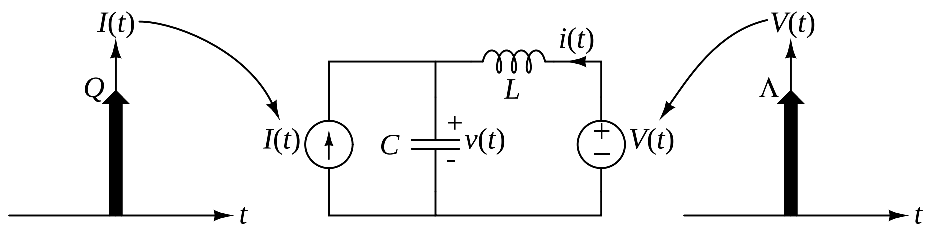
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Spring 2002

Homework #9

Issued 4/10/02 – Due 4/17/02

Exercise 9.1: Determine $i(t)$ and $v(t)$ for $t \geq 0^+$ in the network shown below. Note that the inductor and capacitor both have nonzero states at $t = 0$. Hint: use superposition to establish initial conditions at $t = 0^+$.



Problem 9.1: The network shown below is driven in steady state by the sinusoidal input voltage $v_I(t) = V_I \cos(\omega t)$. The output of the network is the voltage $v_O(t)$, which takes the form $v_O(t) = V_O \cos(\omega t + \phi)$. Find V_O and ϕ as functions of ω as follows.

(A) Using the Taylor Series expansions for e^x, $\cos(x)$ and $\sin(x)$, show that $e^{jx} = \cos(x) + j\sin(x)$. Following this, recognize that $\cos(x) = \Re\{e^{jx}\}$.

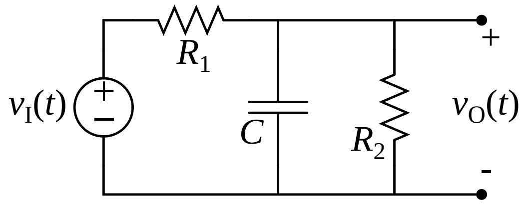
(B) Show that $A + Bj = \sqrt{A^2 + B^2} e^{j \arctan(B/A)}$. Thus, the magnitude and phase of $A + Bj$ are $\sqrt{A^2 + B^2}$ and $\arctan(B/A)$, respectively.

(C) Find a differential equation that can be solved for $v_O(t)$ given $v_I(t)$.

(D) Following Part A, let $v_I(t) = V_I e^{j\omega t}$. Also, let $v_O(t) = \hat{V}_O e^{j\omega t}$ where $\hat{V}_O$ is a complex function of the circuit parameters, ω and V_I. With these substitutions, use the differential equation to find $\hat{V}_O$.

(E) Following Parts A and B, first express v_O from Part (D) in the form $v_O(t) = |\hat{V}_O| e^{j(\omega t + \angle \hat{V}_O)}$, and determine $|\hat{V}_O|$ and $\angle \hat{V}_O$ as functions of the circuit parameters, ω and V_I. Then, find V_O and ϕ for the original cosine input, again both as functions of the circut parameters, ω and V_I.

(F) Sketch and clearly label V_O/V_I and ϕ as functions of ω. Identify the low-frequency and high-frequency asymptotes on the sketch.



Problem 9.2: This problem concerns the sinusoidal-steady-state behavior of the networks shown below, both of which have two ports.

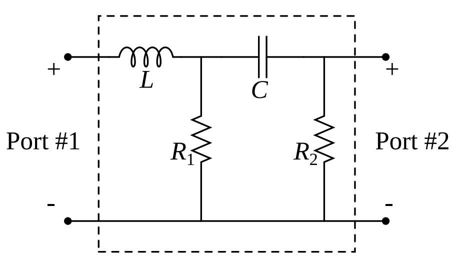
(A) Determine the impedance of each network as viewed into Port #1 under the assumption that Port #2 is open.

(B) Assume that Port #1 of each network is driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is open. Determine the current into the positive terminal of each network at Port #1. Express the current in the form $I_1 \cos(\omega t + \phi_1)$ where I_1 is an amplitude and ϕ_1 is a phase angle.

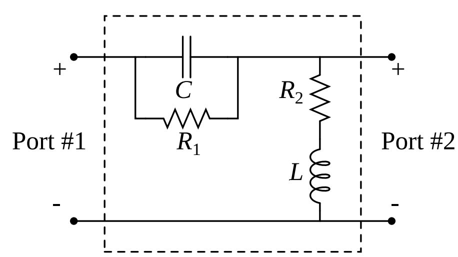
(C) Assume that Port #1 of each network is again driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is again open. Determine the voltage which appears at Port #2. Express the voltage in the form $V_2 \cos(\omega t + \phi_2)$ where V_2 is an amplitude and ϕ_2 is a phase angle.

Note that the results of this problem are useful when completing the pre-lab exercises to Lab #3.

Network (A)



Network (B)



Problem 9.3: Using a 1-mH inductor and two resistors, design a two-port network that has the following input-output relation in the sinusoidal steady state. Note that the relation is defined for the case of an unloaded, or open-circuited, output port.