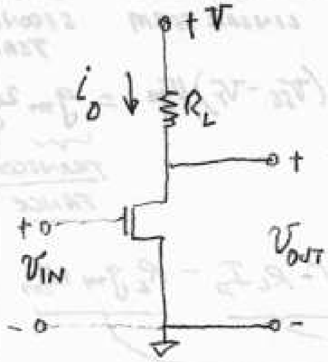
NOTES FOR 6.002 LECTURE #8 TUESDAY, MARCH 4, 2003

READ: 8.1 - 8.2.3

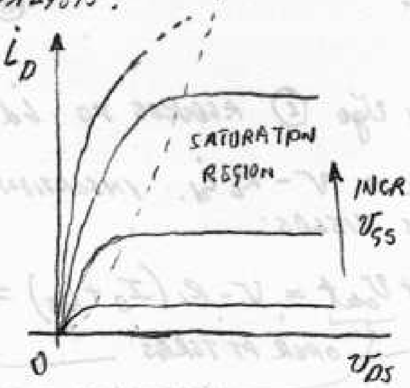
THERE ARE TWO VIDEOS ON THE 6.002 WEB SITE ON THE TOPICS OF LARGE- AND SMALL-SIGNAL ANALYSIS OF FET AMPLIFIERS

A FET AS AN ELECTRONIC VALVE IS AN AMPLIFIER OF SIGNALS
WHAT ABOUT ANALYSIS?

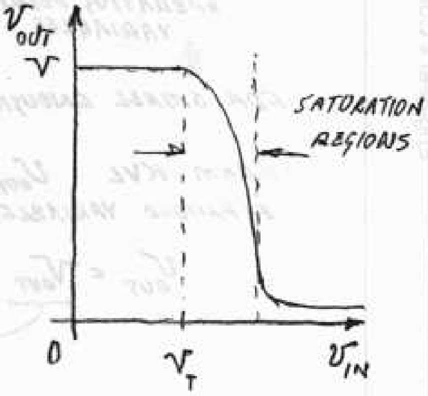


CIRCUIT

$v_{IN} = v_{GS}$, $v_{OUT} = v_{DS}$



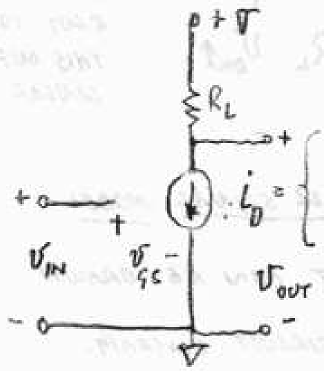
OUTPUT CHARACTERISTICS



TRANSFER CHARACTERISTIC

IN THE SATURATION REGION SMALL CHANGES IN v_{IN} PRODUCE LARGE CHANGES IN v_{OUT} VOILA! VOLTAGE GAIN

INSERT SQUARE-LAW FET MODEL INTO CIRCUIT:



i_D IS A DEPENDENT SOURCE

$$i_D = \begin{cases} 0 & \text{FOR } v_{GS} < V_T \\ \frac{K}{2}(v_{GS} - V_T)^2 & \left[\begin{array}{l} v_{GS} > V_T \\ (v_{GS} - V_T) > v_{DS} \end{array}\right. \end{cases}$$

FOCUS ON THIS DOMAIN WHERE AMPLIFICATION OCCURS.

NOTATION: REPRESENT EVERY VARIABLE AS THE SUM OF TWO COMPONENTS:
INCREMENTAL OR SMALL-SIGNAL VARIABLES MEASURE DEPARTURES FROM AN OPERATING POINT.

$$v_{GS} = V_{GS} + v_{gs}$$

$$v_{DS} = V_{DS} + v_{ds}$$

$$i_d = I_D + i_d$$

QUIESCENT OR OPERATING-POINT VALUES

INSERT THESE VARIABLES INTO SQUARE-LAW MODEL:

$$i_D = \frac{K}{2}\left(v_{GS} - V_T\right)^2 \rightarrow I_D + i_d = \frac{K}{2}\left(V_{GS} - V_T + v_{gs}\right)^2$$

$$(1)\quad I_D + i_d = \frac{K}{2}\left(V_{GS}-V_T\right)^2 + K\left(V_{GS}-V_T\right)v_{gs} + \frac{K}{2}v_{gs}^2$$

THIS EQUATION (1) MUST HOLD FOR ALL VALUES OF v_{gs} INCLUDING $v_{gs}=0$, YIELDING

$$\underbrace{I_D = \frac{K}{2}\left(V_{GS}-V_T\right)^2}_{\text{OPERATING POINT VARIABLES}}$$

SUBTRACT FROM (1)

$$\rightarrow (2)\quad i_d = \underbrace{K\left(V_{GS}-V_T\right)v_{gs}}_{\text{LINEAR TERM}} + \underbrace{\frac{K}{2}v_{gs}^2}_{\text{SECOND ORDER TERM}}$$

FOR SMALL ENOUGH v_{gs} (2) REDUCES TO $i_d \cong K\left(V_{GS}-V_T\right)v_{gs} = \underbrace{g_m}_{\text{TRANSCONDUCTANCE}} v_{gs}$

FROM KVL $v_{OUT} = V - R_L i_d$. INSERTION OF EXPANDED VARIABLES YIELDS:

$$v_{OUT} = V_{OUT} + v_{out} = V - R_L\left(I_D + i_d\right) = V - R_L I_D - R_L g_m v_{in}$$

OPER. PT TERMS

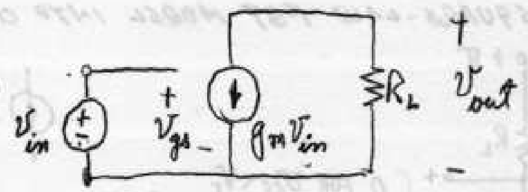
INCREMENTAL TERMS

SUBTRACTING OUT THE OPERATING-POINT TERMS YIELDS:

$$v_{out} = -g_m R_L v_{in}$$

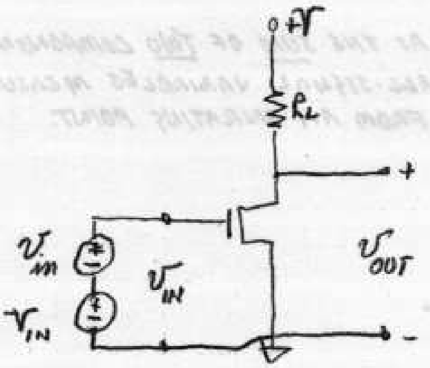
THUS THE SMALL-SIGNAL VOLTAGE GAIN: $A_v = \frac{v_{out}}{v_{in}} = -g_m R_L$

THIS EQUATION CAN BE INTERPRETED BY A CIRCUIT:



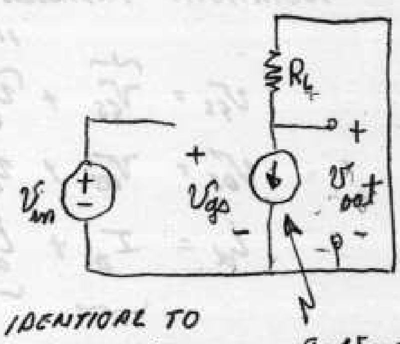
ANOTHER DEPENDENT SOURCE, THIS ONE LINEAR.

THIS IS THE INCREMENTAL OR SMALL SIGNAL MODEL FOR THE AMPLIFIER CIRCUIT. IT CAN BE DRAWN DIRECTLY FROM THE ORIGINAL CIRCUIT DIAGRAM:



REPLACE FET BY $g_m v_{gs}$ DEPENDENT SOURCE

SET ALL FIXED SOURCES TO ZERO



IDENTICAL TO CIRCUIT ABOVE

$g_m v_{gs} = g_m v_{in}$

OBVIOUSLY THE TRANCONDUCTANCE g_m IS DEPENDENT ON THE OPERATING POINT:

$$g_m = K(V_{GS} - V_T) \qquad (V_{GS} - V_T) = \sqrt{\frac{2I_D}{K}}$$ INVERTING THE EQUATION WE STARTED WITH.

OPER. POINT DEPENDENCE $(V_{GS} = V_{IN})$

THUS $\underline{g_m = \sqrt{2KI_D}}$ THE USUAL WAY OF EXPRESSING OPERATING POINT DEPENDENCE

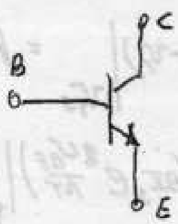
IN TERMS OF v_{gs}, HOW SMALL IS SMALL ENOUGH? FROM EQ. 2

$$\frac{\text{SECOND-ORDER TERM}}{\text{LINEAR TERM}} = \frac{\frac{K}{2} v_{gs}^2}{K(V_{GS}-V_T) v_{gs}} = \frac{v_{gs}}{2(V_{GS}-V_T)}$$

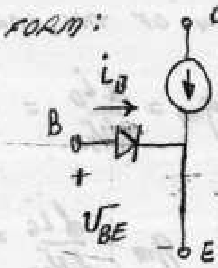
IF $v_{gs} \ll 2(V_{GS} - V_T)$ OR $v_{gs} \ll \frac{2g_m}{K}$ THE LINEAR APPROXIMATION IS OK.

DEMO (BJT)

A SECOND EXAMPLE A BIPOLAR JUNCTION TRANSISTOR, WHICH IS IN A DIFFERENT CLASS OF SEMICONDUCTOR DEVICES HAS A LARGE-SIGNAL MODEL OF THIS FORM:

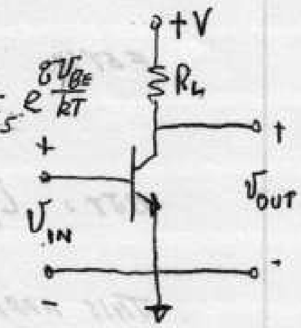


CIRCUIT SYMBOL



LARGE SIGNAL MODEL

$$i_B = I_S\left(e^{q v_{BE}/kT} - 1\right) \cong I_S e^{\frac{q v_{BE}}{kT}}$$



CIRCUIT

AND IS USED IN A CIRCUIT LIKE THIS

USE TAYLOR SERIES TO EXPAND i_B AND LINEARIZE THE $i_B = f(v_{BE})$ RELATIONSHIP:

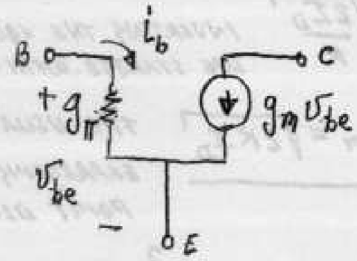
$$I_B + i_b = f(v_{BE})\Big|_{V_{BE}} + \frac{1}{1!}\frac{df}{dv_{BE}}\Big|_{V_{BE}} v_{be} + \frac{1}{2!}\frac{d^2 f}{dv_{BE}^2}\Big|_{V_{BE}} v_{be}^2 + \text{HIGHER ORDER TERMS}$$

$$= \underbrace{I_S e^{\frac{qV_{BE}}{kT}}}_{I_B} + \underbrace{\frac{q}{kT}\left(I_S e^{\frac{qV_{BE}}{kT}}\right) v_{be}}_{i_b} + \underbrace{\frac{1}{2}\left(\frac{q}{kT}\right)^2 \left(I_S e^{\frac{qV_{BE}}{kT}}\right) v_{be}^2 + \cdots}_{\text{NEGLECT}}$$

$$i_b = \frac{q}{kT} I_B v_{be}$$

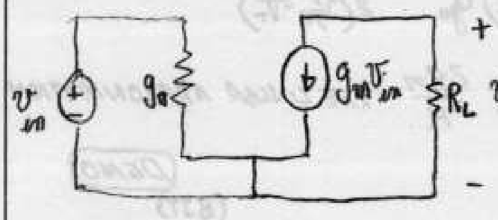
$$= g_\pi v_{be} \quad \text{WHERE } \underline{g_\pi = \left(\frac{q}{kT} I_B\right)}$$

THE INCREMENTAL BJT MODEL IS:



WHERE $g_m = \beta g_\pi = \left(\frac{q}{kT}\right)\beta I_B$ OR $g_m = \left(\frac{q}{kT}\right) I_C$

AND THE INCREMENTAL CIRCUIT IS:



HOW SMALL MUST $v_{in} = v_{be}$ BE TO PERMIT THE LINEAR APPROXIMATION?

$$\frac{\text{SECOND ORDER TERM}}{\text{FIRST ORDER TERM}} = \left(\frac{q}{kT}\right)\frac{v_{be}}{2}$$

CONDITION: $v_{be} \ll \frac{q}{2kT}$

IN EACH OF THESE CASES THE INCREMENTAL PARAMETERS CAN BE OBTAINED BY DIFFERENTIATION OF THE LARGE SIGNAL MODEL.

FET: $i_D = \frac{K}{2}(v_{GS}-V_T)^2 \quad g_m = \frac{di_D}{dv_{GS}} = K(v_{GS}-V_T)\Big|_{V_{GS}} = \sqrt{2KI_D}$

BJT: $i_C = \beta i_B = \beta I_S e^{qv_{BE}/kT} \quad g_m = \frac{di_C}{dv_{BE}} = \left(\frac{q}{kT}\right)\left(\beta I_S e^{qv_{BE}/kT}\right)\Big|_{V_{BE}} = \left(\frac{q}{kT}\right)I_C$

THIS APPROACH GIVES NO GUIDANCE ON THE LIMITS OF LINEARITY.