Notes for 6.002 Lecture #9 Thursday, March 6, 2003

Read: 8.2.4  Error on Lecture 8 Notes: Page 4, the condition for linearity should read: $V_{oc} < \frac{2kT}{q}$

More examples of small-signal analysis:

**Phase-Splitter**

The fixed source $V_g$ establishes an operating point. The signal source is $V_a$

There are two outputs

**Circuit Diagram**

Note use of $\triangledown$ to distinguish the dependent current source,

and note that in this circuit $V_o \neq V_{gd}$, rather $V_o = V_{gd} + R_i i$

where $R_i$ is the drop across the resistor at the source terminal of the FET. The relevant equations are:

$$\begin{align*}
V_{gd} &= V_o - R_i i \\
i_d &= g_m V_{gd} \\
V_o &= \frac{V_o}{1 + g_m R}
\end{align*}$$

Solutions: $V_o = \frac{V_o}{1 + g_m R}$

From which the output voltages can be written:

$$\begin{align*}
V_o &= \frac{V_o}{1 + g_m R} \\
V_o &= \frac{V_o}{1 + g_m R} \\
V_o &= \frac{V_o}{1 + g_m R} \\
V_o &= \frac{V_o}{1 + g_m R}
\end{align*}$$

Outputs are equal in magnitude and opposite in sign. Thus the name “Phase-Splitter”

If $g_m R > > 1$, which is usually the case:

$$\begin{align*}
V_o &= V_o \\
V_o &= V_o \\
V_o &= V_o
\end{align*}$$

A first illustration of the concept of negative feedback.

The equations  describe the open-circuit voltages at the outputs. To create Thevenin equivalent models at the two outputs, either short-circuit currents or Thevenin equivalent resistances must be determined. To do the latter ($R_{th}$) turn off the independent source in the small-signal model. That is, set $V_o = 0$. In terms of the determination of $R_{th}$, where a test source will be applied, $V_o$ is an independent source.
RESULTING SMALL SIGNAL MODEL:

Consider the output first. Apply a voltage \( V_e \) across the terminal. It does not affect the dependent generator and the current is just \( \frac{V_e}{R} \) and \( R_{th} \) at the output is \( R \).

At the output, application of a test voltage \( V_e \) produces \( V_{00} = -V_e \) and the dependent generator is activated.

\[ i_e = \frac{V_e}{R} - g_m V_0 = \frac{V_e}{R} + g_m V_e \]

Thus \( i_e = V_e \left( \frac{1}{R} + g_m R \right) \) and

The Thevenin resistance is \( R_{th} = \frac{V_e}{i_e} = \frac{R}{1 + g_m R} \).

And we choose instead to evaluate the short-circuit currents:

At output, now with \( V_e \) acting:

\[ i_{sc} = -i_e = -V_e \frac{g_m}{1 + g_m R} \hspace{1cm} \text{(1)} \]

At \( b \) output, also with \( V_e \) acting:

\[ i_{sc} = g_m V_0 \text{ by inspection} \]

As a check, form \( R_{th} = \frac{V_e}{i_{sc}} \):

At output:

\[ R_{th} = \frac{V_e}{\frac{V_e}{1 + g_m R}} = R \text{ OK} \]

At \( b \):

\[ R_{th} = \frac{V_e}{\frac{g_m V_0}{1 + g_m R}} = R \text{ OK} \]

Note that \( R_{th} = R \)

which is a consequence of the negative feedback.

NOTE: In evaluating \( i_{sc} \) with a test signal at the terminals, the dependent source must be retained.

In evaluating \( V_00 \) and \( i_{sc} \), the signal source must be retained, and the dependent source as well. Otherwise, \( V_00 \) and \( i_{sc} \) would be zero.
SECOND EXAMPLE: USEFUL FOR MEASURING MINISCULE CURRENTS:

The circuit is symmetrical around the center line and the components, including FETs, are mirror images.

The current source $2I_0$ splits equally so that $I_{dc} = I_{dr} = I_D$

Consider function qualitatively first.

The voltages at the gates are zero when $i = 0$ (think voltage divider).

Set independent sources to zero, replace FETs with small-signal models.

Let $g_m$ be the small-signal conductance of $V_c$. Identify the small-signal source to ground voltage as $V_c$.

**Equations:**

\[ V_{gs} = iR_1 - V_c \]
\[ V_{ds} = -iR_1 - V_c \]
\[ V_o = -i_{dr}R_L \]
\[ i_{dr} = g_mV_{gs} \]

**Solution:**

\[ i_{dc} = g_mV_{gs} \]
\[ i_{dr} = g_mV_{gs} \]

Only solution is $V_c = 0$

\[ V_o = -i_{dr}R_L = -g_mR_LV_{gs} = (g_mR_LR_1)i \]

The transresistance of the amplifier is $\frac{V_o}{i} = g_mR_LR_1$

(A current input produces a voltage output.)
Suppose the FETs are biased at 1 mA (1x10^-3), that \( K = \frac{2mA}{U^2} \), that \( R_1 = 10^7 \Omega \) and \( R_L = 10^4 \Omega \).

\[ g_m = \sqrt{2K I_D} = \sqrt{2 \times 2 \times 10^3 \times 10^{-3}} = 2 \times 10^{-3} \text{ MNOS} \]

\[ U_0 = \frac{2 \times 10^{-3} \times 10^4 \times 10^7}{10^4} = 2 \times 10^8 \text{ MNOS} \]

Thus, a current \( I \) of, for example, 100 picoamperes \( (10^{-12}) \) would produce an output voltage of \( 2 \times 10^8 \times 10^{-12} = 2 \times 10^{-4} = 20 \mu A \).