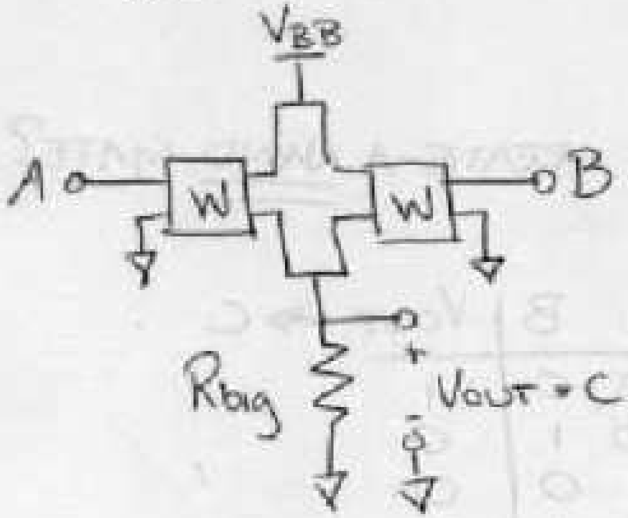
6.002 PROBLEM SET #2 SOLUTIONS

S03-018

PROBLEM 2.1:

(A) CAN WIDGETS BE COMBINED TO CREATE A NAND GATE?

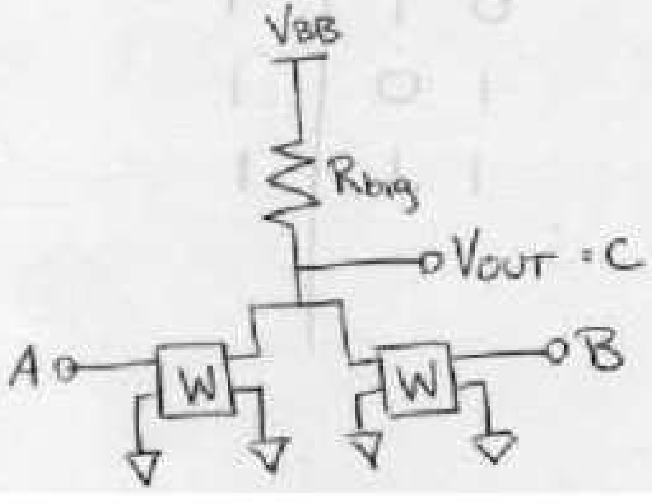
YES!



A	B	$V_{OUT} \Rightarrow C$
0	0	1
0	1	1
1	0	1
1	1	0

(B) CAN WIDGETS BE COMBINED TO CREATE AN AND GATE?

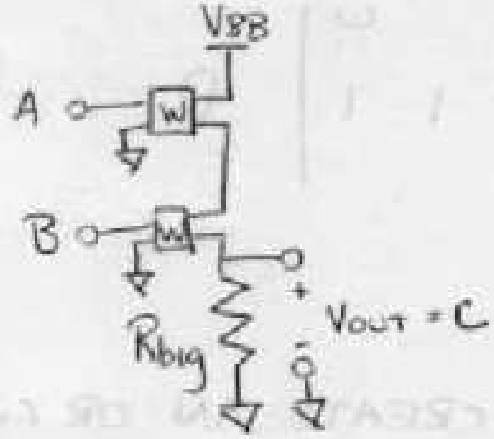
YES!



A	B	$V_{OUT} \Rightarrow C$
0	0	0
0	1	0
1	0	0
1	1	1

(C) CAN WIDGETS BE COMBINED TO CREATE A NOR GATE?

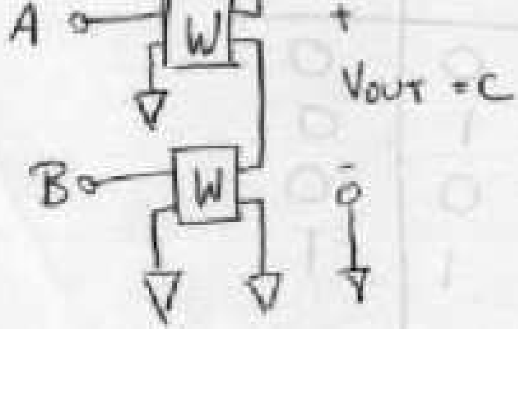
YES!



A	B	Vout ⇒ C
0	0	1
0	1	0
1	0	0
1	1	0

D) CAN WIDGETS BE COMBINED TO CREATE AN OR GATE?

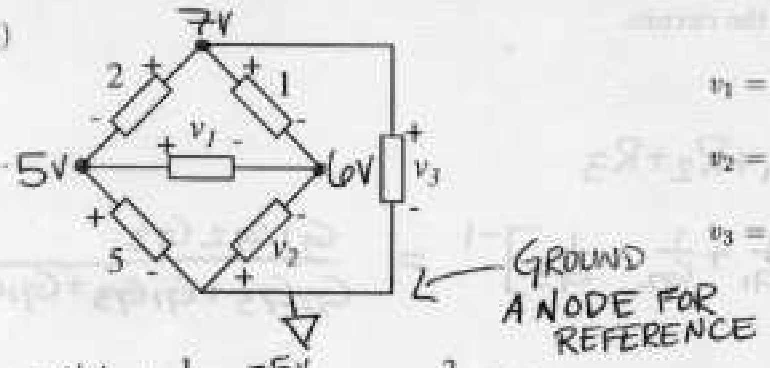
YES!



A	B	Vout ⇒ C
0	0	0
0	1	1
1	0	1
1	1	1

Problem 2.2: In each of the following circuits, determine the values of the indicated voltages and/or currents. (Voltages are in volts and currents are in amperes.)

(a)

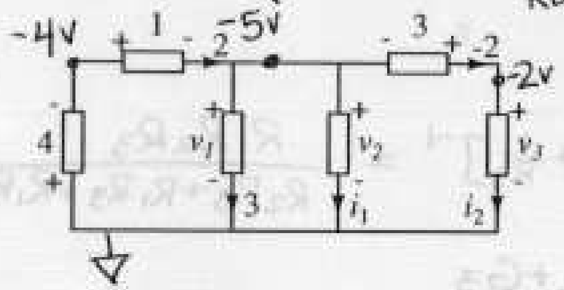


$v_1 =$ 5-6 = -1 V

$v_2 =$ -6 V

$v_3 =$ 7 V

(b)



$v_1 =$ -5V

$v_2 =$ -5V

$v_3 =$ -2V

$i_1 =$ 1A

$i_2 =$ -2A

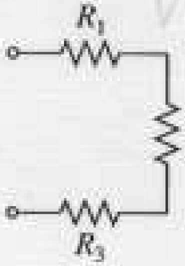
KCL FOR -5V NODE:

$-2+3+i_1-2=0$

$i_1=1$

Problem 2.3: For each of the circuits below express the resistance R, or the equivalent conductance $G = \frac{1}{R}$, at the terminals in terms of the element resistances R_n (or conductances $G_n = \frac{1}{R_n}$). Write your answers next to the circuits.

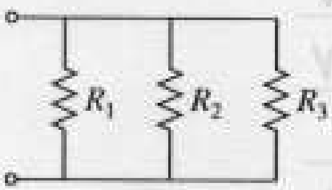
(a)



$$R = R_1 + R_2 + R_3$$

$$G = \left[\frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}\right]^{-1} = \frac{G_1G_2G_3}{G_2G_3 + G_1G_3 + G_1G_2}$$

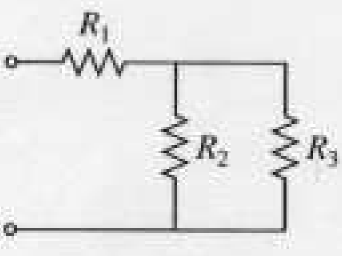
(b)



$$R = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]^{-1} = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$$

$$G = G_1 + G_2 + G_3$$

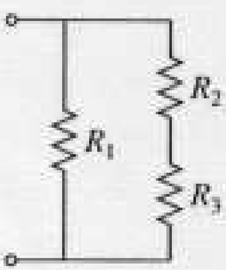
(c)



$$R = R_1 + \frac{R_2R_3}{R_2 + R_3}$$

$$G = \left[\frac{1}{G_1} + \frac{1}{G_2 + G_3}\right]^{-1} = \frac{G_1(G_2 + G_3)}{G_1 + G_2 + G_3}$$

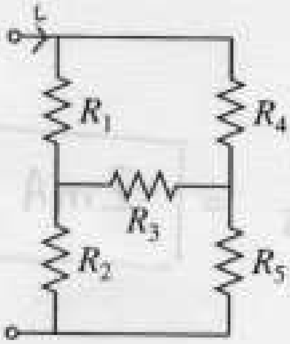
(d)



$$R = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

$$G = G_1 + \frac{G_2G_3}{G_2 + G_3}$$

(e) Can the same methods used in Parts (a) through (d) be used to find the resistance at the terminals of the circuit below? If so, express it; if not, explain why not.

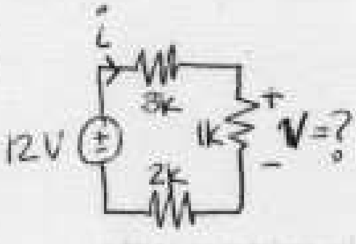


USING SERIES AND PARALLEL COMBINATIONS, IT IS NOT POSSIBLE TO REDUCE THIS CIRCUIT INTO A SIMPLE EQUIVALENT RESISTANCE. IT IS NOT POSSIBLE TO DEFINE CLEAR SETS OF RESISTORS THAT ARE EITHER IN SERIES OR IN PARALLEL. THIS DOES NOT, HOWEVER, MEAN THAT THERE IS NO EQUIVALENT RESISTANCE FOR THE NETWORK. IF YOU APPLY A TEST VOLTAGE AT THE TERMINALS YOU CAN CALCULATE THE CURRENT FLOWING INTO THE RESISTOR NETWORK USING NODE EQUATIONS. THE SAME IS TRUE FOR A TEST CURRENT AND THE RESULTING VOLTAGE ACROSS THE TERMINALS. YOU CAN THEN SOLVE FOR THE EQUIVALENT RESISTANCE $R = V/I$

STUDENTS ARE NOT EXPECTED TO KNOW HOW TO DO THIS YET.

PROBLEM 2.4:

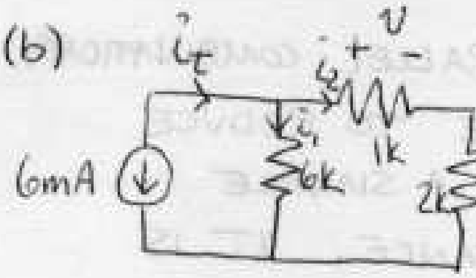
(a)



$$i = \frac{12V}{3k+1k+2k} = \frac{12}{6000} = \boxed{2mA}$$

$$v = iR = 2\times10^{-3}\times1000 = \boxed{2V}$$

(b)



$$i_t = -6\times10^{-3}A$$

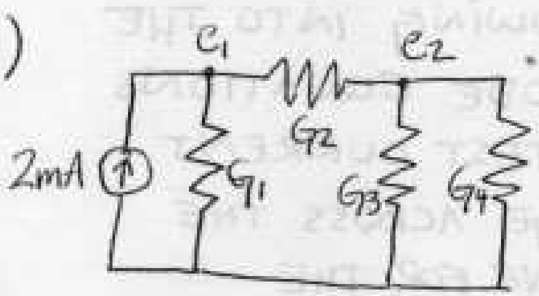
$$i_1 = -6\times10^{-3}\times\frac{3\times10^3}{9\times10^3} = \boxed{-2mA = i}$$

$$i_2 = -4mA$$

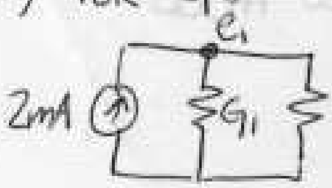
$$v = i_2R = -4\times10^{-3}\times1000 = \boxed{-4V = v}$$

PROBLEM 2.5:

1)



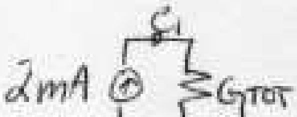
2) FOR e_1:



$\frac{G_2(G_3+G_4)}{G_2+G_3+G_4}$ (from problem 3(c))

$$VG = I$$

$$e_1 = \frac{I}{G_{TOT}}$$

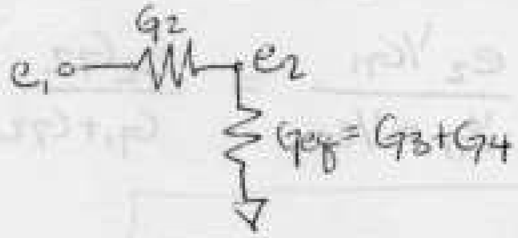


$$G_{TOT} = G_1 + \frac{G_2(G_3+G_4)}{G_2+G_3+G_4}$$

$$= \frac{G_1(G_2+G_3+G_4) + G_2(G_3+G_4)}{G_2+G_3+G_4}$$

$$\boxed{e_1 = \frac{I_1(G_2+G_3+G_4)}{G_1(G_2+G_3+G_4)+G_2(G_3+G_4)}}$$

For e_2:



$$e_2 = \frac{e_1 R_{eq}}{R_2 + R_{eq}} = \frac{e_1 \cdot \frac{1}{G_3+G_4}}{1/G_2 + 1/G_3+G_4}$$

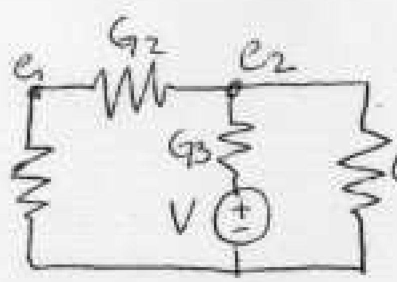
$$e_2 = \frac{e_1 G_2}{G_2+G_3+G_4} =$$

$$\boxed{e_2 = \frac{I_1 G_2}{G_1(G_2+G_3+G_4)+G_2(G_3+G_4)}}$$

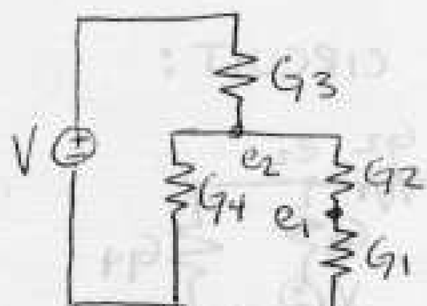
3) $e_1 = \dfrac{2\times10^{-3}(7/2)\times10^{-3}}{7/2\times10^{-6} + 3/2\times10^{-6}} = \dfrac{7}{10/2} = \dfrac{7}{5}\,V = \boxed{1.4V}$

$e_2 = \dfrac{2\times10^{-3}\times\frac{1}{2}\times10^{-3}}{10/2\times10^{-6}} = \dfrac{1}{5}\,V = \boxed{.2V}$

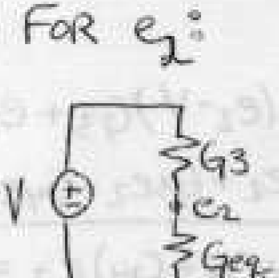
4) 1)



OR



2) For e_2:

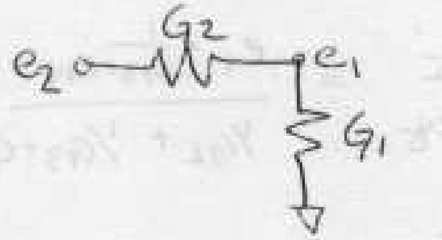


$$G_{eq} = G_4 + \frac{G_2 G_1}{G_2+G_1}$$ ← from problem 2.3 (d)

$$e_2 = \frac{VG_3}{G_3+G_4+\frac{G_2G_1}{G_2+G_1}} = \frac{VG_3(G_2+G_1)}{(G_2+G_1)(G_3+G_4)+G_2G_1}$$

$$e_2 = \frac{V G_3(G_1+G_2)}{G_1(G_2+G_3+G_4)+G_2(G_3+G_4)}$$

FOR e_1:



$$e_1 = \frac{e_2 R_1}{R_1 + R_2} = \frac{e_2 \, ^1/_{G_1}}{^1/_{G_1} + {^1/_{G_2}}} = \frac{e_2 G_2}{G_1 + G_2}$$

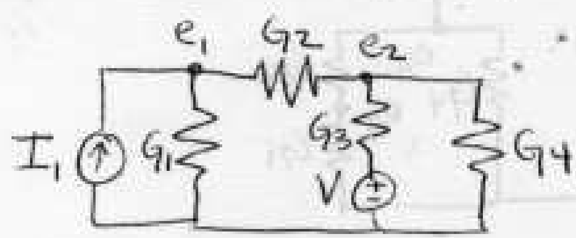
$$\boxed{e_1 = \frac{V G_2 G_3}{G_1(G_2+G_3+G_4) + G_2(G_3+G_4)}}$$

3) $$e_1 = \frac{\frac{3}{2} \times \frac{1}{2} \times 10^{-3} \times 10^{-3}}{10^{-3}(\frac{7}{2} \times 10^{-3}) + \frac{1}{2} \times 10^{-3}(3 \times 10^{-3})}$$

$$e_1 = \frac{3/4}{7/2 + 3/2} = \frac{3/4}{10/2} = \frac{3}{20} V = \boxed{.15 V}$$

$$e_2 = \frac{3/2 \cdot 10^{-3}(3/2 \times 10^{-3})}{10/2 \times 10^{-6}} = \frac{9/4}{10/2} = \frac{9}{20} V = \boxed{.45 V}$$

5) COMPLETE CIRCUIT:



NODE EQUATIONS:

$$-I_1 + e_1 G_1 + (e_1 - e_2) G_2 = 0$$
$$e_1 G_1 + e_1 G_2 - e_2 G_2 = I_1$$
$$\boxed{(G_1 + G_2) e_1 - G_2 e_2 = I_1}$$

$$(e_2 - e_1) G_2 + (e_2 - V) G_3 + e_2 G_4 = 0$$
$$-e_1 G_2 + G_2 e_2 + G_3 e_2 + G_4 e_2 = V G_3$$
$$\boxed{-G_2 e_1 + (G_2 + G_3 + G_4) e_2 = G_3 V}$$

$$e_1 = e_1\Big|_{I_1} + e_1\Big|_V = 1.4V + .15V = \boxed{1.55V}$$

$$e_2 = e_2\Big|_{I_1} + e_2\Big|_V = .2V + .45V = \boxed{.65V}$$

THE ANSWERS ARE THE SAME!