltage V_{IN}.

Exercise 6.2: Find the capacitance of the all-capacitor network, and the inductance of the all-inductor network, shown below.



Problem 6.1: This problem studies the propagation delay of digital signals through the inverter shown below. Assume that the MOSFET in the inverter acts as a switch with on-state resistance R_{ON}. The inverter is loaded with a capacitor, having capacitance C_{G}, that models the combined input capacitance of the logic gates connected to its output. Assume that the inverter obeys the static discipline defined in part by V_{OL} and V_{OH}.

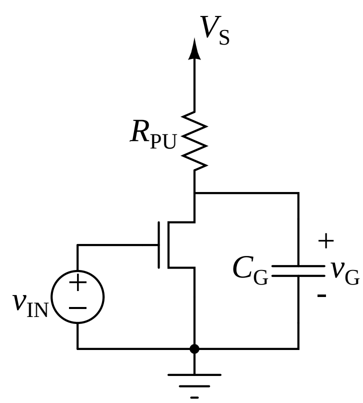
(A) Assume that the MOSFET has been off for a very long time. At $t = 0$, v_{IN} turns the MOSFET on. Determine $v_{\mathrm{G}}(t)$ for $t \geq 0$.

(B) How long does it take $v_{\mathrm{G}}(t)$ to pass by V_{OL}? This delay is the fall time of the inverter.

(C) Assume that the MOSFET has been on for a very long time. At $t = 0$, v_{IN} turns the MOSFET off. Determine $v_{\mathrm{G}}(t)$ for $t \geq 0$.

(D) How long does it take $v_{\mathrm{G}}(t)$ to pass by V_{OH}? This delay is the rise time of the inverter.

(E) How can the fall and rise times be shortened via the design of R_{PU}? What limits the extent to which this design path may be followed?



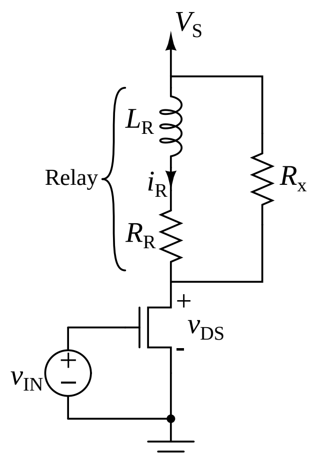
Problem 6.2: In the circuit shown below, a MOSFET and an external resistor having resistance R_X are used to control the current i_R in the winding of a relay. Here, the relay is modeled as a series inductor and resistor having inductance L_R and resistance R_R, respectively. The MOSFET may be modeled as an ideal switch.

(A) At $t = 0$, v_{IN} turns the MOSFET on so that $v_{DS} = 0$. Determine $i_R(t)$ for $t \geq 0$ given that $i_R(t = 0) = 0$.

(B) Next, at $t = T$, v_{IN} turns the MOSFET off. Determine both $i_R(t)$ and $v_{DS}(t)$ for $t \geq T$. Hint: $i_R(t)$ is continuous at $t = T$.

(C) Sketch and clearly label graphs of both $i_R(t)$ and $v_{DS}(t)$ for $t \geq 0$ assuming that $T \approx 5L_R/R_R$ and $R_X = R_R$.

(D) The relay control circuit would be less expensive without the external resistor, which may be "removed" from the circuit by considering the limit $R_X \to \infty$. Why might such a cost reduction be unwise?



Problem 6.3: At $t = 0^-$, the networks shown below have zero initial state. That is, the capacitor voltage $v(t)$ and the inductor current $i(t)$ are both zero at $t = 0^-$. At $t = 0$, the voltage source produces an impulse of area Λ, and the current source produces an impulse of area Q.

(A) Derive the differential equation that relates $v(t)$ to $I(t)$ and $i(t)$ to $V(t)$. Hint: consider using Thevenin or Norton equivalent networks to simplify the work.

(B) Find the capacitor voltage $v(t)$ and the inductor current $i(t)$ at both $t = 0^+$ and $t = \infty$. One way to find the states at $t = 0^+$ is to integrate the corresponding differential equations from $t = 0^-$ to $t = 0^+$ under the assumption that each state remains finite during that time; you should justify this assumption. Then, substitute the initial conditions at $t = 0^-$ into the results to determine the states at $t = 0^+$. Try to determine the states at $t = \infty$ through physical, rather than mathematical, reasoning.

(C) Next, find the time constant by which each state goes from its initial value at $t = 0^+$ to its final value at $t = \infty$.

(D) Using the previous results, and without necessarily solving the differential equations directly, construct $v(t)$ and $i(t)$ for $t \geq 0$.

(E) Verify that the solutions to Part (D) are correct by substituting them into the differential equation found in Part (A).