

Massachusetts Institute of Technology
Department of **Electrical Engineering**¹

6.002 – Circuits and Electronics
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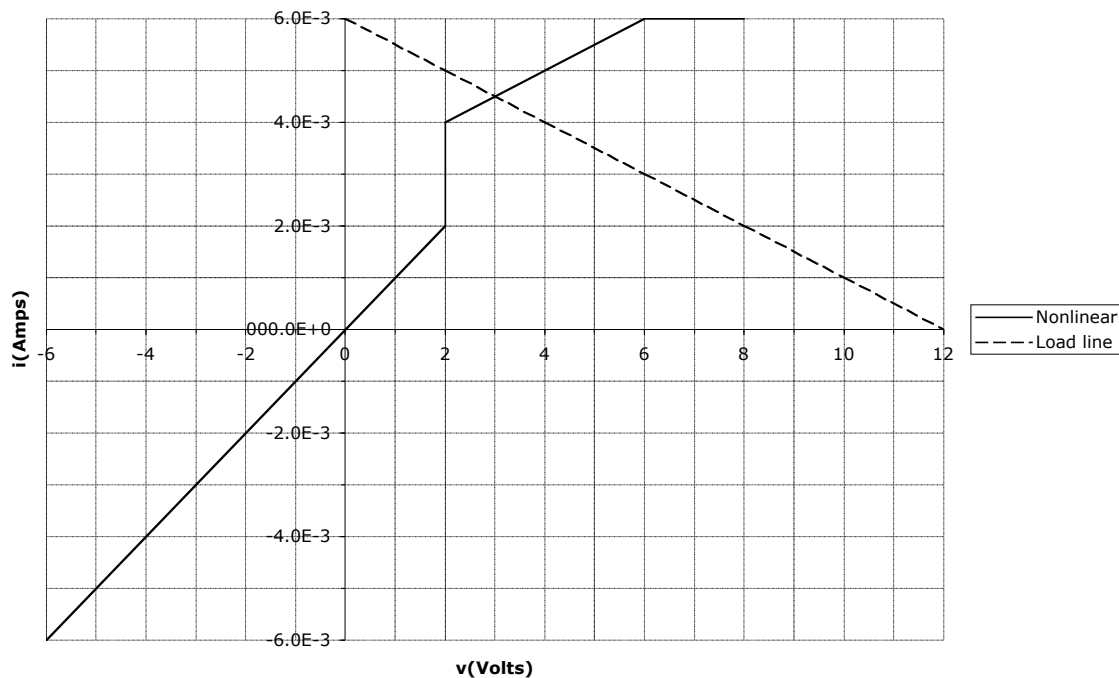
Homework #4 Solutions

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Problem 4.1(A):

The load line for the voltage source and resistor are superposed onto the graph below.

Nonlinear element i-v curve & Load line



The operating point (i, v) of the circuit can be inferred from the above graph to be $(4.5 \text{ mA}, 3 \text{ V})$. This can be confirmed by equating the equations of the nonlinear element and the load line. The $i-v$ relation of the nonlinear element near the region of intersection

can be expressed as: $i = \frac{v}{2k\Omega} + 3 \text{ mA}$.

The load line equation is: $i = \frac{12 \text{ V} - v}{2k\Omega}$.

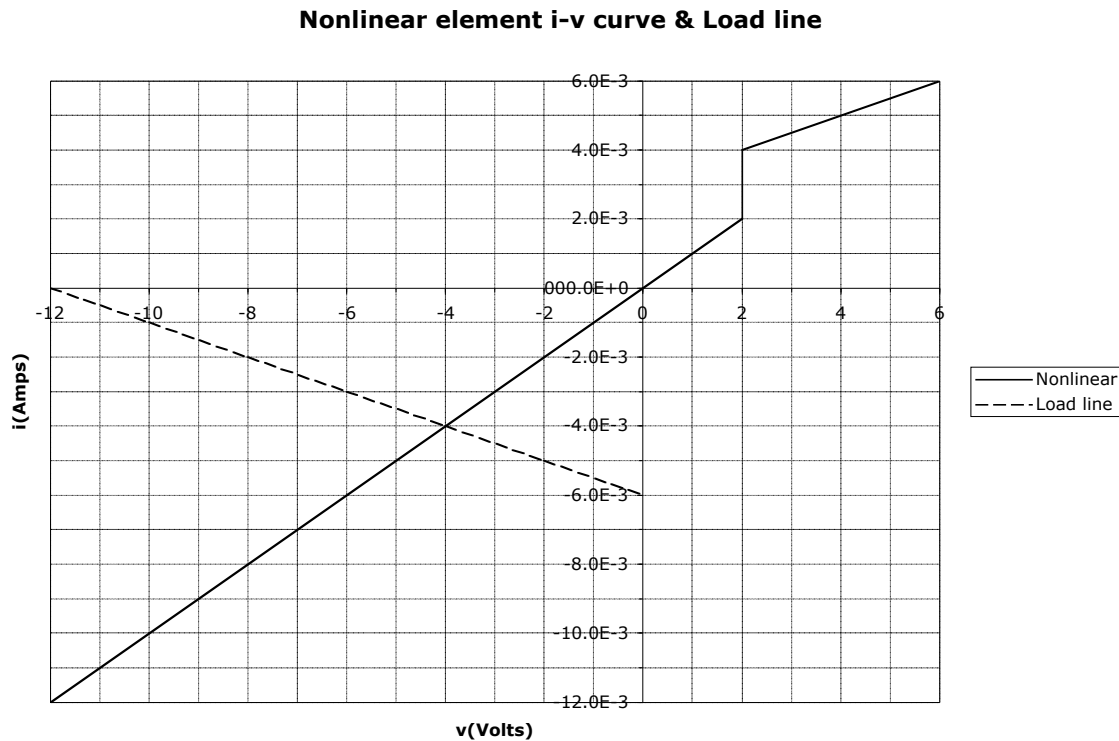
Equating these, we have: $\frac{v}{2k\Omega} + 3 \text{ mA} = \frac{12 \text{ V} - v}{2k\Omega}$. Solving for v , we have $v = 3 \text{ V}$.

Substituting this value of v into the equation for i , we have $i = 4.5 \text{ mA}$.

¹ and Computer Science

Problem 4.1(B):

The i - v curve and load line for this circuit is shown below:



The intersection (i, v) of these two curves is $(-4\text{mA}, -4\text{V})$. This can be confirmed by equating the load line and nonlinear element i - v equations. The load line is: $12V + i2k\Omega + v = 0$. The load line changes in this case because of the change of direction of positive current through the nonlinear element. The equation of the nonlinear element in this region of operation is: $i = \frac{v}{1k\Omega}$.

Equating these, we can solve for the new operating point:

$$12V + \frac{v}{1k\Omega} 2k\Omega + v = 0.$$

Solving for v , and back substituting for i , we have the new operating point (i, v) : $(-4\text{mA}, -4\text{V})$.

Problem 4.2(A):

To find the operating point of the nonlinear element, we can find the Thevenin equivalent circuit seen from the nonlinear element's terminals. Since we have two sources in the circuit, we will use superposition to find each source's contribution to the open-circuit voltage, v_{oc} .

$$v_{oc} = \underbrace{2V \frac{(4+1)k\Omega}{(4+1+1)k\Omega}}_{\text{voltage divider}} + \underbrace{\left[\frac{4k\Omega}{(4+1+1)k\Omega} \right] 1\text{mA} \left[k\Omega \right]}_{\text{current divider}} = 2.33V.$$

The Thevenin resistance, R_{th} , is obtained by turning off the sources (v-sources become short ckts, I-sources become open ckts), and by looking at the equivalent resistance seen from the terminals. For this circuit, we have:

$$R_{th} = 1k\Omega \parallel (1k\Omega + 4k\Omega) = 833.33\Omega$$

Now that the circuit is simplified, we can find the operating point of the nonlinear element by writing the following equation for the simplified circuit:

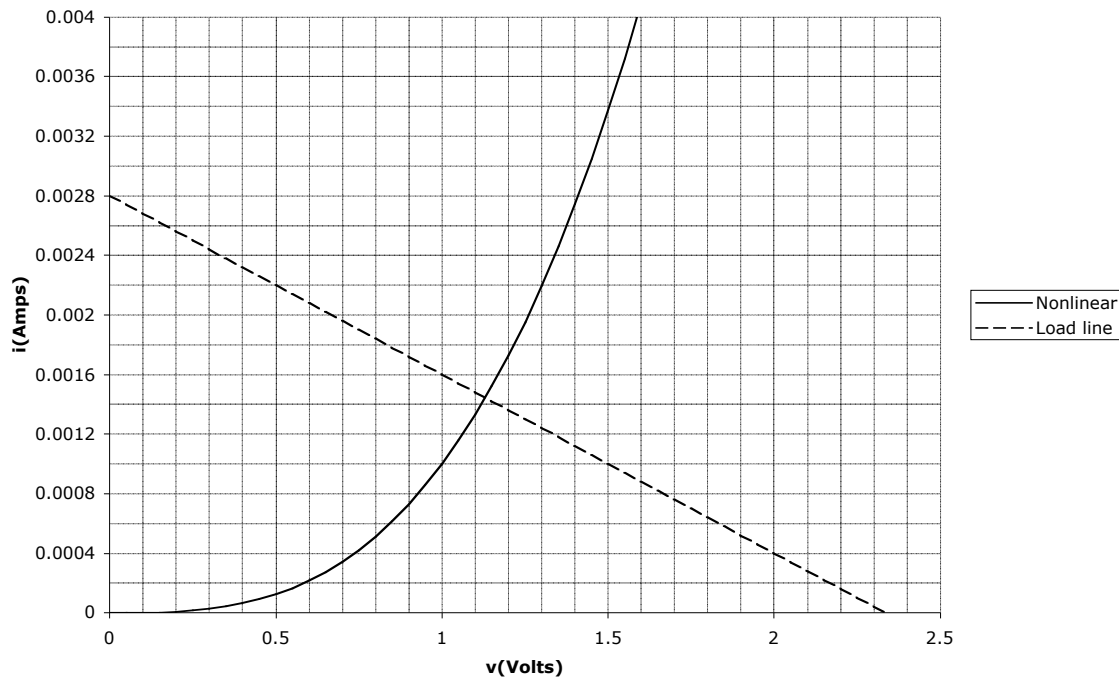
$$v_{oc} - iR_{th} = v.$$

Substituting in the constituent relation for the nonlinear device, we have:

$$v_{oc} - kv^3 R_{th} = v.$$

Solving this by iteration, or MATLAB, yields $v=1.13V$. Back substituting, $i=1.4mA$. Alternatively, this can be estimated graphically, as shown below.

Nonlinear element i-v curve & load line



Problem 4.2(B)

Now that we know the voltage across and the current through the nonlinear element, writing KCL at the node above the nonlinear element gives us the equation need to find v_S .

$$\frac{v_S - v}{1k\Omega} + \frac{2V - v}{1k\Omega} = i$$

$$v_S = i(1k\Omega) + 2v - 2V = 1.7V$$