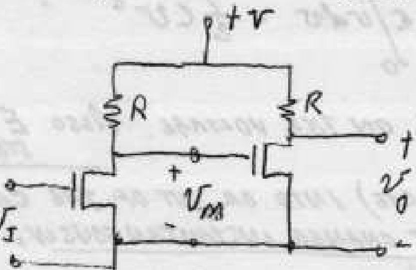
① (503 - 033) PAUL E. GRAY

NOTES FOR 6.002 LECTURE #10, MARCH 11, 2003

READ: 9.1 - 9.3 LAB #2 NEXT WEEK!

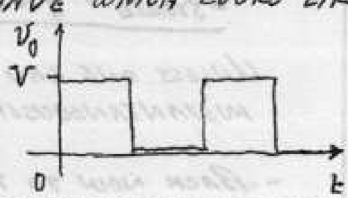
UP TO NOW WE HAVE EXPLORED CIRCUITS CONTAINING, IN ADDITION TO SOURCES (INDEPENDENT AND DEPENDENT) ONLY RESISTORS... NOW CONSIDER ELEMENTS WHICH STORE ENERGY: CAPACITORS AND INDUCTORS.

BEGIN WITH A DEMO. INVOLVING TWO INVERTING GATES:



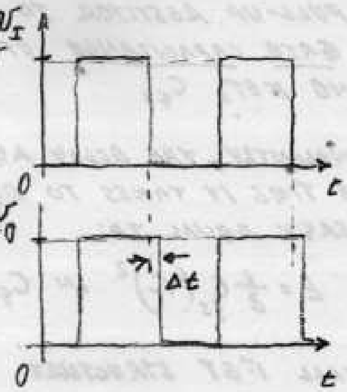
v_I	v_O
0	0
V	V

IF DRIVEN WITH A SQUARE WAVE WHICH LOOKS LIKE:



THE OUTPUT SHOULD REPLICATE THIS

AT LOW PRF (PULSE REPITITION FREQUENCY) THE OUTPUT INDEED REPLICATES THE INPUT. AT HIGHER PRF (AND SHORTER PULSES) IT LOOKS LIKE THIS:



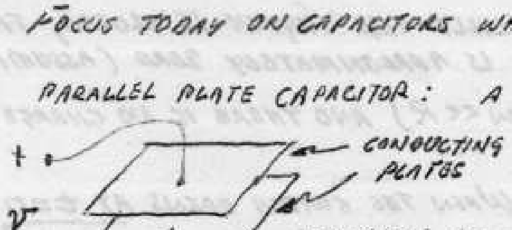
THE TRANSITION FROM V TO ZERO IS DELAYED A BIT, WHICH HAS THE EFFECT OF WIDENING THE OUTPUT PULSE.

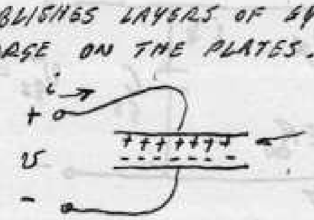
THIS DELAY IS A MANIFESTATION OF ENERGY STORAGE IN THE OXIDE LAYER OF THE SECOND FET.

TWO CIRCUIT ELEMENTS STORE ENERGY: CAPACITORS AND INDUCTORS.

FOCUS TODAY ON CAPACITORS WHERE THE ENERGY IS STORED IN AN ELECTRIC FIELD

PARALLEL PLATE CAPACITOR: A VOLTAGE v APPLIED ACROSS THE PLATES ESTABLISHES LAYERS OF EQUAL BUT OPPOSITE CHARGE ON THE PLATES. FOR $v > 0$





DEFINING RELATIONSHIP: $Q = Cv$ WHERE C IS THE CAPACITANCE IN FARADS

OBVIOUSLY THE TERMINAL CURRENT i IS DETERMINED BY THE RATE OF CHANGE OF CHARGE:

$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

$\underline{i = C\frac{dV}{dt}}$ IS THE CONSTITUENT RELATIONSHIP FOR THE CAPACITOR

CONSIDER AN UNCHARGED CAPACITOR ($V=0$). IF THE VOLTAGE IS INCREASED TO A VALUE V, THE ENERGY STORED IN THE CAPACITOR IS:

$$E = \int_{t_1}^{t_2} i\cdot V\,dt$$

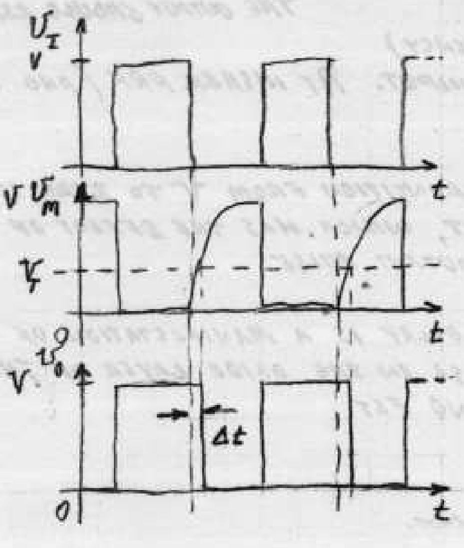
WHERE $i\cdot V$ IS BY DEFINITION THE INSTANTANEOUS POWER INTO C.

$$E = \int_{t_1}^{t_2} CV\frac{dV}{dt}dt = C\int_0^V V\,dV = \frac{1}{2}CV^2$$

$\underline{E_{STORED} = \frac{1}{2}CV^2}$ DEPENDENT ONLY ON THE VOLTAGE. ALSO $\underline{E_{STORED} = \frac{1}{2}\frac{Q^2}{C}}$

UNLESS ONE CAN MOVE ENERGY (OR CHARGE) INTO OR OUT OF THE CAPACITOR INSTANTANEOUSLY, THE VOLTAGE CANNOT CHANGE INSTANTANEOUSLY.

BACK NOW TO THE DEMO. LOOK AT THE VOLTAGE V_M AT THE SECOND GATE.



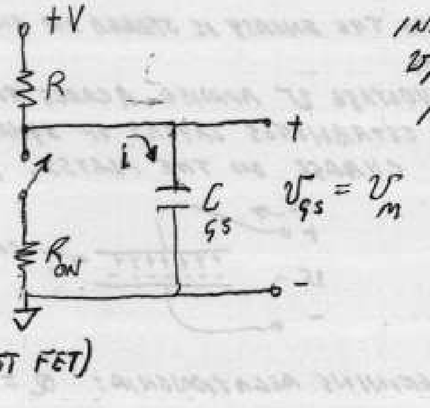
THE VOLTAGE V_M DOES NOT RISE INSTANTANEOUSLY BECAUSE IT TAKES TIME FOR THE CURRENT THROUGH THE PULL-UP RESISTOR TO CHARGE UP THE GATE CAPACITANCE OF THE SECOND FET, C_{gs}

EQUIVALENTLY, THE DELAY Δt REFLECTS THE TIME IT TAKES TO DEPOSIT ENERGY EQUAL TO:

$$E = \frac{1}{2}C_{gs}(V_T)^2 \text{ IN } C_{gs}.$$

RECALL FET STRUCTURE

ANALYSIS OF THE TRANSITION:



(FIRST FET)

INITIALLY THE SWITCH IS CLOSED, THE VOLTAGE V_M IS APPROXIMATELY ZERO (ASSUMING $R_{ON} << R$) AND THERE IS NO CHARGE ON C_{GS}

WHEN THE SWITCH OPENS AT $\underline{t=0}$ CURRENT FLOWS INTO C_{GS} THRU R.

A NODE EQUATION AT THE DRAIN OF THE FIRST (LEFT) FET FOR $t > 0$ IS:
(ADDING CURRENTS OUT OF THE NODE)

$$\frac{v_m - V}{R} + C_{gs}\frac{dv_m}{dt} = 0 \quad \text{OR} \quad \underline{RC_{gs}\frac{dv_m}{dt} + v_m = V}$$

FIRST-ORDER LINEAR ORDINARY CONSTANT COEFFICIENT DIFF. EQ.

IN GENERAL, SOLUTION IS $v_m(t) = \underbrace{v_{mH}(t)}_{\text{HOMOGENEOUS SOLUTION}} + \underbrace{v_{mP}(t)}_{\text{PARTICULAR SOLUTION}}$

HOM. SOLUTION FITS HOM. EQ:

$$\tau\frac{dv_m}{dt} + v_m = 0 \quad \text{AND IS} \quad \underline{v_m(t) = Ae^{-t/\tau}}$$

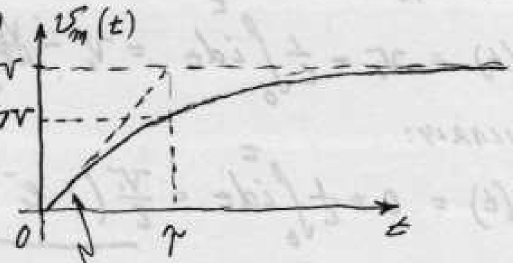
WHERE A IS A CONSTANT DETERMINED BY CONDITIONS AT $t=0$

WHERE $\tau = RC_{gs}$ IS THE TIME CONSTANT

THE PARTICULAR SOLUTION IS $v_{mP}(t) = V$ BY INSPECTION.

THUS $\underline{v_m(t) = V + Ae^{-t/\tau}}$ AT $t=0$, $v_m(t) = 0$ THUS $A = -V$

$$v_m(t) = V(1 - e^{-t/\tau})$$



NOTE EVIDENCE OF TURN-ON OF SECOND FET IN DEMO

WHY?

TO DETERMINE Δt

$$\frac{V_T}{V} = 1 - e^{-\Delta t/\tau}$$

OR $\Delta t = -\tau \ln\left(1 - \frac{V_T}{V}\right)$

FROM THE DEMO: $\tau \cong 15\mu sec$, $R = 10^5\,\Omega$ $\quad C_{gs} = \frac{\tau}{R} = 150\times10^{-12}F$ OR $150\,pF$

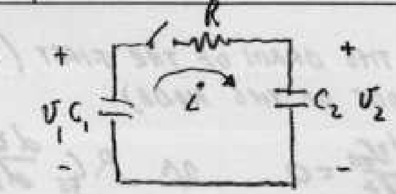
SPEC SHEET SAYS C_{gs} IS 26 - 40 pF. DIFFERENCE IS WIRING CAPACITANCE IN THE DEMO.

WHY IS THERE NO SIMILAR DELAY WHEN THE SWITCH CLOSES?

THE SOLUTION ABOVE CAN BE USED TO DETERMINE THE CHARGING CURRENT:

$$i(t) = C_{gs}\frac{dv_m}{dt} = C_{gs}\frac{V}{\tau}e^{-t/\tau} = \frac{V}{R}e^{-t/\tau}$$

A SECOND EXAMPLE:



FOR $t<0$ THE SWITCH IS OPEN THE CAPACITOR C_1 IS CHARGED TO $v_1 = V_1$ VOLTS AND THERE IS NO CHARGE ON C_2, I.E, $v_2 = 0$. AT $t=0$ THE SWITCH CLOSES

COULD WRITE TWO NODE EQUATIONS, IT IS EASIER TO WRITE KVL AROUND THE LOOP.

$$+\frac{1}{C_1}\int i\,dt + iR + \frac{1}{C_2}\int i\,dt = 0 \qquad \text{DIFFERENTIATE: } \left(\frac{1}{C_1}+\frac{1}{C_2}\right) i + R\frac{di}{dt} = 0$$

TO SIMPLIFY ALGEBRA LET $C_1 = C_2 = C \rightarrow \frac{RC}{2}\frac{di}{dt} + i = 0 \qquad \frac{RC}{2} = \tau$

SOLUTION: $\underline{i(t) = A e^{-t/\tau}}$ WHERE A IS A CONSTANT TO BE DETERMINED

AT $t = 0+$ THE INITIAL VOLTAGE ON C_1 APPEARS ACROSS R BECAUSE THE CHARGES ON C_1, C_2 (AND THE STORED ENERGIES) CANNOT CHANGE INSTANTANEOUSLY.

$$i(0+) = \frac{V_1}{R} = A \quad \text{AND} \quad i(t) = \frac{V_1}{R} e^{-t/\tau}$$

$$v_1(t) = V_1 - \frac{1}{C}\int_0^t i\,dt = V_1 - \frac{V_1}{RC}\int_0^t e^{-t/\tau} dt = \underline{V_1 - \frac{V_1}{2}\left(1 - e^{-t/\tau}\right)}$$

SIMILARLY:

$$v_2(t) = 0 + \frac{1}{C}\int_0^t i\,dt = \underline{\frac{V_1}{2}\left(1 - e^{-t/\tau}\right)}$$

WHAT ABOUT ENERGY BALANCE?

$$t < 0 \quad \text{STORED ENERGY} = \frac{1}{2} C V_1^2$$

$$t \rightarrow \infty \quad \text{STORED ENERGY} = \frac{1}{2}C\left(\frac{V_1}{2}\right)^2 + \frac{1}{2}C\left(\frac{V_1}{2}\right)^2 = \frac{1}{4} C V_1^2$$

WHAT HAPPENED TO THE OTHER HALF OF THE INITIAL STORED ENERGY?