Notes for 6.002 Lecture #8  Tuesday, March 4, 2003

Read: 8.1 - 8.2.3

There are two videos on the 6.002 web site on the topics of large- and small-signal analysis of FET amplifiers.

A FET as an electronic valve is an amplifier of signals. What about analysis?

Circuit
\[ V_{IN} = V_{GS}, \quad V_{OUT} = V_{DS} \]

Output Characteristics
In the saturation region small changes in \( V_{IN} \) produce large changes in \( V_{OUT} \). Voila! Voltage gain!

Transfer Characteristic

Insert square-law FET model into circuit:

\[ I_D = \begin{cases} 0 & \text{for } V_{DS} \leq V_T \\ \frac{K}{2} (V_{DS} - V_T)^2 & \text{for } (V_{DS} - V_T) > V_{GS} \end{cases} \]

Focus on this domain where amplification occurs.

Notation:
Represent every variable as the sum of two components: incremental or small-signal variables measure deviations from an operating point.

\[ V_{GS} = V_{GS}^0 + \Delta V_{GS} \]

\[ V_{DS} = V_{DS}^0 + \Delta V_{DS} \]

\[ I_D = I_D^0 + \Delta I_D \]

Quiescent or operating-point values
INSERT THESE VARIABLES INTO SQUARE-LAW MODEL:

\[ I_d = \frac{K}{2} (V_{gs} - V_T)^2 \]

\[ I_d + I_d' = \frac{K}{2} (V_{gs} - V_T + V_d) \]

1. This equation must hold for all values of \( V_d \) including \( V_d = 0 \), yielding \( I_d \).

2. Subtracting from \( \frac{K}{2} (V_{gs} - V_T)^2 \) gives \( I_d' = K(V_{gs} - V_T) V_d + \frac{K}{2} V_d^2 \).

3. Linear term.

For small enough \( V_d \), \( \frac{K}{2} V_d^2 \) reduces to \( \frac{K}{2} (V_{gs} - V_T) V_d \).

4. From KVL, \( V_{out} = V - R_L I_d \). Insertion of expanded variables yields:

\[ V_{out} = \frac{V_{out} + I_d + V_{in}}{R_L} = \frac{V - R_L I_d}{R_L} = \frac{-R_L V_{in}}{R_L} \]

5. Incremental terms.

Subtracting out the operating-point terms yields:

\[ V_{out} = -9 \text{m} R_L V_{in} \]

Thus, the voltage gain: \( Av = \frac{V_{out}}{V_{in}} = -9 \text{m} R_L \)

6. This equation can be interpreted by a circuit:

7. This is the incremental or small signal model for the amplifier circuit. It can be drawn directly from the original circuit diagram.

8. Replace FET by \( g_m V_d \) dependent source.

9. Set all fixed sources to zero.

10. Identical to circuit above.

\[ g_m V_d = g_m V_{in} \]
Obviously the transconductance $g_m$ is dependent on the operating point:

$$g_m = k(V_{ES} - V_T) \left( \frac{-V_T}{V_{ES} - V_T} \right) = \frac{2I_D}{K}$$

Inverting the equation we started with:

Thus $g_m = \frac{1}{2kI_D}$

The usual way of expressing operating point dependence

In terms of $V_{GS}$, how small is small enough? From Eq. 2

$$\frac{\text{second-order term}}{\text{linear term}} = \frac{\frac{k}{2}V_{GS}^2}{K(V_{GS} - V_T)\frac{V_{GS}}{2(V_{GS} - V_T)}}$$

If $V_{GS} \ll 2(V_{GS} - V_T)$ or $2V_{GS} \ll \frac{2g_m}{k}$ the linear approximation is ok

A second example. A bipolar junction transistor, which is in a different class of semiconductor devices has a large-signal model of this form:

Circuit symbol

Large-signal model

And is used in a circuit like this:

Use Taylor series to expand $I_B$ and linearize the $I_B = f(V_{BE})$ relationship:

$$I_B + i_B = I_0 \left( \frac{V_{BE}}{V_{BE}} \right) \left( \frac{1}{2!} \right) \frac{d^2f}{dx^2} \left( \frac{V_{BE}}{V_{BE}} \right)$$

Higher order terms

$$= I_0 \left( 1 + \frac{V_{BE}}{V_{BE}} + \frac{1}{2!} \frac{d^2f}{dx^2} \left( \frac{V_{BE}}{V_{BE}} \right) V_{BE} + \cdots \right)$$

$$I_B \approx I_0 \left( \frac{V_{BE}}{V_{BE}} \right)$$

$$i_B = \frac{I_B}{V_{BE}}$$

$$= q \pi V_{BE}$$

Where $q \pi = \frac{2}{kT}$
The incremental BJT model is:

\[ g_m = \beta \frac{g_m}{T} \] \( \text{where } g_m = \beta \frac{g_m}{T} \) or \( g_m = \frac{g_m}{T} E \)

And the incremental circuit is:

How small must \( V_{BE} \) be to permit the linear approximation?

\[
\begin{align*}
\text{Second order term} & = \frac{g_m}{kT} \frac{V_{BE}}{2} \\
\text{First order term} & = \frac{g_m}{kT} \frac{V_{BE}}{2}
\end{align*}
\]

Condition: \( V_{BE} < \frac{Z}{2kT} \)

In each of these cases the incremental parameters can be obtained by differentiation of the large signal model.

For: \( I_d = \frac{k}{2} (V_{BE} - V_s)^2 \)

\[ g_m = \frac{dI_d}{dV_{BE}} = k \left( \frac{V_{BE} - V_s}{V_s} \right) = \frac{1}{2} \frac{1}{kT} \]

But: \( I_C = \beta I_E \quad g_m = \frac{dI_C}{dV_{BE}} = \frac{\beta E}{kT} \frac{V_{BE}}{V_s} \)

This approach gives no guidance on the limits of linearity.