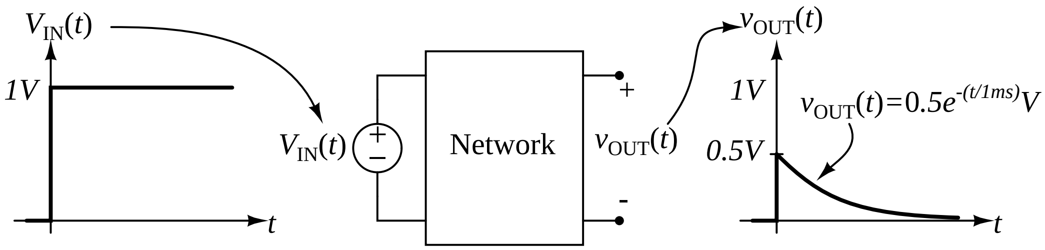
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

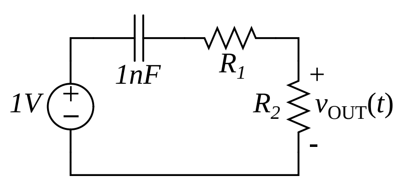
6.002 – Electronic Circuits
Spring 2002

Homework #8 Solutions

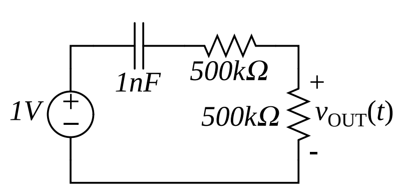
Exercise 8.1: Using one 1-nF capacitor and two resistors, construct a two-port network that has the following response to a 1-V step input; assume that the capacitor voltage is zero prior to the step. Provide a diagram of the network, and specify the values of the two resistors.



Answer: By examining the information given in the figure, we can see that τ is equal to 1 ms for this network. Since it's an RC network, that means that $R_{\text{eq}}C = 1ms$ or $R_{\text{eq}} = 1M\Omega$. Given that the output voltage starts at half the input voltage at $t = 0$, there must be a voltage divider at the output port. A good guess for the placement of the capacitor is shown in the following figure.

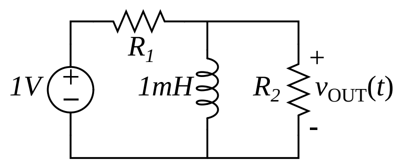


At $t = 0$, the capacitor looks like a short circuit. If this is the case $v_{\text{OUT}} = 1 \cdot \frac{R_2}{R_1+R_2}[\text{V}]$. This satisfies the $v_{\text{OUT}}(t) = 0.5[\text{V}]$ if $R_1 = R_2$. When $t = \infty$, the capacitor looks like an open circuit, and $v_{\text{OUT}} = 0[\text{V}]$. This circuit completely satisfies the initial and final values for v_{OUT}. Combining this with our earlier constraint on the equivalent resistance means that $R_1 = R_2 = 500k\Omega$. The final circuit is then

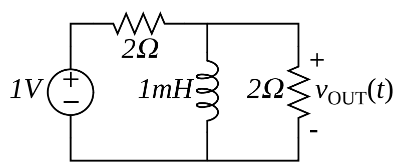


Exercise 8.2: Repeat Exercise 8.1 given that the allowable components are now one 1-mH inductor and two resistors; assume that the inductor current is zero prior to the step.

Answer: Given that the output voltage is 0[V] at $t = \infty$, it is a good guess that the inductor (a short at $t = \infty$) should appear across the output terminals. Let's guess this circuit:



At $t = 0$, the inductor looks like an open circuit, so $v_{\text{OUT}} = 1 \cdot \frac{R_2}{R_1+R_2}[\text{V}]$. This satisfies the $v_{\text{OUT}}(t) = 1/2[\text{V}]$ if $R_1 = R_2$. When $t = \infty$, the inductor looks like a short circuit, and $v_{\text{OUT}} = 0[\text{V}]$. This circuit completely satisfies the initial and final values for v_{OUT}. The equivalent resistance for the inductor is R_1 in parallel with R_2. We know that $\frac{L}{R_{\text{eq}}} = 1ms$. This means that $R_1 = R_2 = 2\Omega$. The final circuit is then



Problem 8.1: The network shown below includes a switch with three positions: A, B and C. Prior to $t = 0$, the switch is in Position B, and the inductor current $i(t)$ and capacitor voltage $v(t)$ are both zero.

(A) At $t = 0$ the switch moves to Position A, and it remains there until $t = T_1$. Find $i(t)$ and $v(t)$ for $0 \leq t \leq T_1$.

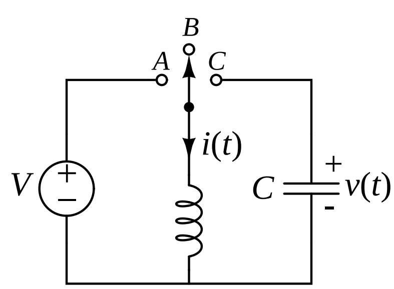
(B) At $t = T_1$ the switch moves to Position C, and it remains there until $i(t)$ goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ goes to zero as $t = T_2$. Determine T_2, and find both $i(t)$ and $v(t)$ for $T_1 \leq t \leq T_2$.

(C) The switch remains in Position B until $t = T_3$. Find both $i(t)$ and $v(t)$ for $T_2 \leq t \leq T_3$.

(D) At $t = T_3$ the switch moves again to Position A, and it remains there until $t = T_4$. Find $i(t)$ and $v(t)$ for $T_3 \leq t \leq T_4$.

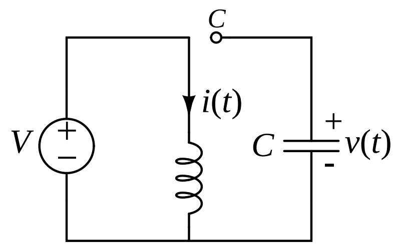
(E) Finally, at $t = T_4$ the switch moves to Position C, and it remains there until $i(t)$ again goes to zero, at which time the switch moves back to Position B. Define the time at which $i(t)$ again goes to zero as T_5. Determine T_5, and find both $i(t)$ and $v(t)$ for $T_4 \leq t \leq T_5$.

(F) Sketch and clearly label $i(t)$ and $v(t)$ for $0 \leq t \leq T_5$.



Answer:

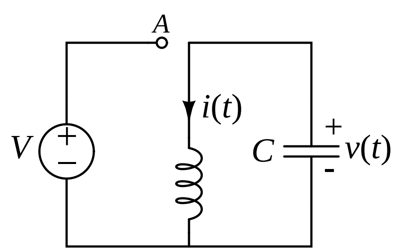
(A) For this case, the circuit looks like the following:



Examining this circuit, we can see that $V = L\frac{di}{dt}$. Integrating this function gives $i(t) = \frac{Vt}{L} + c$, where c is a constant of integration. Since $i(0) = 0$, c must be zero. Since $v(0) = 0$ and the capacitor is disconnected from the circuit, there's no way for the capacitor to charge, and $v(t)$ must be zero during this time period. Therefore, our answers to Part (A) are:

$$\left.\begin{array}{rcl} i(t) & = & \frac{Vt}{L} \\ v(t) & = & 0 \end{array}\right\} \quad \text{when} \quad 0 \leq t \leq T_1$$

(B) Now, the switch moves from A to C, and the circuit looks like the following:



The only part we have to worry about is the loop with the inductor and the capacitor. We know that the inductor will be discharging the energy it has stored in part (A) into the capacitor in this circuit configuration. The voltage across both elements is the same, and we can write $v = L\frac{di}{dt}$ and $-i = C\frac{dv}{dt}$. For practice, let's plod through this solution all the way from the differential equation step-by-step. Combining the information above, we can write

$$LC\frac{d^2v}{dt^2} + v = 0 \quad \text{or} \quad \frac{d^2v}{dt^2} + \frac{1}{LC}v = 0$$

A good guess for the solution would be something of the form $v = Ae^{s(t-T_1)} + Be^{-s(t-T_1)}$. We need the $t - T_1$ term because we're starting at time T_1 in this part of the problem. You only need to substitute one term or the other into the equation to get the right quadratic equation (do the math to convince yourself!), so let's use $v = Ae^{s(t-T_1)}$ for the substitution. Substituting this "solution" and canceling out terms gives a quadratic equation in terms of s:

$$s^2 + \frac{1}{LC} = 0$$

Therefore, we can write that our solution is of the form

$$v(t) = Ae^{j\sqrt{\frac{1}{LC}}(t-T_1)} + Be^{-j\sqrt{\frac{1}{LC}}(t-T_1)}$$

Hopefully, by now you are experienced enough to know that the above behavior is oscillatory, but let's actually work through the math to get this expression into sines and cosines.

Let's rewrite $v(t)$ in the following form:

$$v(t) = \left(\frac{A+B}{2}\right)\left(e^{j\sqrt{\frac{1}{LC}}(t-T_1)} + e^{-j\sqrt{\frac{1}{LC}}(t-T_1)}\right) + \left(\frac{A-B}{2}\right)\left(e^{j\sqrt{\frac{1}{LC}}(t-T_1)} - e^{-j\sqrt{\frac{1}{LC}}(t-T_1)}\right)$$

By now, you should see that it's clear how we can use Euler's relationships to transform $v(t)$ into sines and cosines.

$$v(t) = (A+B)\cos\left(\frac{(t-T_1)}{\sqrt{LC}}\right) + j(A-B)\sin\left(\frac{(t-T_1)}{\sqrt{LC}}\right)$$

Now let's use our initial conditions to solve for A and B. We know that $v(T_1) = 0$ and $i(T_1) = \frac{VT_1}{L}$. Plugging our initial condition for the voltage into the equation for the voltage tells us that $A + B = 0$. Our other condition tells us something about the current, so we need to somehow transform our voltage expression into a current expression. Remember that $-i = C\frac{dv}{dt}$. So, let's take the derivative of the voltage expression and multiply by $-C$.

$$i(t) = \sqrt{\frac{C}{L}}(A+B)\sin\left(\frac{(t-T_1)}{\sqrt{LC}}\right) - j\sqrt{\frac{C}{L}}(A-B)\cos\left(\frac{(t-T_1)}{\sqrt{LC}}\right)$$

Plugging in the initial condition gives $A - B = j\frac{VT_1}{\sqrt{LC}}$.

Now let's rewrite $v(t)$ and $i(t)$ using what we know about $A + B$ and $A - B$.

$$\begin{aligned} v(t) &= -\frac{VT_1}{\sqrt{LC}}\sin\left(\frac{(t-T_1)}{\sqrt{LC}}\right) \\ i(t) &= \frac{VT_1}{L}\cos\left(\frac{(t-T_1)}{\sqrt{LC}}\right) \end{aligned}$$

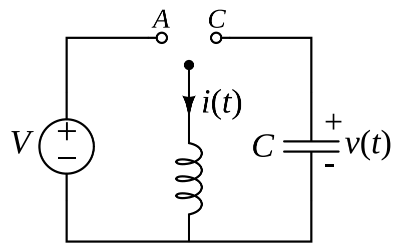
This would be our final answer if we allowed the switch to remain closed for all time. However, the switch goes back to being open as soon as $i(t)$ reaches zero for the first time. Since $i(t)$ is a cosine function, the switch opens when the argument of the cosine function is equal to $\frac{\pi}{2}$, or $\frac{t-T_1}{\sqrt{LC}} = \frac{\pi}{2}$. Solving this equation for t gives us the value of T_2.

$$T_2 = T_1 + \frac{\pi}{2}\sqrt{LC}$$

Therefore, our answer to (B) is the following:

$$\left.\begin{aligned} v(t) &= -\tfrac{VT_1}{\sqrt{LC}}\sin\left(\tfrac{(t-T_1)}{\sqrt{LC}}\right) \\ i(t) &= \tfrac{VT_1}{L}\cos\left(\tfrac{(t-T_1)}{\sqrt{LC}}\right) \end{aligned}\right\} \quad \text{when} \quad T_1 \leq t \leq T_2 = T_1 + \frac{\pi}{2}\sqrt{LC}$$

(C) For Part (C) our circuit looks like the drawing below:



During time T_1 to T_2, the current through the inductor drops to zero and the capacitor gets charged. Therefore, the initial conditions for Part (C) are $i(T_2) = 0$ and $v(T_2) = -\frac{VT_1}{\sqrt{LC}}$ (The sine term is equal to 1 when evaluated at T_2).

Since there is no current flowing in the circuit for Part (C), there is no way to discharge the charge on the capacitor, and therefore the circuit continues to look as it did at $t = T_2$ during the entire interval from T_2 to T_3.

So, the answer to (C) is the following:

$$\left.\begin{array}{l} v(t) = -\frac{VT_1}{\sqrt{LC}} \\ i(t) = 0 \end{array}\right\} \quad \text{when} \quad T_2 \leq t \leq T_3$$

(D) Now our circuit looks like the one from Part (A), except with slightly different initial conditions. From Part (C), we know that $i(T_3) = 0$ and $v(T_3) = -\frac{VT_1}{\sqrt{LC}}$. Since the initial condition for $i(t)$ is the same as the one for Part (A), the solution for $i(t)$ here is the same for Part (A), compensated for the time shift: $i(t) = \frac{V(t-T_3)}{L}$. Since there is still no current path for the capacitor to discharge its stored charge, the voltage across the capacitor remains the same for the entire interval T_3 to T_4.

Therefore, our answer for Part (D) is the following:

$$\left.\begin{array}{lcl} v(t) & = & -\frac{VT_1}{\sqrt{LC}} \\ i(t) & = & \frac{V(t-T_3)}{L} \end{array}\right\} \quad \text{when} \quad T_3 \leq t \leq T_4$$

(E) After the switch at T_4, the circuit looks again like it did in part (B). Since none of the elements have changed, we know that the solution will be of the same form as (B), but with different values for the coefficients. Therefore, we can directly write

$$v(t) = (A + B)\cos\left(\frac{t - T_4}{\sqrt{LC}}\right) + j(A - B)\sin\left(\frac{t - T_4}{\sqrt{LC}}\right)$$

$$i(t) = \sqrt{\frac{C}{L}}(A + B)\sin\left(\frac{t - T_4}{\sqrt{LC}}\right) - j\sqrt{\frac{C}{L}}(A - B)\cos\left(\frac{t - T_4}{\sqrt{LC}}\right)$$

However, the A and B here are different than those in Part (B) because of the differing initial conditions. Our initial conditions are now $v(T_4) = -\frac{VT_1}{\sqrt{LC}}$ and $i(T_4) = \frac{V(T_4-T_3)}{L}$.

These initial conditions allow us to solve for $A+B$ and $A-B$ in the same way we did in Part (B), giving the expressions for $v(t)$ and $i(t)$.

$$v(t) = -\frac{VT_1}{\sqrt{LC}}\cos\left(\frac{t-T_4}{\sqrt{LC}}\right) - \frac{V(T_4-T_3)}{\sqrt{LC}}\sin\left(\frac{t-T_4}{\sqrt{LC}}\right)$$

$$i(t) = -\frac{VT_1}{L}\sin\left(\frac{t-T_4}{\sqrt{LC}}\right) + \frac{V(T_4-T_3)}{L}\cos\left(\frac{t-T_4}{\sqrt{LC}}\right)$$

We need to find the time when $i(t) = 0$, which is defined to be T_5. Setting the above equation equal to zero and simplifying gives

$$\tan\left(\frac{T_5-T_4}{\sqrt{LC}}\right) = \frac{T_4-T_3}{T_1}$$

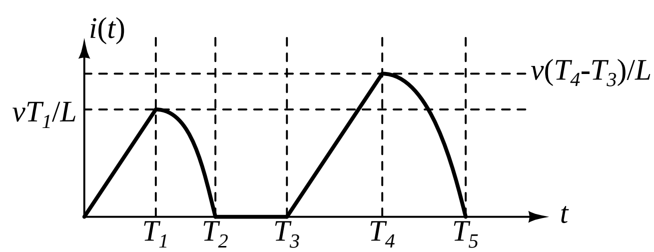
Solving for T_5 gives the following:

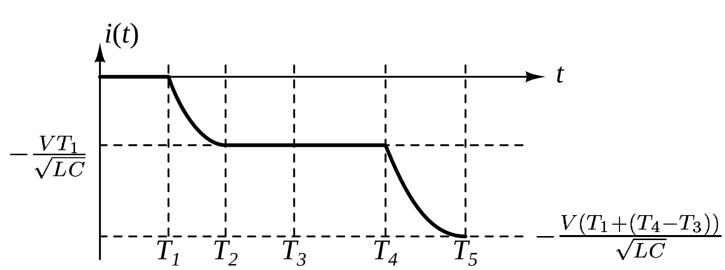
$$T_5 = T_4 + \sqrt{LC}\arctan\left(\frac{T_4-T_3}{T_1}\right)$$

Our final answer for Part (E) is the following:

$$\left.\begin{array}{lcl} v(t) & = & -\frac{VT_1}{\sqrt{LC}}\cos\left(\frac{t-T_4}{\sqrt{LC}}\right) - \frac{V(T_4-T_3)}{\sqrt{LC}}\sin\left(\frac{t-T_4}{\sqrt{LC}}\right) \\ i(t) & = & -\frac{VT_1}{L}\sin\left(\frac{t-T_4}{\sqrt{LC}}\right) + \frac{V(T_4-T_3)}{L}\cos\left(\frac{t-T_4}{\sqrt{LC}}\right) \end{array}\right\} \quad \text{when} \quad T_4 \le t \le T_5$$

(F) The graphs of $v(t)$ and $i(t)$ appear below.





Problem 8.2: This problem is a continuation of Problem 8.1. It explores the use of energy conservation to analyze the operation of the network described therein.

(A) Determine the energy stored in the inductor at $t = T_1$.

(B) The energy stored in the inductor at $t = T_1$ is transferred to the capacitor at $t = T_2$. Use this fact to determine $v(T_2)$. This answer should match the answer to Part B of Problem 8.1.

(C) Determine the energy stored in the inductor at $t = T_4$.

(D) Use energy conservation to determine the energy stored in the capacitor at $t = T_5$, and then determine $v(T_5)$. This answer should match the answer to Part E of Problem 8.1.

(E) Now let the switch move repetitively through the cycle of Positions B to A to C to B. Assume that in each cycle the switch remains in Position A for the duration T. Further, assume that switch always moves from Position C to Position B when $i(t)$ reaches zero. Assuming that v and i are initially zero, determine v at the end of the nth switching cycle in terms of n, C, L, T and V.

Answer:

(A) The energy stored in an inductor is given by $E_{\text{L}} = \frac{1}{2}Li^2$. From Problem 8.1 Part (A), we have $i(T_1) = \frac{VT_1}{L}$. Substituting for i in the energy equation and simplifying gives the energy stored in the inductor at $t = T_1$:

$$E_{\text{L}}(T_1) = \frac{V^2T_1^2}{2L}$$

(B) The energy in a capacitor is given by the equation $E_{\text{C}} = \frac{1}{2}CV^2$. Since the energy stored in the inductor at $t = T_1$ is transferred to the capacitor at $t = T_2$, the energy in the capacitor at $t = T_2$ is equal to the energy in the inductor at $t = T_1$:

$$\frac{1}{2}Cv^2 = \frac{V^2T_1^2}{2L}$$

Isolating v gives the voltage across the capacitor at $t = T_2$:

$$v = \pm\frac{VT_1}{\sqrt{LC}}$$

Here we choose the minus sign due to the direction of $i(t)$ at T_1.

From Problem 8.1 Part (B), we have

$$v(t) = -\frac{VT_1}{\sqrt{LC}}\sin\left(\frac{t - T_1}{\sqrt{LC}}\right)$$

We also know that $T_2 = T_1 + \frac{\pi}{2}\sqrt{LC}$. Substituting this in for t in the equation for $v(t)$ gives $v(T_2) = -\frac{VT_1}{\sqrt{LC}}$, which matches what we found by solving for the energy across the capacitor.

(C) From Problem 8.1 Part (D), we have that $i(t) = \frac{V(t-T_3)}{L}$ for $T_3 < t < T_4$. Therefore, $i(T_4) = \frac{(T_4-T_3)}{L}$. The energy stored in an inductor is $\frac{1}{2}Li^2$, so the energy at $t = T_4$ is the following:

$$E_{\mathrm{L}}(T_4) = \frac{V^2 (T_4 - T_3)^2}{2L}$$

(D) Conservation of energy requires that the energy stored in the capacitor at $t = T_5$ must be equal to the energy stored in the capacitor at $t = T_2$ plus the energy stored in the inductor at $t = T_4$. Therefore, the energy stored in the capacitor at $t = T_5$ is given by the following:

$$E_{\mathrm{C}}(T_5) = \frac{V^2 T_1^2}{2L} + \frac{V^2 (T_4 - T_3)^2}{2L}$$

Since the energy stored in the capacitor at $t = T_5$ is $\frac{1}{2}Cv^2$, we can set the above equation equal to that and solve for v.

Solving for $v(T_5)$ gives:

$$v(T_5) = \pm \frac{V}{\sqrt{LC}} \sqrt{T_1^2 + (T_4 - T_3)^2}$$

Again we choose the minus sign.

From Problem 8.1 Part (E) the voltage across the capacitor at $t = T_5$ is equal to $-\frac{VT_1}{\sqrt{LC}} \cos(\theta) - \frac{V(T_4-T_3)}{\sqrt{LC}} \sin(\theta)$, where $\theta = \arctan\left(\frac{T_4-T_3}{T_1}\right)$. Going back to basic trigonometry, we can write that $\cos(\theta) = \frac{T_1}{\sqrt{T_1^2+(T_4-T_3)^2}}$ and $\sin(\theta) = \frac{(T_4-T_3)}{\sqrt{T_1^2+(T_4-T_3)^2}}$. Substituting these values for the sine and cosine functions in our expression shows that the voltage across the capacitor is the same as we calculated above:

$$v(T_5) = -\frac{V}{\sqrt{LC}} \sqrt{T_1^2 + (T_4 - T_3)^2}$$

(E) Since there is no way for the capacitor to dissipate its energy during the switching cycle, its charge and voltage will keep building up in the same way it did between $t = 0$ and $t = T_2$ in the work we did above. Therefore, v at the end of the nth switching cycle is given by the following:

$$v = -\frac{VT\sqrt{n}}{\sqrt{LC}}$$

Problem 8.3: In the network shown below, the inductor and capacitor have zero current and voltage, respectively, prior to $t = 0$. At $t = 0$, a step in voltage from 0 to $V_\circ$ is applied by the voltage source as shown.

(A) Find v_C, v_L, v_R, i and $\frac{di}{dt}$ just after the step at $t = 0$.

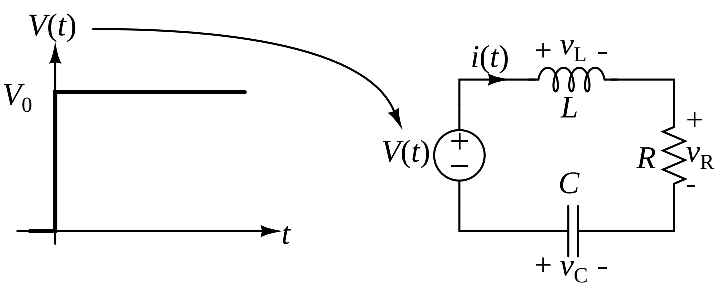
(B) Argue that $i = 0$ at $t = \infty$ so that $i(t)$ has no constant component.

(C) Find a second-order differential equation which describes the behavior of $i(t)$ for $t \geq 0$.

(D) Following (B) the current $i(t)$ takes the form $i(t) = I \sin(\omega t + \phi)e^{-\alpha t}$. Find I, ω, ϕ and α. Hint: first find ω and α from the differential equation, and then find I and ϕ from the initial conditions; alternatively, solve this problem by any method you wish.

(E) Suppose that the input is a voltage impulse with area $\Lambda_\circ$ where $\Lambda_\circ = \tau V_\circ$, $V_\circ$ is the amplitude of the voltage step shown below, and τ is a given time constant. Find the response of the network shown below to the impulse. Hint: before solving this problem directly, consider the relation between step and impulse responses.

Save a copy of your answers to this problem. They will be useful during the pre-lab exercises for Lab #3.



(A) We know that for short lengths of time, an inductor looks like an open circuit and a capacitor looks like a short circuit. Since there is an open circuit, there is no current flowing, and $i(0) = 0$. Also, since the capacitor looks like a short, $v_C(0) = 0$. Since there is no current flowing in the circuit, $v_R(0) = 0$. That means that $v_L(0) = V_o$ in order to satisfy KVL. We know that $v_L = L\frac{di}{dt}$; therefore, $\frac{di}{dt} = \frac{V_o}{L}$ at $t = 0$.

In summary, the following equations are true at $t = 0$:

$$
\begin{aligned}
v_C(0) &= 0 \\
v_L(0) &= V_o \\
v_R(0) &= 0 \\
i(0) &= 0 \\
\frac{di}{dt}(0) &= \frac{V_o}{L}
\end{aligned}
$$

(B) At $t = \infty$, the inductor will look like a short circuit and the capacitor will look like an open circuit. Because there is an open circuit in the loop, there can again be no current flow. Therefore, $i = 0$ at $t = \infty$, and $i(t)$ has no constant component.

(C) We know that $i = C\frac{dv_C}{dt}$ and $v_L = L\frac{di_L}{dt}$. Also we can write a KVL equation around the loop: $V_o = v_L + v_R + v_C$. Using all of this knowledge, we can write the following equation:

$$V_o - v_R - v_C = L\frac{di}{dt}$$

Now let's express v_C and v_R in terms of i and substitute:

$$V_o - iR - \frac{1}{C}\int idt = L\frac{di}{dt}$$

Differentiating both sides with respect to t and regrouping gives us the answer:

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

(D) If we assume a solution to the differential equation of the form Ae^{st}, and plug that into the the equation, a quadratic equation in terms of s appears:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Solving gives our roots for s

$$s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Examining these roots tells us that $\alpha = \frac{R}{2L}$, $\omega_o = \sqrt{\frac{1}{LC}}$, and $\omega = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$. Because the solution is a sinusoid, we know that $\alpha < \omega_o$.

So let's rewrite $i(t)$ given what we know to this point:

$$i(t) = I\sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t + \phi\right)e^{-\frac{R}{2L}t}$$

Now we need to use the initial conditions for $i(t)$ and $\frac{di}{dt}$ that we calculated in Part (A): $i(0) = 0$ and $\frac{di}{dt} = \frac{V_o}{L}$ at $t = 0$. Substituting $t = 0$ into the equation for $i(t)$ tells us that $0 = I\sin\phi$, which means that $\phi = 0$. Let's rewrite $i(t)$ one more time.

$$i(t) = I\sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t\right)e^{-\frac{R}{2L}t}$$

$$\frac{di}{dt} = -\frac{RI}{2L}\sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t\right)e^{-\frac{R}{2L}t} + Ie^{-\frac{R}{2L}t}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t\right)$$

At $t = 0$, $\frac{di}{dt} = \frac{V_o}{L}$, so $I = \frac{V_o}{L\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$

Combining all of this information:

$$i(t) = I \sin(\omega t + \phi) e^{-\alpha t}$$

where

$$\begin{aligned} I &= \frac{V_o}{L\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \\ \omega &= \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \\ \phi &= 0 \\ \alpha &= \frac{R}{2L} \end{aligned}$$

(E) We have just solved the step response for this system. Since our input was V_o times the unit step, if we divide our output by V_o, we would have the unit step response.

The unit impulse response is simply the derivative of the unit step response. However, our impulse has area $\Lambda_\circ = \tau V_o$. Therefore the response for this impulse would be $\Lambda_\circ$ times the unit impulse.

In summary, the response to an impulse of area $\Lambda_\circ$ is the derivative of the response to a step of V_o multiplied by $\frac{\Lambda_\circ}{V_o} = \tau$.

Our answer:

$$i(t) = -\frac{\tau R V_o}{2L^2}\left(\frac{1}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}\right) \sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t\right) e^{-\frac{R}{2L}t} + \frac{\tau V_o}{L} \cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\quad t\right) e^{-\frac{R}{2L}t}$$