S03-014 6.002

HW 1 Solutions

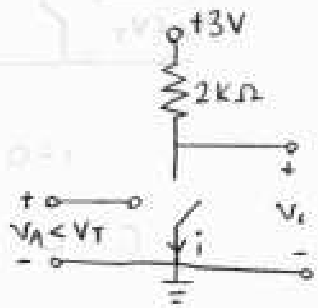
1.1

(i) inverter

1)

A	C
0	1
1	0

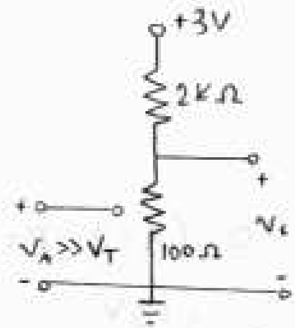
2) $C = 1$ $(A = 0)$:



$i = 0$

$\Rightarrow V_c = \boxed{3V}$

$C = 0$ $(A = 1)$:



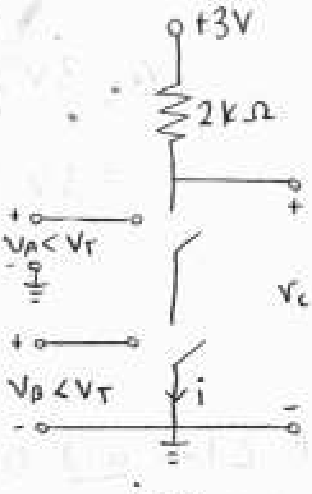
$$V_c = 3V \frac{100\Omega}{2100\Omega} = \boxed{\tfrac{1}{7}V}$$

(ii) nand gate

1)

A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

2) $C = 1$ $(A = B = 0)$:

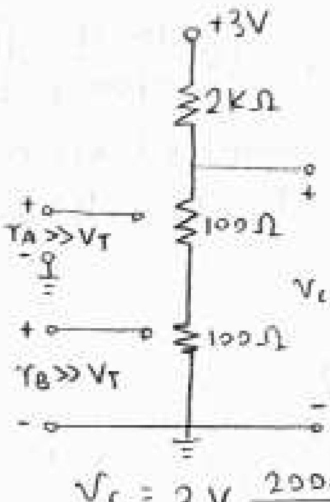


$i = 0$

$\Rightarrow V_c = \boxed{3V}$

same situation with $A = 1$ or $B = 1$ (but not both)

$C = 0$ $(A = B = 1)$:



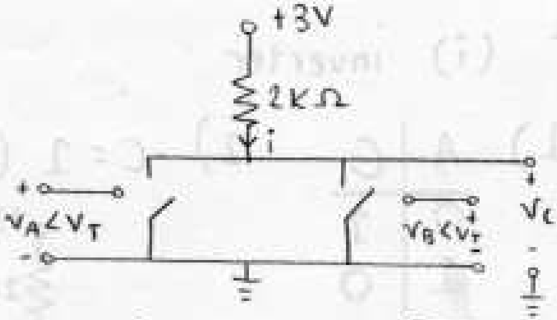
$$V_c = 3V \frac{200\Omega}{2200\Omega} = \boxed{\tfrac{3}{11} V}$$

(iii) nor gate

1)

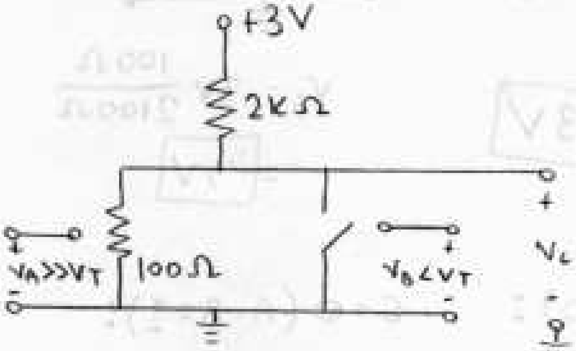
A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

2) $C = 1$ $(A = B = 0)$:



$i = 0 \Rightarrow V_C = \boxed{3V}$

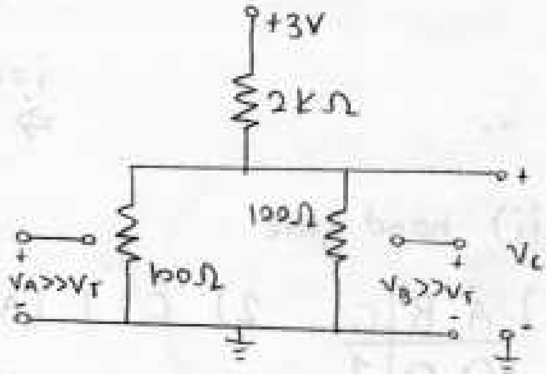
$C = 0$ $(A = 1, B = 0)$:



$V_C = 3V \frac{100\Omega}{2100\Omega} = \boxed{\tfrac{1}{7} V}$

same situation for $A = 0, B = 1$

$C = 0$ $(A = B = 1)$:



$V_C = 3V \frac{100 \| 100\Omega}{2k\Omega + 100 \| 100\Omega}$

$= 3V \frac{50\Omega}{2050\Omega} = \boxed{\frac{3}{41} V}$

1.2

1) Logic: X is high if C low <u>and</u> D high <u>and</u> (A low <u>or</u> B high):

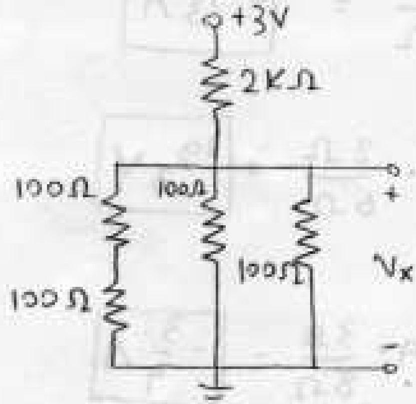
$X = \bar{C} \wedge D \wedge (\bar{A} \vee B)$

$= \overline{C \vee \bar{D}} \wedge \overline{A \wedge \bar{B}}$

A	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
B	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
C	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
D	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\bar{A} \vee B$	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
$\bar{C} \wedge D$	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
X	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0

2) Lowest value:

$A = \bar{B} = C = \bar{D} = 1$



$$V_x = \frac{200 \| 100 \| 100\ \Omega}{2k\Omega + 200\|100\|100\ \Omega} \cdot 3V$$

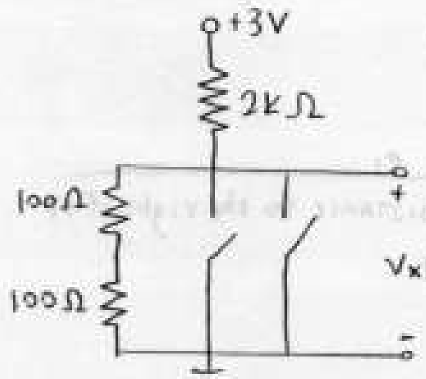
$$= \frac{200 \| 50\ \Omega}{2k\Omega + 200\|50\ \Omega} \cdot 3V$$

$$= \frac{40\Omega}{2040\Omega} \cdot 3V = 1/_{17}\ V$$

Highest value:

$A = \bar{B} = 1$

$C = \bar{D} = 0$



$$V_x = \frac{200\Omega}{2200\Omega} \cdot 3V$$

$$= 3/_{11}\ V$$

$$\boxed{1/_{17}\ V \le V_x \le 3/_{11}\ V}$$

3) Power: $P = IV = \frac{V^2}{R}$

V: voltage across circuit
I: total current thru circuit
R: total resistance of circuit

$V = +3V$

$\Rightarrow$ minimum R produces max. power

from (2) minimum R is $2040\,\Omega$. $(A = \bar{B} = C = \bar{D} = 1)$

$$\Rightarrow P = \frac{(3V)^2}{2040\,\Omega} = \boxed{3/680\,W} \approx 4.4\,mW$$

1.3

(i)

$$i_Y = -\frac{10V}{4\Omega + 3\|6\Omega} = \frac{-10V}{4\Omega + 2\Omega} = \boxed{-5/3\,A}$$

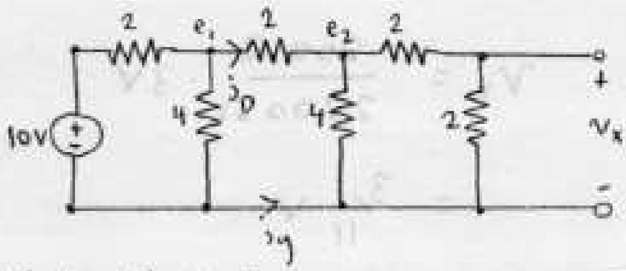
$$v_x = 10V \cdot \frac{3\|6\,\Omega}{4\Omega + 3\|6\,\Omega} = 10V \cdot \frac{2\Omega}{6\Omega} = \boxed{10/3\,V}$$

(ii)

$$i_Y = -2A \cdot \frac{1\Omega + 2\Omega}{5\Omega + 1\Omega + 2\Omega} = -2A \cdot \frac{3\Omega}{8\Omega} = \boxed{-\frac{3}{4}A}$$

$$v_x = 2\Omega \cdot (-2A - i_Y) = (2\Omega)(-5/4\,A) = \boxed{-5/2\,V}$$

(iii)



Note that $i_y = -i_D$. $i_D = \frac{e_1}{\text{resistance to the right of } e_1}$

$e_1 = 10V \frac{\text{resistance to the right of } e_1}{\text{total resistance}}$

total resistance $R_{TOT} = 2 + (4 \| (2 + (4 \| (2+2))))$

$= 2 + (4 \| (2 + (4 \| 4)))$

$= 2 + (4 \| (2+2))$

$= 2 + 2 = 4\Omega$

resistance right of e_1 $\quad R_{e_1} = 2 + 4 \| (2+2) = 2+2 = 4\Omega$

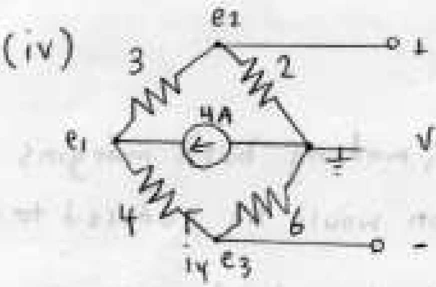
$e_1 = 10V \dfrac{R_{e_1}}{R_{TOT}} = 10V \dfrac{2\Omega}{4\Omega} = 5V$

$j_y = -j_0 = -\dfrac{e_1}{R_{e_1}} = -\dfrac{5V}{4\Omega} = \boxed{-1.25A}$

$v_x = e_2 \dfrac{2\Omega}{4\Omega} = \frac{1}{2} e_2$. $\quad e_2 = e_1 \dfrac{4 \| (2+2)\Omega}{2 + 4\|(2+2)\Omega} = e_1 \dfrac{2\Omega}{4\Omega} = \frac{1}{2} e_1$

$v_x = \frac{1}{2} e_2 = \frac{1}{4} e_1 = \boxed{\frac{5}{4} V}$

(iv)



First find voltages e_1, e_2, e_3

$e_1 = (4A) R_{TOT}$

$= (4A)\left[(3+2) \| (4+6)\Omega\right]$

$= (4A)(5 \| 10\Omega)$

$= 4A \cdot {}^{50}/_{15}\Omega = {}^{40}/_{3} V$

$e_2 = e_1 \cdot \dfrac{2\Omega}{(2+3)\Omega} = \dfrac{2}{5} \cdot \dfrac{40}{3} V = \dfrac{16}{3} V$

$e_3 = e_1 \cdot \dfrac{6\Omega}{(4+6)\Omega} = \dfrac{6}{10} \cdot \dfrac{40}{3} V = 8V$

$v_x = e_2 - e_3 = \dfrac{16}{3} V - 8V = \boxed{-\frac{8}{3} V}$

$j_Y = -\dfrac{e_1}{(4+6)\Omega} = -\dfrac{40}{30} A = \boxed{-\frac{4}{3} A}$

1.4 1) If $v_I < v_{IL}$, $v_o = +6V$ (switch open)

no dependence on R

If $v_I > v_{IH}$, $V_o = +6V \cdot \frac{R_{ON}}{R_{ON} + R}$ (switch closed)

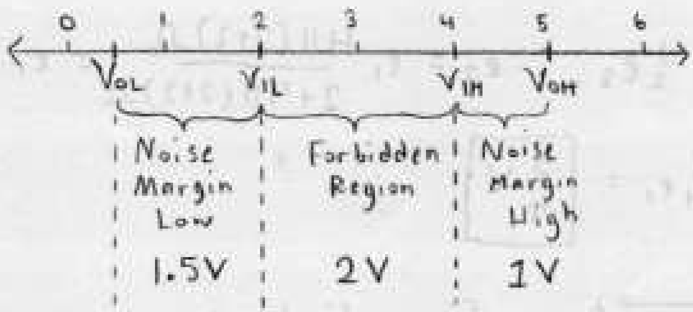
need $V_o < V_{OL}$: $6V \cdot \frac{200}{200+R} < 0.5V$

$1200 < 0.5(200+R)$

$1100 < .5R$ $\boxed{R > 2200}$

2)

3)



4) - Could reduce V_{IH} to 3.5V, making both margins 1.5V wide. Forbidden region would be reduced to 1.5V

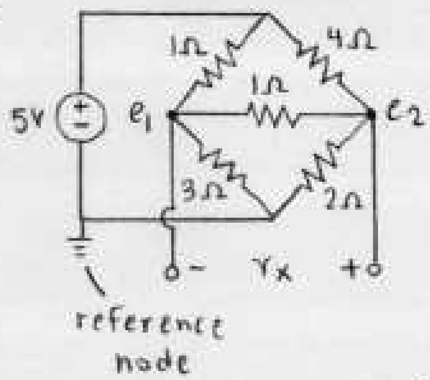
- Could reduce V_{IL} to 1.5V, making both margins 1V wide. Forbidden region would be enlarged to 2.5V

- Could raise V_{OH} to 5.5V, making both margins 1.5V wide. Forbidden region would be unchanged

- Could raise V_{OL} to 1V, making both margins 1V wide. Forbidden region unchanged.

- Other possibilities as well

1.5



Nodes e_1, e_2

KCL at e_1:

$$\frac{5V-e_1}{1\Omega}+\frac{e_2-e_1}{1\Omega}+\frac{-e_1}{3\Omega}=0$$

KCL at e_2:

$$\frac{5V-e_2}{4\Omega}+\frac{e_1-e_2}{1\Omega}+\frac{-e_2}{2\Omega}=0$$

$$5-e_1+e_2-e_1-\frac{e_1}{3}=0$$

$$5-\frac{7}{3}e_1+e_2=0$$

$$e_2=\frac{7}{3}e_1-5$$

substitute

$$\frac{5}{4}-\frac{e_2}{4}+e_1-e_2-\frac{e_2}{2}=0$$

$$\frac{5}{4}-\frac{7}{4}e_2+e_1=0$$

$$\frac{5}{4}-\frac{49}{12}e_1+\frac{35}{4}+e_1=0$$

$$10=\frac{37}{12}e_1 \quad e_1=\frac{120}{37}V$$

$$e_2=\frac{7}{3}\cdot\frac{120}{37}-5=95/37\,V$$

$$v_x=e_2-e_1=\boxed{-25/37\,V}$$

1.6 KCL at e_1: $I_1+(-G_1e_1)+G_2(e_2-e_1)=0$

$$e_1(-G_1-G_2)+e_2G_2=-I_1 \quad ①$$

KCL at e_2: $G_2(e_1-e_2)+G_3(V_1-e_2)+G_2(e_3-e_2)=0$

$$e_1G_2+e_2(-G_3-2G_2)+e_3G_2=-G_3V_1 \quad ②$$

KCL at e_3: $-I_2+(-G_4e_3)+G_2(e_2-e_3)=0$

$$e_2G_2+e_3(-G_2-G_4)=I_2 \quad ③$$

$$\begin{cases} ① & (-G_1-G_2)e_1+(G_2)e_2+ \qquad = -I_1 \\ ② & (G_2)e_1+(-G_3-2G_2)e_2+(G_2)e_3=-G_3V_1 \\ ③ & (G_2)e_2+(-G_2-G_4)e_3=I_2 \end{cases}$$