Notes for E.002 Lecture #10, March 11, 2003

Read: 9.1-9.3 Lab #2 next week!

Up to now we have explored circuits containing, in addition to sources (independent and dependent) only resistors. Now consider elements which store energy: capacitors and inductors.

BEGIN WITH A DEMO. INVOLVING TWO INVERTING GATES:

![Circuit Diagram]

At low PRF (Pulse Repetition Frequency) the output indeed replicates the input. At higher PRF (and shorter pulses) it looks like this:

![Waveform Diagram]

The transition from V to 0 to 2 V is delayed a bit, which was the effect of widening the output pulse.

This delay is a manifestation of energy stored in the oxide layer of the second FET.

Two circuit elements store energy: capacitors and inductors.

Focus today on capacitors where the energy is stored in an electric field.

Parallel Plate Capacitor: A voltage $V$ applied across the plates establishes layers of equal but opposite charge on the plates. For $V > 0$

![Capacitor Diagram]

Defining relationship: $Q = CV$ where $C$ is the capacitance in farads.
Obviously the terminal current $I$ is determined by the rate of change of charge:

$$I = \frac{dQ}{dt} = \frac{d}{dt} (CV) = CV \frac{dV}{dt}$$

This is the constituent relationship for the capacitor.

Consider an uncharged capacitor ($V = 0$). If the voltage is increased to a value $V$, the energy stored in the capacitor is:

$$E = \int_{t_1}^{t_2} iU dt$$

where $iU$ is by definition the instantaneous power put into $C$.

$$E = \int_{t_1}^{t_2} CV \frac{dV}{dt} dt = CV^2 - 0 = \frac{1}{2} CV^2$$

$E$ stored = $\frac{1}{2} CV^2$ dependent only on the voltage. Also $E = \frac{1}{2} C U^2$

Unless one can move energy (or charge) into or out of the capacitor instantaneously, the voltage cannot change instantaneously.

Back now to the demo. Look at the voltage $V_m$ at the second gate.

The voltage $V_m$ does not rise instantaneously because it takes time for the current through the pull-up resistor to charge up the gate capacitance of the second FET, $C_{gs}$.

Equivalently, the delay at reflects the time it takes to absorb energy equal to:

$$E = \frac{1}{2} C_{gs} (V_m)^2$$

Recall FET structure

Analysis of the transition:

Initially the switch is closed, the voltage $V_m$ is approximately zero (assuming $R_{on} \ll R$) and there is no charge on $C_{gs}$.

When the switch opens at $t = 0$, current flows into $C_{gs}$ through $R$.

(First FET)
A more equation at the drain of the first (left) FET for \( t > 0 \) is:

\[
\frac{v_m}{R} - v + c_s \frac{dv_m}{dt} = 0 \quad \text{or} \quad R \frac{dv_m}{dt} + v_m = \frac{v}{R}
\]

In general, solution is \( v_m(t) = v_{m1}(t) + v_{m2}(t) \)

Nonhensive solution:

\[
\frac{dv_m}{dt} + v_m = 0 \quad \text{and is} \quad v_m(t) = A e^{-\gamma t}
\]

Where \( \gamma = R C_s \) is the time constant.

The particular solution is \( v_{mp}(t) = \frac{v}{R} \) by inspection.

Thus \( v_m(t) = v + Ae^{-\gamma t} \) at \( t = 0 \), \( v_m(t) = 0 \) thus \( A = -v \)

\( v_m(t) = v \left(1 - e^{-\gamma t}\right) \)

To determine \( \gamma \):

\[
\frac{v}{\gamma} = 1 - e^{-\gamma t}
\]

Or \( \gamma = \frac{v}{1 - v/V} \)

\[
\text{Initial slope} = \frac{v}{\gamma}
\]

From the demo: \( \gamma = \frac{V}{\text{time} \times 10^5} \), \( C_s = \frac{V}{\gamma} = 150 \times 10^{-12} \) or 150 pF

Spec sheet says \( C_s \leq 25 \times 40 \) for difference is charging capacitance in the demo.

Why is there no similar delay when the switch closes?

The solution above can be used to determine the charging current:

\[
i(t) = c_s \frac{dv_m}{dt} = c_s \frac{v}{R} e^{-\gamma t} = \frac{v}{R} e^{-\gamma t}
\]
A second example:

For \( t < 0 \) the switch is open. The capacitor \( C_1 \) is charged to \( V_1 \) volts and there is no charge on \( C_2 \). i.e., \( V_2 = 0 \)

At \( t = 0 \) the switch closes.

Could write two more equations, it is easier to write KVL around the loop:

\[ \frac{1}{C_1} \int i dt + iR + \frac{1}{C_2} \int i dt = 0 \]

Differentiate:

\[ \frac{1}{C_1} \frac{dV}{dt} + iR + \frac{1}{C_2} \frac{dV}{dt} = 0 \]

To simplify algebra let \( C_1 = C_2 = C \rightarrow V_1 \frac{dV}{dt} + i = 0 \]

\[ \frac{R}{C_1} \frac{dV}{dt} + i = 0 \quad \Rightarrow \quad \frac{V_1}{C} \frac{dV}{dt} = -i \]

**Solution:**

\[ E(t) = A e^{-\frac{t}{\tau}} \]

where \( A \) is a constant to be determined.

At \( t = 0^+ \) the initial voltage on \( C_1 \) appears across \( R \) because the charges on \( C_1, C_2 \) (and the stored energies) cannot change instantaneously.

\[ i(0^+) = \frac{V_1}{R} = i_0 \quad \text{and} \quad i(t) = \frac{V_1}{R} e^{-\frac{t}{\tau}} \]

\[ V_1(t) = V_1 - \frac{1}{\tau} \int_0^t i \, dt = V_1 - \frac{V_1}{R} \int_0^t e^{-\frac{t}{\tau}} \, dt = V_1 - \frac{V_1}{\tau} (1 - e^{-\frac{t}{\tau}}) \]

Similarly:

\[ V_2(t) = 0 + \frac{1}{\tau} \int_0^t i \, dt = \frac{V_1}{\tau} (1 - e^{-\frac{t}{\tau}}) \]

**What about energy balance?**

\[ E < 0 \quad \text{stored energy} = \frac{1}{2} CV_1^2 \]

\[ E \rightarrow 0 \quad \text{stored energy} = \frac{1}{2} C \left( \frac{N_1}{2} \right)^2 + \frac{1}{2} C \left( \frac{N_2}{2} \right)^2 = \frac{1}{2} CV_1^2 \]

**What happened to the other half of the initial stored energy?**