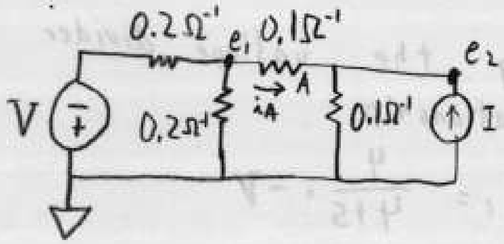
Problem 3.1 Spring 03 - 022
6.002

A.) First label ground and unknown voltages, e1 and e2.



Then, write KCL for each node with positive currents leaving a node.

node 1: $0.2(e_1 + V) + 0.1(e_1 - e_2) + 0.2e_1 = 0$

node 2: $0.1(e_2 - e_1) + 0.1e_2 - I = 0$

$\Rightarrow$ $\boxed{\begin{array}{l} 0.5e_1 - 0.1e_2 = -0.2V \\ -0.1e_1 + 0.2e_2 = I \end{array}}$

B.) $2(0.5e_1 - 0.1e_2 = -0.2V)$

$+ \quad (-0.1e_1 + 0.2e_2 = I)$

$0.9e_1 = -0.4V + I$

$\Rightarrow e_1 = -\frac{4}{9}V + \frac{10}{9}I$

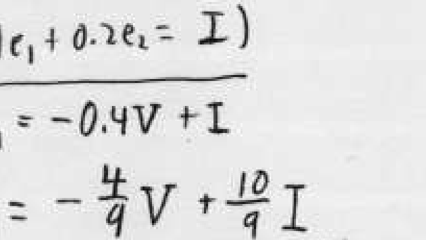
$-0.1\left(-\frac{4}{9}V + \frac{10}{9}I\right) + 0.2e_2 = I$

$e_2 = 5I + \frac{1}{2}\left(-\frac{4}{9}V + \frac{10}{9}I\right)$

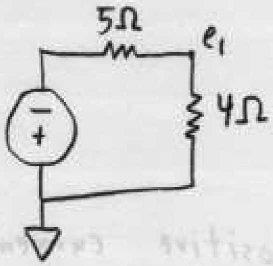
$e_2 = -\frac{2}{9}V + \frac{50}{9}I$

$i_A = 0.1(e_1 - e_2) = \frac{1}{10}\left(-\frac{2}{9}V - \frac{40}{9}I\right) \Rightarrow \boxed{i_A = -\frac{1}{45}V - \frac{4}{9}I}$

C.) When current source is off, it acts as an open circuit so we are left with the following. Note that conductances have been changed to resistances.



D.) Let's combine the two 10Ω resistors to make a 20Ω resistor, which is in parallel with a 5Ω resistor. If we combine the 20Ω resistor and the 5Ω in parallel, we get a 4Ω resistor and the following circuit:



Using the voltage divider relationship

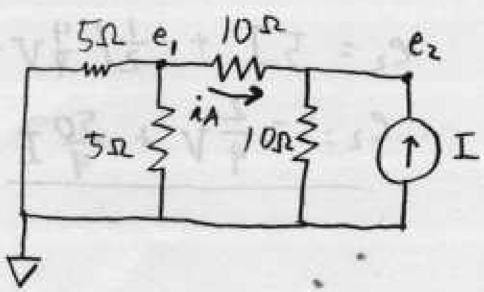
$$e_1 = \frac{4}{4+5} \cdot -V$$

$$e_1 = -\frac{4}{9}V$$

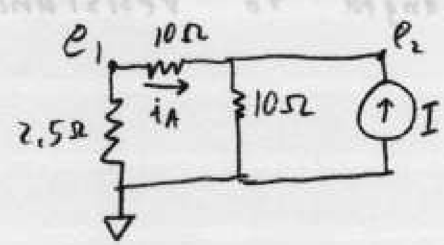
From the figure in part C.) we find.

$$i_A = \frac{e_1}{10+10} \Rightarrow \boxed{i_A = -\frac{1}{45}V}$$

E.) When a voltage source is off, it acts as a short circuit.



F.) First combine the two 5Ω resistors to form a 2.5Ω resistor. This results in the circuit below. Since i_A flows through the 2.5Ω and 10Ω resistor in series we can use a current divider to get i_A.



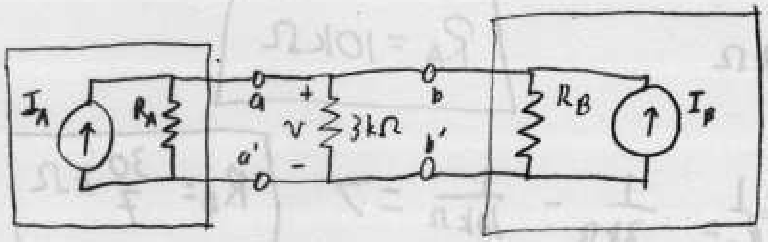
$$i_A = -\frac{10\Omega}{(2.5\Omega + 10\Omega) + 10\Omega} I$$

$$i_A = -\frac{20}{45} I \Rightarrow \boxed{i_A = -\frac{4}{9} I}$$

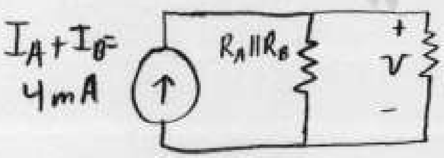
G.) $\underline{i_A = -\frac{1}{45}V - \frac{4}{9}I}$ The answers agree.

Problem 3.2

1) Replace each box with its Norton equivalent circuit.



When the 3kΩ resistor is replaced by a short circuit, $i_a = -I_A$ and $i_b = -I_B$; therefore, $I_A = 6\,mA$ and $I_B = -2\,mA$. Combining the two current sources and the two unknown resistors results in the following circuit.



We know that with the 3kΩ resistor in the circuit $v = 6V$. From this we can determine the parallel combination of R_A and R_B.

$$6V = 4mA\left[3k\Omega \,\|\, (R_A \| R_B)\right] \Rightarrow \left(\frac{1}{3k\Omega} + \frac{1}{R_A \| R_B}\right)^{-1} = \frac{3}{2}k\Omega$$

$$\Rightarrow \frac{1}{R_A \| R_B} = \frac{2}{3}k\Omega - \frac{1}{3}k\Omega \Rightarrow \underline{R_A \| R_B = 3\,k\Omega}$$

Therefore, when the 3kΩ resistor is replaced by an open circuit,

$$v = 4mA\,(R_A \|_B) \Rightarrow \boxed{v = 12\,V}$$

B.) No. As seen from above, we have enough information to find the Norton currents and the parallel combination of their resistances, but not individual resistances.

C.) Find the open circuit voltage of either box. This would give you enough information to find the Norton resistance of that box $\left(R_N = \frac{V_{oc}}{I_N}\right)$. Since we know the parallel combination value we can find the other Norton resistance.

).) Assume the open circuit voltage of box A (from a to a') is 60V. Then,

$$R_A = \frac{V_{oc}}{I_A} = \frac{60V}{6mA} = 10k\Omega \qquad \boxed{R_A = 10k\Omega}$$

and

$$\frac{1}{R_B} = \frac{1}{R_A \| R_B} - \frac{1}{R_A} \Rightarrow \frac{1}{R_B} = \frac{1}{3k\Omega} - \frac{1}{10k\Omega} \Rightarrow \boxed{R_B = \frac{30}{7}k\Omega}$$

Problem 3.3

The voltage at terminal a is:

$$V_a = \frac{4k\Omega}{2k\Omega + 4k\Omega}(-12V) = -8V$$

The current at terminal b can be found by a current divider:

$$i_{6k\Omega} = \frac{3k\Omega}{3k\Omega + 6k\Omega}(1mA)$$

$$i_{6k\Omega} = \frac{1}{3} mA$$

The voltage at b can be found by multiplying $i_{6k\Omega}$ by the resistance.

$$V_b = 6k\Omega\,(1/3\,mA$$

$$V_b = 2V$$

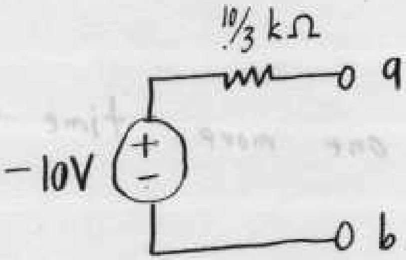
The open circuit voltage is the difference of these two.

$$V_{oc} = -10V$$

To determine the equivalent resistance, we turn off the internal, independent sources so the voltage source becomes a short circuit and the current source becomes an open circuit. The result is that we have 2 parallel combinations acting in series

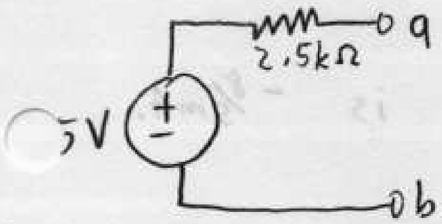
$$R_T = 2\|4 + 6\|3 = \frac{4}{3} + 2 = \frac{10}{3} k\Omega$$

The Thevenin equivalent is then as follows

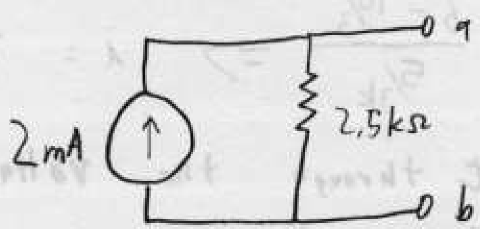


Problem 3.4

The open circuit (ie, zero current) voltage is $v_{oc} = 5V$. The short circuit (ie zero voltage) current is $i_{sc} = -i(v=0) = 2mA$ Therefore $R_T = \frac{v_{oc}}{i_{sc}} = 2.5\,k\Omega$. Thus, the following are the Thevenin and Norton equivalents



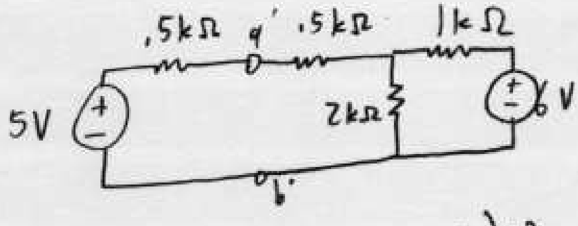
Thevenin Equivalent



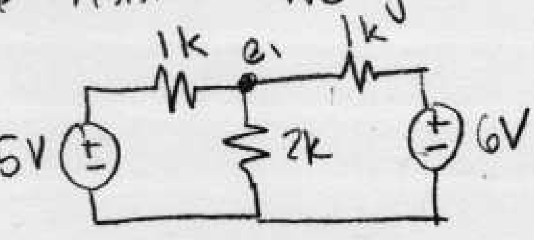
Norton Equivalent

Problem 3.5

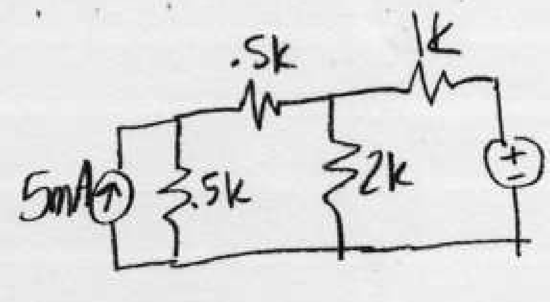
Switching to the Thevenin equivalent; we have the following circuit:



Combining the resistors we get:

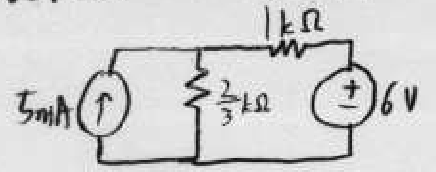


thévenin

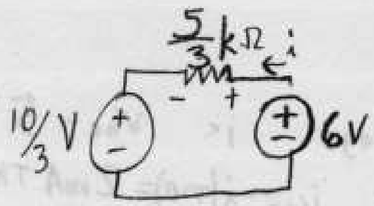


NORTON

We can combine the two parallel resistors to get this circuit (NORTON METHOD)



We can switch to Thevenin equivalent one more time and add the two resistors in series



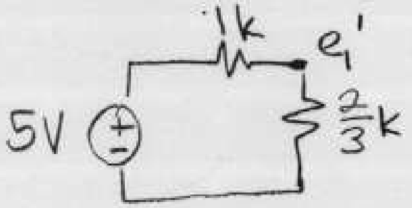
Now we can define polarities and a current going through the resistor

$$i = \frac{6 - 10/3}{5/3k} \Rightarrow i = \frac{8}{5}\,mA$$

The current through the voltage source is $-8/5\,mA$. Therefore power is

$$P = vi = 6V\left(-\tfrac{8}{5}mA\right) \Rightarrow \boxed{P = \frac{-48}{5}mW}$$ power is produced by the voltage supply

Now we use superposition to find the voltage e_1 (THÉVENIN METHOD)



$1k\Omega // 2k\Omega = \frac{2}{3}k\Omega$

$$e_1' = \frac{5 \cdot \frac{2}{3}}{1 + 2/3} = \frac{5 \cdot 2}{3+2} = 2V$$

$$e_1'' = \frac{6 \cdot 2}{3+2} = \frac{12}{5}$$

$$e_1 = \frac{10}{5} + \frac{12}{5} = \frac{22}{5} V$$

$$i(\text{through 6V power supply}) = \frac{\frac{22}{5}V - 6V}{1k\Omega}$$

$$i = \frac{-8}{5 \cdot 1k} = -\frac{8}{5} mA$$

$$P = I \cdot V = -\frac{8}{5} \times 10^{-3} \times 6 = \boxed{\frac{-48}{5} mW}$$

Both methods are equivalent!

Power produced (negative power absorbed)