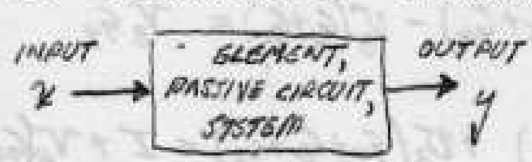
NOTES FOR 6.002 LECTURE #4 FEBRUARY 13, 2003

P.E. GRAY'S OFFICE HOURS: EVERY WEDNESDAY 3-5 PM IN 38-344
OR BY ARRANGEMENT (3-4665)

READINGS THIS WEEK: SECTIONS 3.1 - 3.3, 3.5

NOW, FOLLOWING TOOLS FOR ANALYSIS OF LUMPED ELECTRIC CIRCUITS, TAKE UP TWO THEOREMS APPLICABLE TO NETWORKS COMPRISED OF LINEAR ELEMENTS.

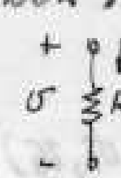
STRICT DEFINITION OF LINEARITY:



IN	OUT
x_1 →	y_1
x_2 →	y_2

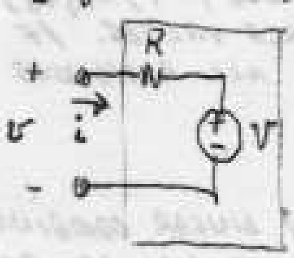
THE BOX IS LINEAR ⟶ IF: $ax_1 + bx_2 \rightarrow ay_1 + by_2$

WHERE a, b ARE CONSTANTS



$v = iR$

OBVIOUSLY LINEAR



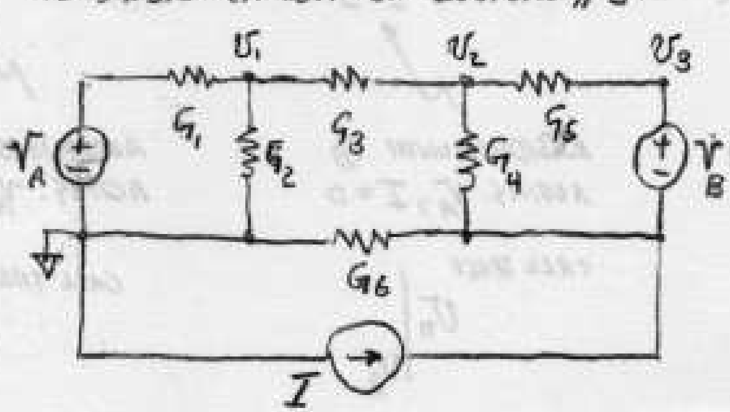
$v = iR + V$ NOT LINEAR (INNARDS NOT PASSIVE - AN INDEPENDENT SOURCE IS PRESENT

WE WILL SOON INTRODUCE OTHER LINEAR ELEMENTS: INDUCTORS, CAPACITORS, DEPENDENT SOURCES.

DEVELOP THE SUPERPOSITION PRINCIPLE, FORMALLY A THEOREM

WHAT FOLLOWS IS A PLAUSIBILITY ARGUMENT

CONSIDER CIRCUIT OF LECTURE #3:



THE 3 KCL EQUATIONS ARE:

$$(v_1 - V_A)G_1 + v_1 G_2 + (v_1 - v_2)G_3 = 0$$

$$(v_2 - v_1)G_3 + (v_2 - v_3)G_5 + [v_2 - (v_3 - V_B)]G_4 = 0$$

$$(v_3 - v_2)G_5 + (v_3 - V_B)G_6 - I + [(v_3 - V_B) - v_2]G_4 = 0$$

REWRITTEN, COLLECTING TERMS:

$$v_1(G_1 + G_2 + G_3) - v_2(G_3) = V_A G_1$$

$$-v_1(G_3) + v_2(G_3 + G_5 + G_4) - v_3(G_4 + G_5) = V_B G_4$$

$$-v_2(G_4 + G_6) + v_3(G_4 + G_5 + G_6) = I + V_B(G_4 + G_6)$$

NOTE SYMMETRY ACROSS THE DIAGONAL AND NATURE OF COEFFICIENTS.

NOTE ALSO THAT THE NETWORK RESPONSES (v_1, v_2, v_3) ARE LINEARLY RELATED TO THE THREE SOURCE VALUES. IF ALL SOURCE VALUES ARE MULTIPLIED BY k, ALL RESPONSES ARE k TIME AS BIG.

BECAUSE THE SOURCES APPEAR ONLY IN LINEAR COMBINATIONS (AT MOST) THE RESPONSES FOR EACH SOURCE ACTING ALONE ARE READILY DETERMINED. THREE SINGLE-SOURCE EQUATION SETS:

①

$$\begin{bmatrix} \text{CONDUCTANCE} \\ \text{MATRIX} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} V_A G_1 \\ 0 \\ 0 \end{bmatrix}$$

②

$$\begin{bmatrix} \text{DITTO} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ V_B G_4 \\ V_B(G_4+G_6) \end{bmatrix}$$

③

$$\begin{bmatrix} \text{DITTO} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

RESPONSES WITH V_A ACTING. $V_B, I = 0$

CALL THESE $v_n\big|_{V_A}$

RESPONSES WITH V_B ACTING. $V_A, I = 0$

CALL THESE $v_n\big|_{V_B}$

RESPONSES WITH I ACTING. $V_A, V_B = 0$

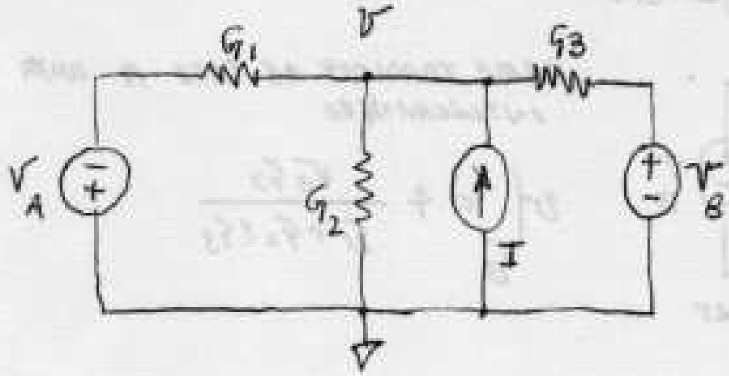
CALL THESE $v_n\big|_{I}$

CLEARLY THE COMPLETE RESPONSES ARE: $v_n = v_n\Big|_{V_A} + v_n\Big|_{V_B} + v_n\Big|_I$

I.E. THE SUM OF THE COMPONENT RESPONSES

SUPERPOSITION PRINCIPLE: THE RESPONSE OF A NETWORK COMPRISED OF LINEAR ELEMENTS WHICH CONTAINS n INDEPENDENT SOURCES IS EQUAL TO THE RESPONSE OF n DIFFERENT NETWORKS EACH OF WHICH HAS ONLY ONE SOURCE ACTING, WITH THE OTHER SOURCES SHUT OFF.

CONSIDER AN ALGEBRAICALLY-SIMPLER EXAMPLE:



KCL AT v: $(v + V_A)G_1 + vG_2 + (v - V_B)G_3 - I = 0$

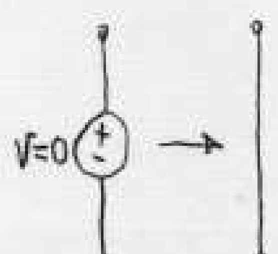
$$(G_1 + G_2 + G_3)\,v = -V_A G_1 + V_B G_3 + I$$

$$v = \frac{-V_A G_1 + V_B G_3 + I}{G_1 + G_2 + G_3} \quad \text{THE NETWORK RESPONSE}$$

NOW CONSIDER THE NETWORK UNDER THREE CONDITIONS, EACH WITH ONE SOURCE ACTING, THE OTHER TWO TURNED OFF.

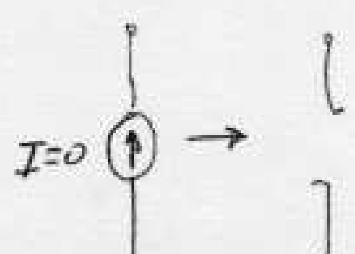
WHAT DOES IT MEAN TO TURN A SOURCE OFF?

VOLTAGE SOURCE



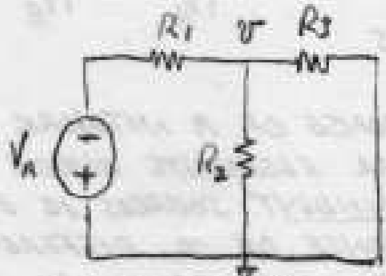
A SHORT CIRCUIT

CURRENT SOURCE



AN OPEN CIRCUIT

CASE A: V_A ACTIVE, V_B, I OFF $(R_n = 1/G_n)$



I REMOVED, V_B REPLACED BY A SHORT

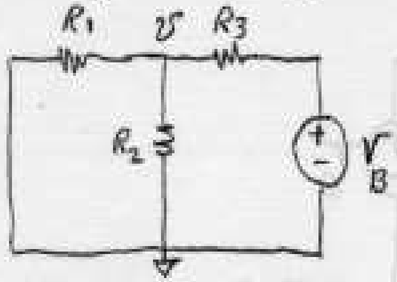
BY INSPECTION: $v = -V_A \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} = -V_A \frac{\frac{R_2 R_3}{R_2 + R_3}}{\frac{R_2 R_3}{R_2 + R_3} + R_1}$

$$v\Big|_{V_A} = -V_A \frac{R_2 R_3}{R_2 R_3 + R_1 R_2 + R_1 R_3}$$

MULTIPLY NUMERATOR AND DENOMINATOR BY $G_1 G_2 G_3$

$$= \frac{-V_A G_1}{G_1 + G_3 + G_2}$$

CASE B: V_B ACTIVE, V_A, I OFF



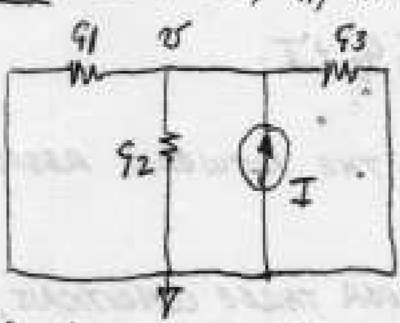
V_A REPLACED BY A SHORT
I REMOVED

SAME TOPOLOGY AS CASE A WITH R_1 & R_3 INTERCHANGED

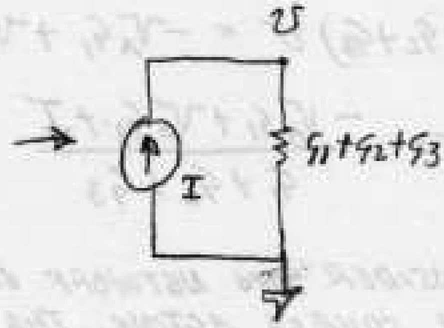
$$v\Big|_{V_B} = + \frac{V_B G_3}{G_1 + G_2 + G_3}$$

CASE C: I ACTIVE, V_A, V_B OFF

THREE CONDUCTANCES IN PARALLEL



V_A, V_B REPLACED BY SHORTS



$$v\Big|_I = \frac{I}{G_1 + G_2 + G_3}$$

THE THREE COMPONENTS ADDED TOGETHER GIVE THE COMPLETE RESPONSE