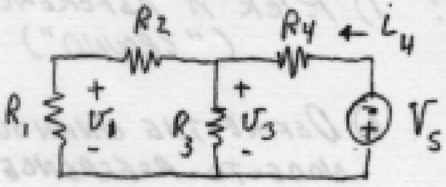
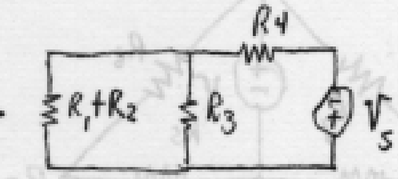
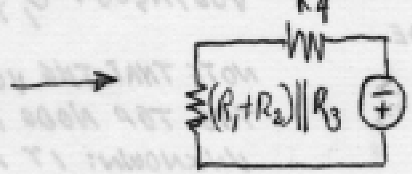
NOTES FOR 6.002 LECTURE #3 · FEBRUARY 11, 2003

CIRCUIT ANALYSIS: ANALYSIS OF A CIRCUITS MEANS DETERMINING THE COMPLETE DISTRIBUTION OF VOLTAGES AND CURRENTS.

IN MANY RESISTIVE CIRCUITS SERIES-PARALLEL REDUCTIONS SUFFICE. FOR EXAMPLE, DETERMINE v_1, v_3, i_4





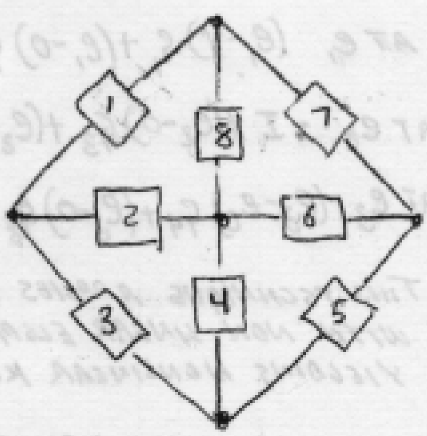


WITH THIS REDUCED CIRCUIT i_4 CAN BE WRITTEN AS: $i_4 = -\dfrac{V_S}{R_4 + (R_1+R_2)\|R_3}$

v_3 MUST THEN BE $v_3 = -V_S - i_4 R_4$ (KVL)

AND $v_1 = v_3 \times \dfrac{R_1}{R_1+R_2}$

THIS TECHNIQUE FAILS WHEN THE CIRCUIT IS NOT "LADDER-LIKE."



REPRESENTS ANY TWO-TERMINAL LUMPED CIRCUIT ELEMENT, I.E., A RESISTOR.

THE EIGHT ELEMENTS IN THIS CIRCUIT REPRESENT 16 ELECTRICAL VARIABLES: 8 CURRENTS AND 8 VOLTAGES.

THE CIRCUIT HAS 5 NODES ON ANY 4 OF WHICH KCL STATEMENTS CAN BE WRITTEN. THE KCL STATEMENT FOR THE 5TH NODE IS DERIVABLE FROM THE OTHERS AND IS NOT INDEPENDENT.

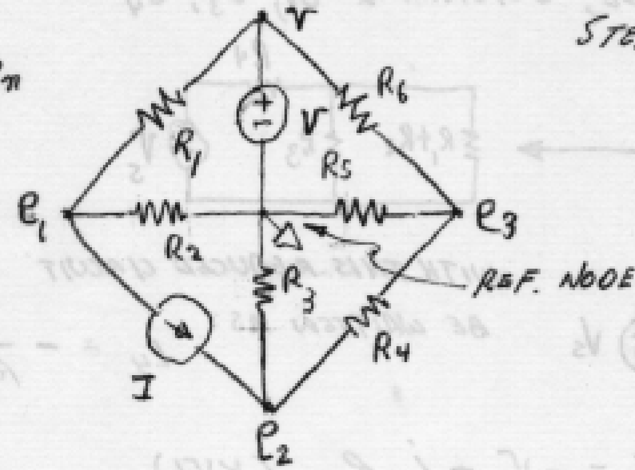
THE CIRCUIT HAS FOUR SIMPLE LOOPS OR MESHES (THINK FISHNET). THERE ARE MANY OTHER MORE COMPLEX CLOSED LOOPS, NONE OF WHICH ARE INDEPENDENT.

EACH ELEMENT HAS A CONSTITUENT RELATIONSHIP. THAT IS $i_n = f(v_n)$ OR $v_n = g(i_n)$.

SUMMING UP, 4 KVL EQUATIONS PLUS 4 KCL EQUATIONS PLUS 8 CONSTITUENT RELATIONSHIPS AMOUNTS TO 16 EQUATIONS, WHICH ARE SUFFICIENT TO SOLVE FOR THE UNKNOWN CURRENTS AND VOLTAGES, OF WHICH THERE ARE 16
Q.E.D.

A BETTER APPROACH TO SYSTEMATIC CIRCUIT ANALYSIS: EMPLOY NODE-TO-REFERENCE VOLTAGES

$G_n = 1/R_n$



STEPS: 1) PICK A REFERENCE NODE ("GROUND")

2) DEFINE THE UNKNOWN NODE-TO-REFERENCE VOLTAGES: e_1, e_2, e_3

NOTE THAT THE VOLTAGE AT THE TOP NODE IS NOT UNKNOWN: IT IS CONSTRAINED BY THE VOLTAGE SOURCE BECAUSE OF MY CHOICE OF REFERENCE NODE.

3) WRITE KCL STATEMENTS AT THE NODES WHERE e_1, e_2, e_3 ARE DEFINED USING CONDUCTANCES.

SUMMING CURRENTS OUT YIELDS:

AT e_1 $\quad (e_1 - V)G_1 + (e_1 - 0)G_2 + I = 0$

AT e_2 $\quad -I + (e_2 - 0)G_3 + (e_2 - e_3)G_4 = 0$

AT e_3 $\quad (e_3 - e_2)G_4 + (e_3 - 0)G_5 + (e_3 - V)G_6 = 0$

THESE THREE KCL EQUATIONS CAN BE SOLVED FOR THE THREE NODE-TO-REFERENCE VOLTAGES. THE CURRENTS ARE OBTAINED FROM THE VOLTAGE DIFFERENCES AND OHMS LAW.

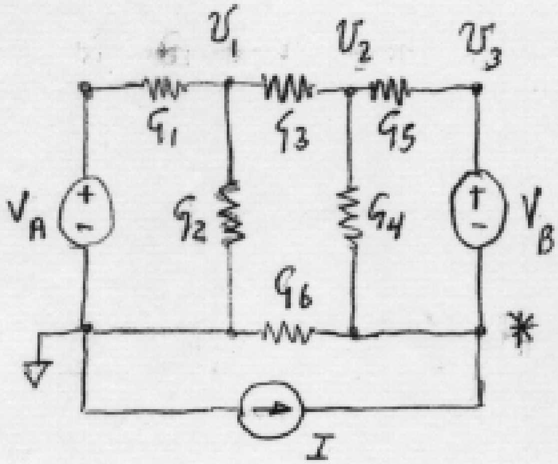
THIS TECHNIQUE APPLIES TO CIRCUITS WITH NON-LINEAR ELEMENTS AS WELL, YIELDING NONLINEAR KCL STATEMENTS

A NOTE ON CHOICES: ADD TOGETHER THE THREE KCL STATEMENTS: (SLIGHTLY REARRANGED)

$$\underbrace{\underbrace{\underbrace{(e_1 - V)G_1}_{\text{CURRENT UP IN } R_1} + \underbrace{(e_3 - V)G_6}_{\text{CURRENT UP IN } R_6}}_{\text{CURRENT DOWN IN } V} + \underbrace{(e_1 - 0)G_2}_{\text{CURRENT TO RIGHT IN } R_2} + \underbrace{(e_2 - 0)G_3}_{\text{CURRENT UP IN } R_3} + \underbrace{(e_3 - 0)G_5}_{\text{CURRENT TO LEFT IN } R_5}}_{\text{CURRENTS INTO THE CENTRAL NODE}} = 0$$

KCL, OF COURSE, APPLIES THERE TOO!

SECOND EXAMPLE



1) HOW MANY NODES?

2) HOW MANY INDEPENDENT NODES?

3) WHERE TO PUT THE REFERENCE NODE?

4) HOW MANY NODE-TO-REFERENCE VOLTAGES?

THE LABELING SHOWS ONE POSSIBLE CHOICE - THERE ARE FIVE MORE.

NODE EQUATIONS (SUMMING CURRENTS OUT OF THE NODES)

AT v_1: $(v_1 - V_A)G_1 + v_1 G_2 + (v_1 - v_2)G_3 = 0$

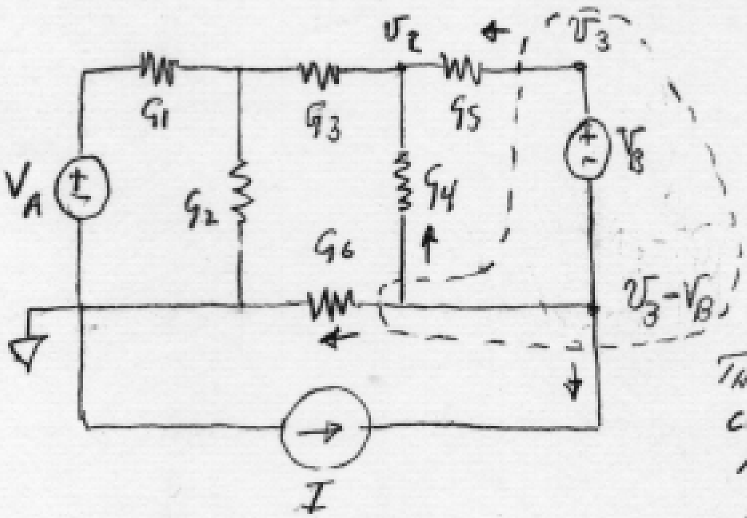
AT v_2: $(v_2 - v_1)G_3 + (v_2 - v_3)G_5 + v_2 - \underbrace{(v_3 - V_B)}_{\text{VOLTAGE AT THE NODE MARKED *}} G_4 = 0$

AT v_3: $(v_3 - v_2)G_5 + \underbrace{[(v_3 - V_B) - 0]G_6 - I + [(v_3 - V_B) - v_2]G_4}_{\text{COMPONENTS OF CURRENT FLOWING OUT OF *,}} = 0$

WHICH ADDED TOGETHER EQUAL THE CURRENT DOWN THROUGH V_B.

THIS "REACH" TO THE NEXT NODE (*) IS NECESSARY BECAUSE THE CURRENT OUT THROUGH V_B CANNOT BE SPECIFIED A PRIORI; IT IS KNOWN ONLY WHEN THE ANALYSIS IS FINISHED

ANOTHER WAY OF THINKING ABOUT CURRENT COMPONENTS AT v_3 IS TO LINK THE NODES * AND AT WHICH v_3 IS DEFINED



SOMETIMES CALLED A "SUPER NODE"

THE FOUR COMPONENTS OF CURRENT OUT OF THE NODE AT V_B ARE MARKED WITH ARROWS.