

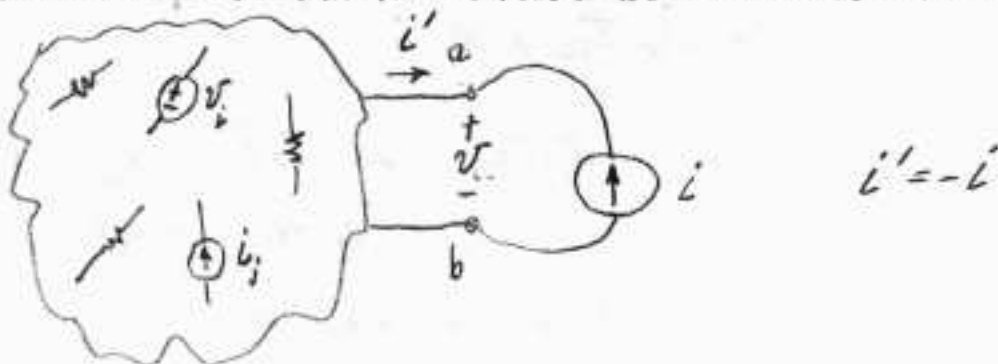
NOTES FOR 6.002 LECTURE #5 FEBRUARY 20, 2003

P.E. GRAY'S OFFICE HOURS: EVERY WEDNESDAY 3-5 (38-344)  
OR BY ARRANGEMENT

READING THIS WEEK: 3.6

IN A CIRCUIT COMPRISED OF LINEAR ELEMENTS AND INDEPENDENT SOURCES, SUPERPOSITION APPLIES. THE COMPLETE DISTRIBUTION OF VOLTAGES AND CURRENTS CAN BE FOUND BY ADDING COMPONENT DISTRIBUTIONS FOR EACH OF WHICH ONLY ONE SOURCE IS ACTING.

CONSIDER A NETWORK WITH  $n$  INDEPENDENT VOLTAGE SOURCES AND  $m$  INDEPENDENT CURRENT SOURCES: PULL OUT A PAIR OF TERMINALS. HOW CAN THE NETWORK BE CHARACTERIZED AT THE TERMINALS?



① THE TERMINALS REPRESENT ANY TWO NODES EXCEPT THOSE ACROSS A VOLTAGE SOURCE

② CONNECT A CURRENT SOURCE  $i$  AT THE TERMINALS

USING THE SUPERPOSITION PRINCIPLE AN EXPRESSION FOR  $v_{ab}$  IS:

$$v = \underbrace{\sum_n A_n v_n + \sum_m R_m i_m}_{\text{COMPONENT OF } v \text{ CAUSED BY ALL INTERNAL SOURCES}} + \underbrace{R_T i}_{\text{COMPONENT OF } v \text{ CAUSED BY SOURCE } i}$$

$A_n$  ARE DIMENSIONLESS CONSTANTS

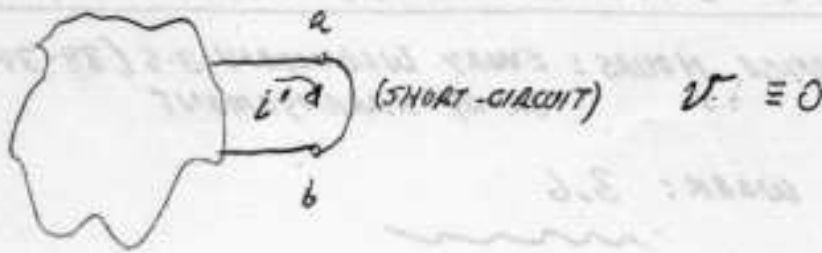
COMPONENT OF  $v$  CAUSED BY ALL INTERNAL SOURCES

COMPONENT OF  $v$  CAUSED BY SOURCE  $i$

$R_m, R_T$  ARE CONSTANTS WITH DIMENSION OF RESISTANCE

③ NOW ADJUST  $i$  SO THAT  $v \equiv 0$ . UNDER THIS CONDITION

$i' (= -i)$  CAN BE INTERPRETED AS THE SHORT-CIRCUIT CURRENT AT THE TERMINALS  $a, b$ .



FROM THE EQUATION FOR  $V$ , WITH  $V = 0$   $i' = - \frac{\sum A_n V_n + \sum R_m i_m}{R_T}$

AND  $i' = -i$  IS  $i' = + \frac{\sum A_n V_n + \sum R_m i_m}{R_T}$

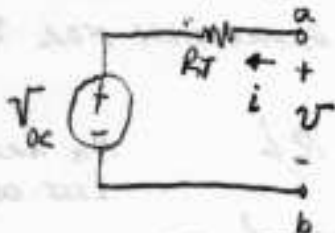
CALL THIS THE SHORT-CIRCUIT CURRENT  $I_{sc}$

THEN  $V = R_T I_{sc} + R_T i$

WHEN  $i = 0$   $V = R_T I_{sc}$  LET  $R_T I_{sc} = V_{oc}$  THE OPEN-CIRCUIT VOLTAGE AT  $a, b$

THE LINEAR EQUATION CAN BE WRITTEN AS  $V = V_{oc} + R_T i$  WHERE  $V_{oc}$  IS THE OPEN-CIRCUIT VOLTAGE AND  $R_T$  IS THE EQUIVALENT RESISTANCE SEEN AT  $a, b$  WITH ALL INTERNAL SOURCES TURNED OFF

THE EQUATION CAN BE REPRESENTED BY A CIRCUIT:



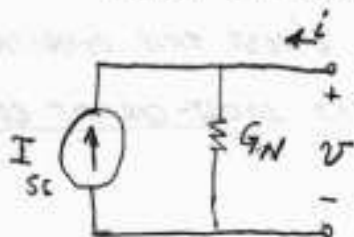
THIS IS A THEVENIN EQUIVALENT CIRCUIT

IT IS A COMPLETE MODEL OF THE ORIGINAL CIRCUIT ONLY AS SEEN AT THE TERMINALS a, b

BY KVL:  $V = V_{oc} + i R_T$

THE EQUATION CAN BE REWRITTEN AS:  $i = -\frac{V_{oc}}{R_T} + \frac{V}{R_T}$  OR  $i = -I_{sc} + \frac{V}{R_T}$

WHICH CAN ALSO BE REPRESENTED AS A CIRCUIT:



BY KCL  $i = -I_{sc} + G_N V$

$G_N = 1/R_T$

THIS IS A NORTON EQUIVALENT CIRCUIT

### THEVENIN EQUIVALENT CIRCUIT: REPRISE

SUPPOSE THAT THE VOLTAGE  $V$  IS FORCED TO BE ZERO BY SHORTING THE TERMINALS. THEN REPLACE THE SHORT CIRCUIT WITH A CURRENT SOURCE ( $\hat{I}$ ) HAVING A VALUE EQUAL TO THE SHORT-CIRCUIT CURRENT. BEFORE AND AFTER ARE EQUIVALENT: THE CURRENT IS THE SAME AND THE VOLTAGE IS ZERO IN BOTH CASES.

THE ZERO-VOLTAGE CONDITION CAN BE INTERPRETED, USING SUPERPOSITION AS RESULTING FROM THE SUM OF TWO EQUAL BUT OPPOSITE VOLTAGES, ONE RESULTING FROM THE ORIGINAL INTERNAL SOURCES THE OTHER FROM THE CURRENT SOURCE  $\hat{I}$

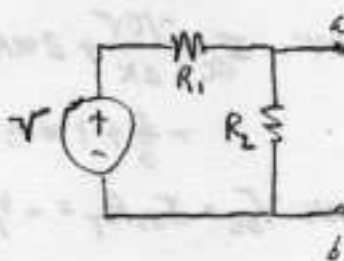
THEREFORE, THE VOLTAGE  $V_{oc}$  PRODUCED BY THE INTERNAL SOURCES MAY ALTERNATIVELY BE PRODUCED BY CONNECTING A CURRENT SOURCE OF VALUE  $-I_{sc}$  AND TURNING OFF THE INDEPENDENT INTERNAL SOURCES. THE RELATIONSHIP IS

$$V_{oc} = I_{sc} R_T \quad \text{OR} \quad I_{sc} = V_{oc} G_T$$

WHERE  $R_T = 1/G_T$  IS THE RESISTANCE SEEN AT THE TERMINALS WITH THE INTERNAL SOURCES TURNED OFF.

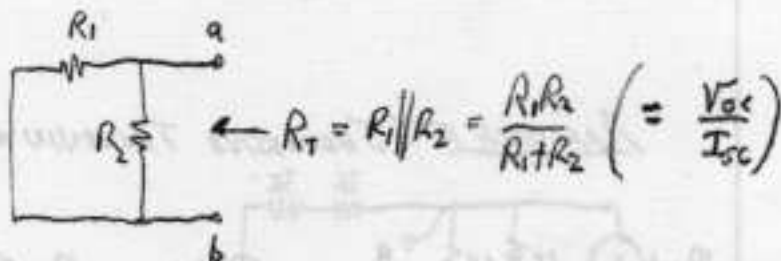
#### EXAMPLE 1

FIND THEVENIN & NORTON EQUIVALENTS AT a-b.



$$V_{oc} = V \frac{R_2}{R_1 + R_2} \quad I_{sc} = \frac{V}{R_1}$$

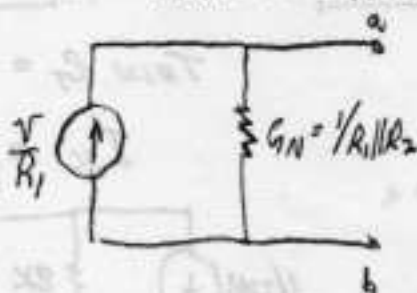
WITH INTERNAL SOURCE TURNED OFF:



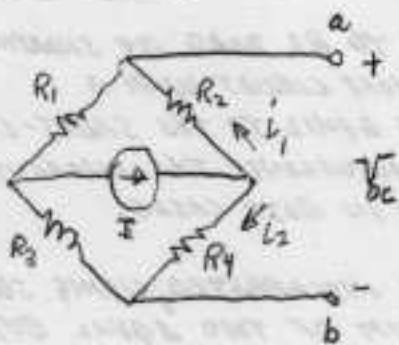
THEVENIN EQUIVALENT:



NORTON EQUIVALENT



EXAMPLE 2



$$i_1(R_1 + R_2) = i_2(R_3 + R_4) \quad i_1 + i_2 = I \quad (\text{CURRENT DIVIDER})$$

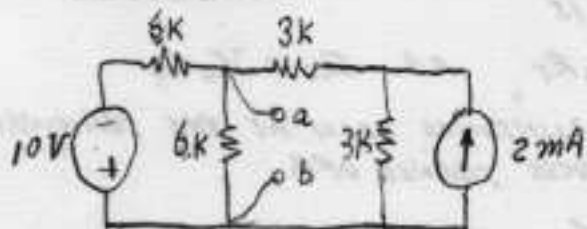
$$i_1 = I \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4}, \quad i_2 = I \frac{R_1 + R_2}{R_1 + R_2 + R_3 + R_4}$$

$$V_{oc} = -i_1 R_2 + i_2 R_4 \quad (\text{ADDING DROPS})$$

$$V_{oc} = I \frac{R_4(R_1 + R_2) - R_2(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} = \frac{R_1 R_4 - R_2 R_3}{R_1 + R_2 + R_3 + R_4}$$

$$R_T = \cancel{(R_1 + R_2)} \parallel \cancel{(R_3 + R_4)} \quad (R_1 + R_2) \parallel (R_3 + R_4)$$

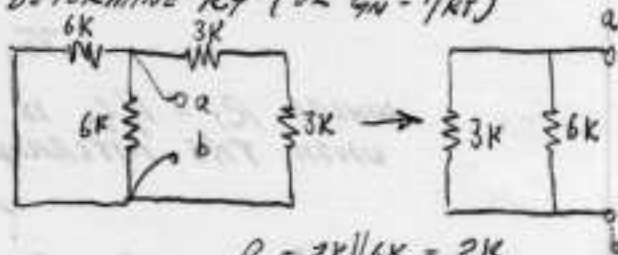
EXAMPLE 3



FIND SHORT CIRCUIT CURRENT:



APPROACH A SHUT SOURCES OFF TO DETERMINE  $R_T$  (OR  $G_N = 1/R_T$ )

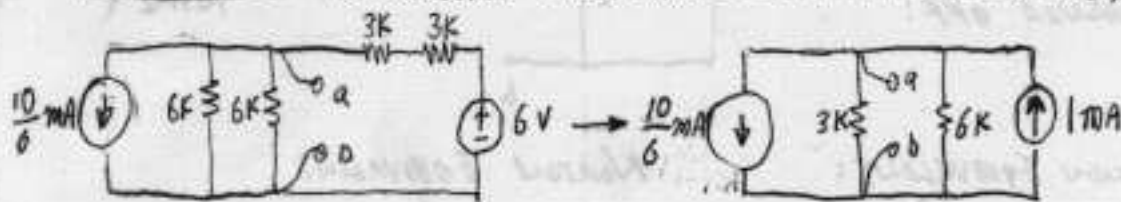


$$R_T = 3k \parallel 6k = 2k$$

USING SUPERPOSITION:  $I_{sc} = \frac{-10V}{6k} + 2mA \frac{3k}{3k+3k}$   
 $-\frac{5}{3} + 1 = -\frac{2}{3} mA$

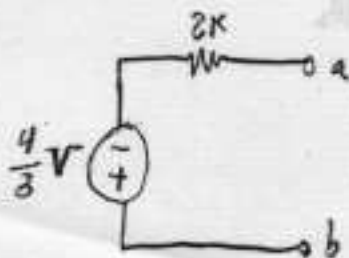
THUS  $V_{oc} = I_{sc} R_T = -\frac{4}{3} V$

APPROACH B SUCCESSIVE THEVENIN ↔ NORTON TRANSFORMATIONS:



$$THEN R_T = 3k \parallel 6k = 2k$$

$$I_{sc} = 1mA - \frac{5}{3} mA = -\frac{2}{3} mA$$



THEVENIN



NORTON

EQUIVALENCE APPLIES ONLY AT DEFINED TERMINALS