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NOTES FOR 6.002 LECTURE #9 THURSDAY, MARCH 6, 2003

READ: 8.2.4

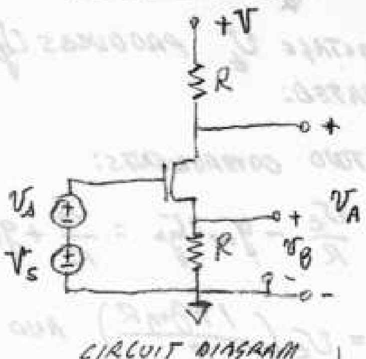
ERROR ON LECTURE 8 NOTES: PAGE 4, THE CONDITION FOR LINEARITY SHOULD READ: $v_{be} \ll \frac{2kT}{q}$ HAD I CHECKED DIMENSIONS WOULD HAVE BEEN OK!

MORE EXAMPLES OF SMALL-SIGNAL ANALYSIS:

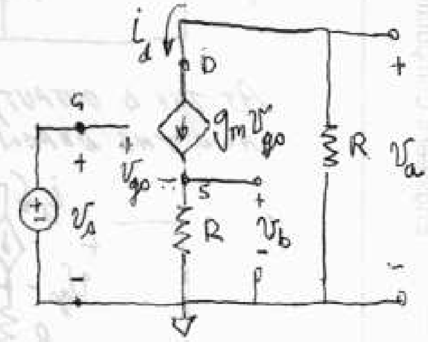
PHASE-SPLITTER

THE FIXED SOURCE V_S ESTABLISHES AN OPERATING POINT. THE SIGNAL SOURCE IS v_A

THERE ARE TWO OUTPUTS



CIRCUIT DIAGRAM



SMALL-SIGNAL MODEL

NOTE USE OF TO DISTINGUISH THE DEPENDENT CURRENT SOURCE, AND NOTE THAT IN THIS CIRCUIT $v_A \neq v_{gs}$. RATHER $v_A = v_{gs} + R i_d$ WHERE $R i_d$ IS THE DROP ACROSS THE RESISTOR AT THE SOURCE TERMINAL OF THE FET. THE RELEVANT EQUATIONS ARE:

$$\left.\begin{aligned} v_{gs} &= v_A - R i_d \\ i_d &= g_m v_{gs} \end{aligned}\right\} \text{SOLUTION: } v_{gs} = \frac{v_A}{1 + g_m R}$$

FROM WHICH THE OUTPUT VOLTAGES CAN BE WRITTEN:

$$\begin{aligned} v_b &= i_d R \\ v_a &= -i_d R \end{aligned} \quad \text{OR:} \quad (1) \quad \left.\begin{aligned} v_b &= v_A \frac{g_m R}{1 + g_m R} \\ v_a &= -v_A \frac{g_m R}{1 + g_m R} \end{aligned}\right\}$$

OUTPUTS ARE EQUAL IN MAGNITUDE AND OPPOSITE IN SIGN. THUS THE NAME "PHASE-SPLITTER"

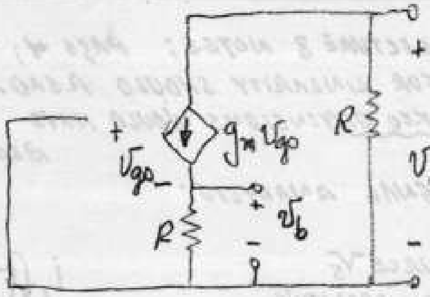
IF $g_m R \gg 1$, WHICH IS USUALLY THE CASE: $\left\{\begin{aligned} v_b &\cong v_A \\ v_a &\cong -v_A \end{aligned}\right.$ INDEPENDENT OF g_m!

A FIRST ILLUSTRATION OF THE CONCEPT OF NEGATIVE FEEDBACK.

THE EQUATIONS (1) DESCRIBE THE OPEN-CIRCUIT VOLTAGES AT THE OUTPUTS. TO CREATE THEVENIN EQUIVALENT MODELS AT THE TWO OUTPUTS, EITHER SHORT-CIRCUIT CURRENTS OR THEVENIN EQUIVALENT RESISTANCES MUST BE DETERMINED. TO DO THE LATTER (R_{TH}) TURN OFF THE INDEPENDENT SOURCE IN THE SMALL-SIGNAL MODEL. THAT IS, SET $v_A = 0$. IN TERMS OF THE DETERMINATION OF R_{TH}, WHERE A TEST SOURCE WILL BE APPLIED, v_A IS AN INDEPENDENT SOURCE

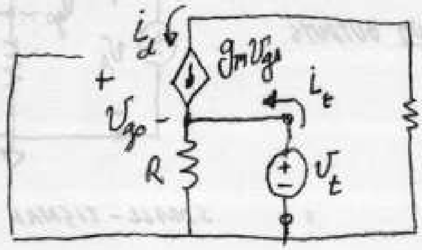
(2)

RESULTING SMALL SIGNAL MODEL:



CONSIDER THE a OUTPUT FIRST. APPLY A VOLTAGE v_t ACROSS THE TERMINAL. IT DOES NOT AFFECT THE DEPENDENT GENERATOR AND THE CURRENT IS JUS v_t/R AND $R_{TH}|_a$ AT THE a OUTPUT IS R

AT THE b OUTPUT, APPLICATION OF A TEST VOLTAGE v_t PRODUCES $v_{gp} = -v_t$ AND THE DEPENDENT GENERATOR IS ACTIVATED:



i_t HAS TWO COMPONENTS:

$$i_t = \frac{v_t}{R} - g_m v_{gp} = \frac{v_t}{R} + g_m v_t$$

THUS $i_t = v_t\left(\frac{1+g_mR}{R}\right)$ AND

THE THEVENIN RESISTANCE IS $R_{TH}\big|_b = \frac{v_t}{i_t} = \frac{R}{1+g_mR}$

HAD WE CHOSEN INSTEAD TO EVALUATE THE SHORT-CIRCUIT CURRENTS:

AT a OUTPUT, NOW WITH v_s ACTING $I_{SC}\big|_a = -i_d = -v_s\frac{g_m}{1+g_mR}$ FROM EQUATIONS (1)

AT b OUTPUT, ALSO WITH v_s ACTING: $I_{SC}\big|_b = g_m v_s$ BY INSPECTION

AS A CHECK, FORM $R_{TH} = \frac{V_{OC}}{I_{SC}}$:

AT a $R_{TH}\big|_a = \frac{-v_s g_m R}{1+g_mR} \Big/ \frac{v_s g_m}{1+g_mR} = R$ OK

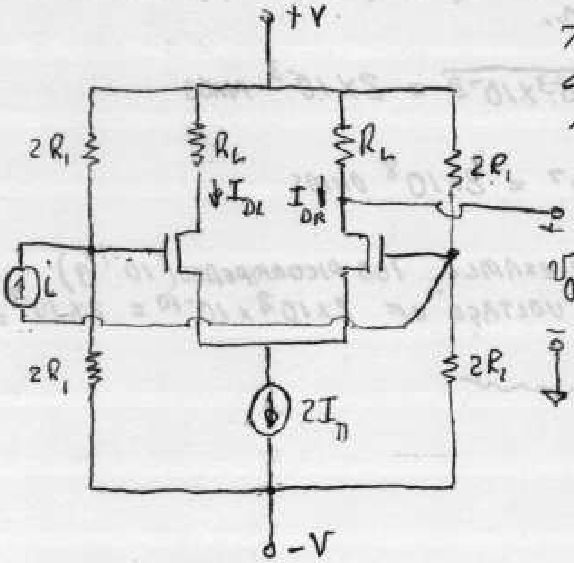
AT b $R_{TH}\big|_b = \frac{+v_s g_m R}{1+g_mR} \Big/ g_m v_s = \frac{R}{1+g_mR}$ OK

NOTE THAT $R_{TH}\big|_b \ll R_{TH}\big|_a$ WHICH IS A CONSEQUENCE OF THE NEGATIVE FEEDBACK.

NOTE: IN EVALUATING R_{TH} WITH A TEST SIGNAL AT THE TERMINALS THE DEPENDENT SOURCE MUST BE RETAINED.

IN EVALUATING V_{OC} AND I_{SC} THE SIGNAL SOURCE MUST BE RETAINED, AND THE DEPENDENT SOURCE AS WELL. OTHERWISE V_{OC} AND I_{SC} WOULD BE ZERO.

SECOND EXAMPLE: A DIFFERENTIAL AMPLIFIER USEFUL FOR MEASURING MINISCULE CURRENTS:



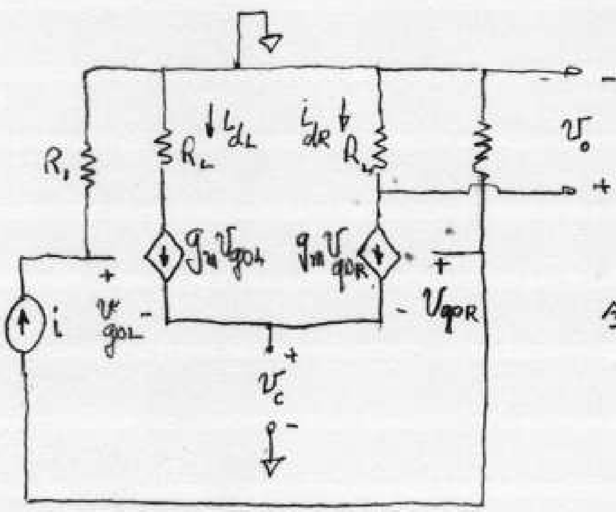
THE CIRCUIT IS SYMMETRICAL AROUND THE CENTER LINE AND THE COMPONENTS, INCLUDING FETS ARE MIRROR IMAGES

THE CURRENT SOURCE $2I_0$ SPLITS EQUALLY SO THAT $I_{DL} = I_{DR} = I_D$

CONSIDER FUNCTION QUALITATIVELY FIRST

THE VOLTAGES AT THE GATES ARE ZERO WHEN $i = 0$ (THINK VOLTAGE DIVIDERS)

SET INDEPENDENT SOURCES TO ZERO, REPLACE FETS WITH SMALL-SIGNAL MODELS LET v_0 BE THE SMALL-SIGNAL COMPONENT OF V_0. IDENTIFY THE SMALL-SIGNAL SOURCE TO GROUND VOLTAGE AS v_c.



EQUATIONS:

$$v_{goL} = iR_1 - v_c$$

$$v_{goR} = -iR_1 - v_c$$

$$v_o = -i_{dR} R_L$$

AND $\quad i_{dL} = -i_{dR}$ FORCED BY THE CIRCUIT CONFIGURATION

SOLUTION $\left.\begin{array}{l} i_{dL} = g_m v_{goL} \\ i_{dR} = g_m v_{goR} \end{array}\right\} \rightarrow g_m(iR_1 - v_c) = -g_m(-iR_1 - v_c)$

ONLY SOLUTION IS $\underline{v_c = 0}$

$$v_o = -i_{dR} R_L = -g_m R_L v_{goR} = (g_m R_L R_1)\, i$$

THE TRANSRESISTANCE OF THE AMPLIFIER IS $\underline{\dfrac{v_o}{i} = g_m R_L R_1}$

(A CURRENT INPUT PRODUCES A VOLTAGE OUTPUT)

SUPPOSE THE FETS ARE BIASED AT 1 mA (1×10^{-3}), THAT $K = 2\frac{mA}{V^2}$, THAT $R_1 = 10^7\,\Omega$ AND $R_L = 10^4\,\Omega$.

$$g_m = \sqrt{2KI_D} = \sqrt{2\times2\times10^{-3}\times10^{-3}} = 2\times10^{-3} \text{ MHOS}$$

$$\frac{v_o}{i} = 2\times10^{-3}\times10^4\times10^7 = 2\times10^8 \text{ OHMS}$$

THUS A CURRENT i OF, FOR EXAMLE, 100 PICOAMPERES (10^{-10} A) WOULD PRODUCE AN OUTPUT VOLTAGE OF $2\times10^8\times10^{-10} = 2\times10^{-2} = 20$ mV