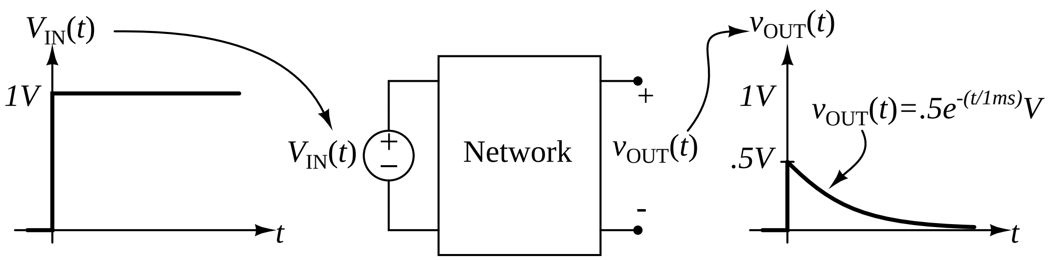
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Spring 2002

Homework #8

Issued 4/3/02 – Due 4/10/02

Exercise 8.1: Using one 1-nF capacitor and two resistors, construct a two-port network that has the following response to a 1-V step input; assume that the capacitor voltage is zero prior to the step. Provide a diagram of the network, and specify the values of the two resistors.



Exercise 8.2: Repeat Exercise 8.1 given that the allowable components are now one 1-mH inductor and two resistors; assume that the inductor current is zero prior to the step.

Problem 8.1: The network shown below includes a switch with three positions: A, B and C. Prior to $t = 0$, the switch is in Position B, and the inductor current $i(t)$ and capacitor voltage $v(t)$ are both zero.

(A) At $t = 0$ the switch moves to Position A, and it remains there until $t = T_1$. Find $i(t)$ and $v(t)$ for $0 \leq t \leq T_1$.

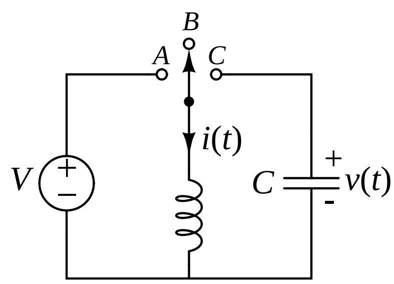
(B) At $t = T_1$ the switch moves to Position C, and it remains there until $i(t)$ goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ goes to zero as $t = T_2$. Determine T_2, and find both $i(t)$ and $v(t)$ for $T_1 \leq t \leq T_2$.

(C) The switch remains in Position B until $t = T_3$. Find both $i(t)$ and $v(t)$ for $T_2 \leq t \leq T_3$.

(D) At $t = T_3$ the switch moves again to Position A, and it remains there until $t = T_4$. Find $i(t)$ and $v(t)$ for $T_3 \leq t \leq T_4$.

(E) Finally, at $t = T_4$ the switch moves to Position C, and it remains there until $i(t)$ again goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ again goes to zero as T_5. Determine T_5, and find both $i(t)$ and $v(t)$ for $T_4 \leq t \leq T_5$.

(F) Sketch and clearly label $i(t)$ and $v(t)$ for $0 \leq t \leq T_5$.



Problem 8.2: This problem is a continuation of Problem 8.1. It explores the use of energy conservation to analyze the operation of the network described therein.

(A) Determine the energy stored in the inductor at $t = T_1$.

(B) The energy stored in the inductor at $t = T_1$ is transferred to the capacitor at $t = T_2$. Use this fact to determine $v(T_2)$. This answer should match the answer to Part B of Problem 8.1.

(C) Determine the energy stored in the inductor at $t = T_4$.

(D) Use energy conservation to determine the energy stored in the capacitor at $t = T_5$, and then determine $v(T_5)$. This answer should match the answer to Part E of Problem 8.1.

(E) Now let the switch move repetitively through the cycle of Positions B to A to C to B. Assume that in each cycle the switch remains in Position A for the duration T. Further, assume that switch always moves from Position C to Position B when $i(t)$ reaches zero. Assuming that v and i are initially zero, determine i at the end of the nth switching cycle in terms of n, C, L, T and I.

Problem 8.3: In the network shown below, the inductor and capacitor have zero current and voltage, respectively, prior to $t = 0$. At $t = 0$, a step in voltage from 0 to $V_\circ$ is applied by the voltage source as shown.

(A) Find v_C, v_L, v_R, i and $\frac{di}{dt}$ just after the step at $t = 0$.

(B) Argue that $i = 0$ at $t = \infty$ so that $i(t)$ has no constant component.

(C) Find a second-order differential equation which describes the behavior of $i(t)$ for $t \geq 0$.

(D) Following (B) the current $i(t)$ takes the form $i(t) = I\sin(\omega t + \phi)e^{-\alpha t}$. Find I, ω, ϕ and α. Hint: first find ω and α from the differential equation, and then find I and ϕ from the initial conditions; alternatively, solve this problem by any method you wish.

(E) Suppose that the input is a voltage impulse with area $\Lambda_\circ$ where $\Lambda_\circ = \tau V_\circ$, $V_\circ$ is the amplitude of the voltage step shown below, and τ is a given time constant. Find the response of the network shown below to the impulse. Hint: before solving this problem directly, consider the relation between step and impulse responses.

Save a copy of your answers to this problem. They will be useful during the pre-lab exercises for Lab #3.