PROBLEM 5.1 (503-038)

Parts A and B

First let's define the thermal voltage, $V_{TH} = \frac{kT}{q}$.

Assuming $e^{v_D/V_{TH}} \gg 1$ allows us to simplify the diode equation:

$$i_D = I_S e^{v_D/V_{TH}}$$

To find the small signal equation, we rewrite in Taylor series format:

$$I_D + i_d = I_S e^{v_D/V_{TH}}\Big|_{v_D = V_D} + \frac{\left(I_S e^{v_D/V_{TH}}\Big|_{v_D=V_D}\right) v_d}{(1!)\, V_{TH}} + \frac{\left(I_S e^{v_D/V_{TH}}\Big|_{v_D}\right) v_d^2}{(2!)\, V_{TH}^2} + \ldots$$

substituting I_D for $I_S e^{V_D/V_{TH}}$ we get

$$I_D + i_d = I_D + \frac{I_D v_d}{V_{TH}} + \frac{I_D v_d^2}{2 V_{TH}^2} + \ldots$$

we can subtract I_D from each side and write in summation form:

$$i_d = \sum_{n=1}^{\infty} \frac{I_D}{n!}\left(\frac{v_d}{V_{TH}}\right)^n = v_d I_D \sum_{n=1}^{\infty} \frac{(v_d)^{n-1}}{n!(V_{TH})^n}$$

$$\boxed{g = \frac{i_d}{v_d} = I_D \sum_{n=1}^{\infty} \frac{(v_d)^{n-1}}{n!(V_{TH})^n}}$$

But we only want a linear model, so we can neglect all but the first term $(n=1)$. We also note that $I_D = I$ for this problem.

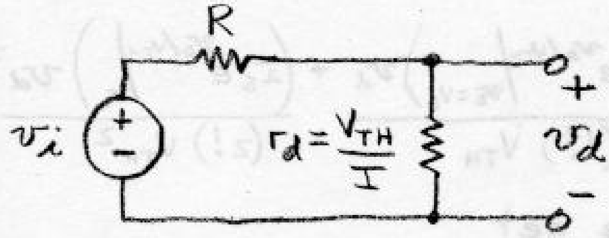
$$g = \frac{i_d}{v_d} = I_D \frac{1}{V_{TH}} = I\left(\frac{1}{V_{TH}}\right)$$

$$\boxed{g = \frac{I}{V_{TH}} \;;\; V_{TH} = \frac{kT}{q}}$$

Part C

For the small signal model, we replace the diode with an equivalent resistance, $r_d = \frac{1}{g}$.

The current source is constant... the small signal component is equal to zero. This is equilalent to an open circuit.



Part D

The small signal model is just a voltage divider (attenuator).

$$v_d = v_i \left(\frac{r_d}{R + r_d} \right)$$

Assuming $R \gg r_d$ allows us to neglect r_d in the denominator.

$$v_d = v_i \left(\frac{r_d}{R} \right) = \frac{V_{TH}}{IR}$$

$$\boxed{A = \frac{v_d}{v_i} = \frac{V_{TH}}{IR}}$$

Part E

We have neglected the higher order terms in the Taylor expansion, assuming they were small compared to the linear term. This assumption holds as long as the quadratic term is much smaller than the linear term.

$$\left(\frac{I}{V_{TH}} \right) v_d \gg \left(\frac{I}{2 V_{TH}^2} \right) v_d^2$$

cancelling like terms and solving for v_d, we get

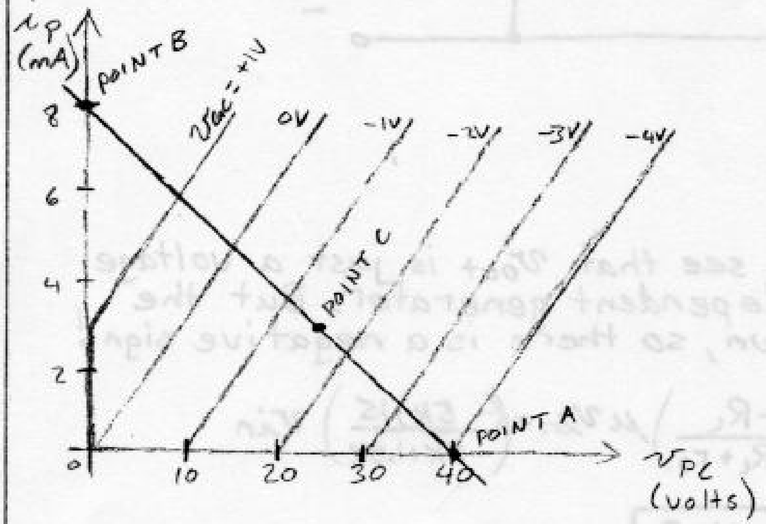
$$\boxed{1 \gg \frac{v_d}{2V_{TH}}} \quad \text{or} \quad \boxed{v_d \ll 2 V_{TH}}$$

PROBLEM 5.2

Part A

We want to superimpose a load line on the given graph. We see that when there is zero current through the load resistor v_{PC} must be equal to the power supply, or $v_{PC} = +40V$. This is Point A.

When $v_{PC} = 0$ there is 40V across the load resistor. This gives $i_P = \frac{40V}{5k\Omega} = 8mA$. This is Point B. We can now draw the load line between points A and B.



The circuit constrains the solution to lie on the loadline. We want $I_P = 3mA$, which is Point C.

Point C lies halfway between the curves for $V_{GS} = -1V$ and $V_{GC} = -2V$, so V_{GS} must be $-1.5V$.

$V_{IN} = V_{GC}$, so $\boxed{V_{IN} = -1.5V}$

Part B

Referring to the graph from Part A, we see that $V_{PC} = 25V$ at Point C.

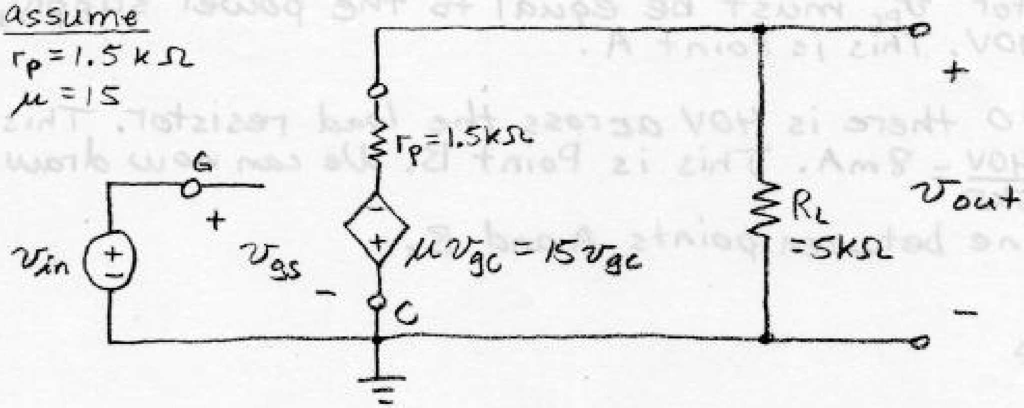
$\boxed{V_{PC} = 25V}$

Part C

no work for this part...

Part D

assume
$r_p = 1.5\,k\Omega$
$\mu = 15$



Part E

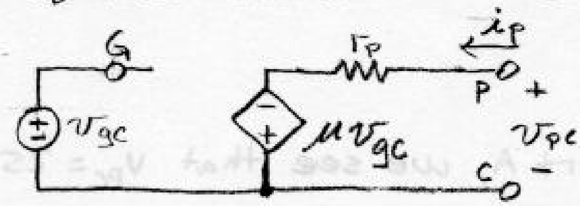
From the small signal we see that v_{out} is just a voltage divided fraction of the dependent generator. But the generator is upside down, so there is a negative sign!

$$v_{out} = \left(\frac{-R_L}{R_L + r_p}\right)\mu v_{gc} = \left(\frac{-R_L}{R_L + r_p}\right)\mu v_{in} = \left(\frac{(-5k)15}{5k + 1.5k}\right) v_{in}$$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = \frac{-75}{6.5} \approx -11.5}$$

Part F

For just the GIZMO, we have the following small signal model. Choosing any point on the $V_{GC}=0$ curve forces the dependent generator to equal zero and allows us to easily find r_p.



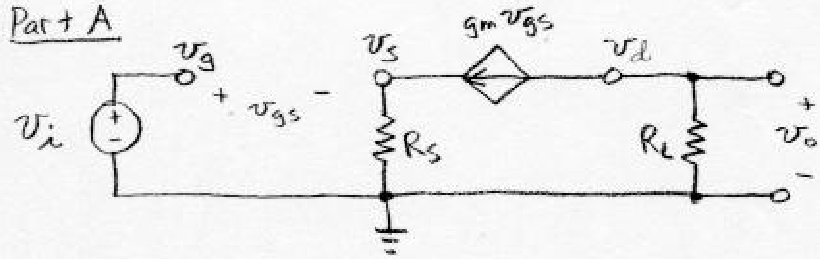
$$r_p = \frac{v_{pc}}{i_p} = \frac{10V}{3mA} \qquad \boxed{r_p = 3\tfrac{1}{3}\,k\Omega}$$

Choosing a point where $i_p = 0$ forces $v_{PC} = -\mu v_{gs}$. If we choose $i_p = 0$, $V_{PC} = 10$ then $V_{GC} = -1V$.

$$\mu = -\frac{V_{PC}}{V_{GS}} = \frac{-10V}{-1V} \qquad \boxed{\mu = 10}$$

PROBLEM 5.3

Part A



Part B

We start by writing $v_o = R_L(-g_m v_{gs})$. Note that v_{gs} is not equal to v_i in this circuit. We can find v_{gs} using KVL:

$$v_i = v_{gs} + v_s = v_{gs} + R_s g_m v_{gs} = v_{gs}(1 + g_m R_s)$$

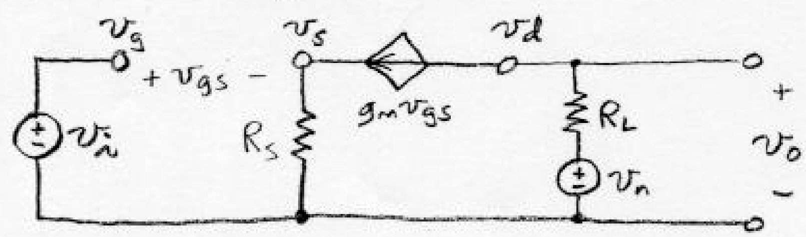
$$v_{gs} = v_i\left(\frac{1}{1 + g_m R_s}\right)$$

Substituting this into $(v_o = -g_m R_L v_{gs})$ and solving for $\frac{v_o}{v_i}$, we get

$$\boxed{\frac{v_o}{v_i} = \frac{-g_m R_L}{1 + g_m R_s}}$$

Part C

Now the power supply includes a small signal component v_n, so we can no longer call it incremental ground. We have to include v_n in the small signal model.



Part D

We're only interested in the component of v_o due to v_n. Superposition says we can set $v_i = 0$ and compute v_o/v_n separately. But remember that v_{gs} and v_i are not the same in this circuit!

$$v_{gs} = v_i - v_s = -v_s = -R_s g_m v_{gs}$$

solving for v_{gs} we get $v_{gs} = \frac{0}{1 + g_m R_s}$

But we know that $g_m R_s$ is positive, so v_{gs} must equal zero afterall!

Now we write an equation for v_o, using KVL

$$v_o = v_n - R_L g_m v_{gs}$$

But $v_{gs} = 0$, so $\boxed{v_o = v_n}$