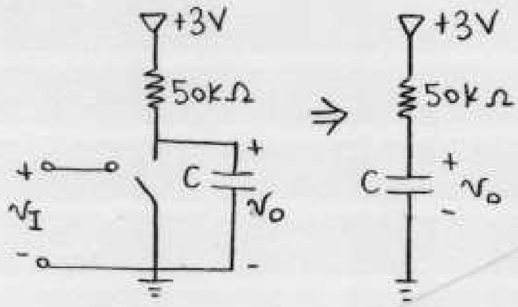
S03-S041

HW 6 Solutions

6.1 (A) since $v_I = 0$ for all $t<0$, circuit is (for $t<0$):

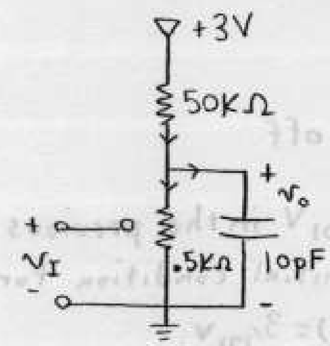


$\boxed{v_o = 3V}$ for $t<0$

(capacitor has been charged to 3V since $t = -\infty$)

(B) $v_I = 3V > V_T$ for $t>0$, $t'<0$

⇒ MOSFET is on:



(C) KCL:

$$\underbrace{\frac{3V - v_o}{50K\Omega}}_{\text{current thru } R_{PU}} = \underbrace{\frac{v_o}{.5K\Omega}}_{\text{current thru } R_{ON}} + \underbrace{\frac{dv_o}{dt}(10pF)}_{\text{current thru cap}}$$

Note that v_o is the node voltage.

(D) Rearranging + dropping units: $\frac{dv_o}{dt} + (2.02\times10^8)v_o = 6\times10^6$

<u>Homogenous</u> v_{oH}

$$\frac{dv_o}{dt} + (2.02\times10^8)v_o = 0$$

$$v_{o_H} = Ae^{-(2.02\times10^8)t}$$

<u>Particular</u> v_{op}

Guess $v_{op} = c$, then

$$0 + (2.02\times10^8)c = 6\times10^6$$

$$c = \frac{6\times10^6}{2.02\times10^8} = 3/_{101}V$$

$$v_o(t) = v_{oH} + v_{op}$$

$$= Ae^{-(2.02\times10^8)t} + 3/_{101}V$$

<u>Initial Condition</u> $v(0^-) = 3V$

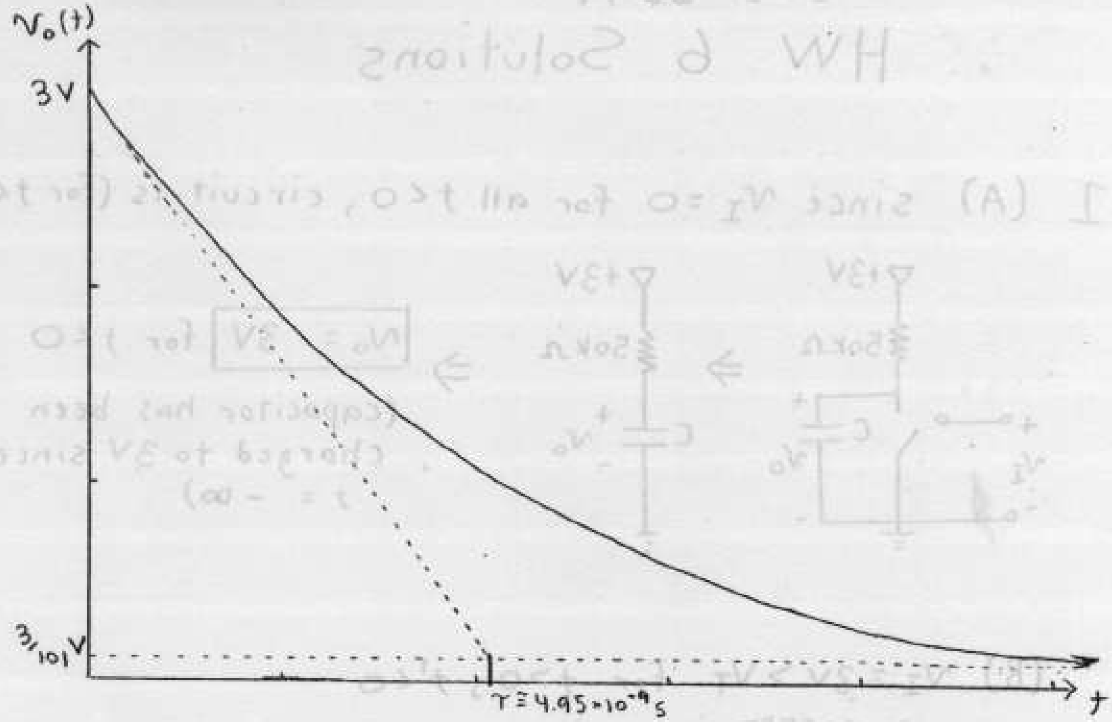
$$Ae^0 + 3/_{101}V = 3V$$

$$A = 300/_{101}V$$

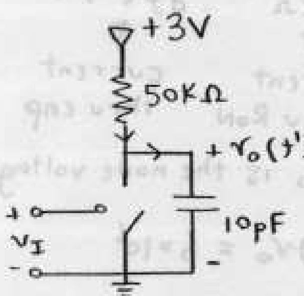
So,

$$\boxed{v_o(t) = \frac{300}{101}e^{-(2.02\times10^8)t} + 3/_{101}V}$$

Note that $\tau = \frac{1}{2.02\times10^8} \approx 4.95\times10^{-9}s$



(E) $v_I = 0V < V_T$ for $t' > 0$ $\Rightarrow$ MOSFET is off



Remember that $v_o \to 3/_{101}$ V in the previous problem; thus the initial condition for this part is $v_o(0^-) = 3/_{101}$ V

KCL: $\dfrac{3V - v_o}{50K\Omega} = (10pF)\dfrac{dv_o}{dt}$

Rearranging: $\dfrac{dv_o}{dt} + (2\times10^6)v_o = 6\times10^6$

<u>Homogenous</u> v_{oH}

$\dfrac{dv_o}{dt} + (2\times10^6)v_o = 0$

$v_{oH} = Ae^{-(2\times10^6)t}$

<u>Particular</u> v_{op}

guess $v_{op} = c$

$0 + (2\times10^6)c = 6\times10^6$

$c = 6/2 = 3V$

<u>Initial Condition</u> $v_o(0^-) = 3/_{101}$ V

$v_{oH}(0) + v_{op}(0) = 3/_{101}$ V

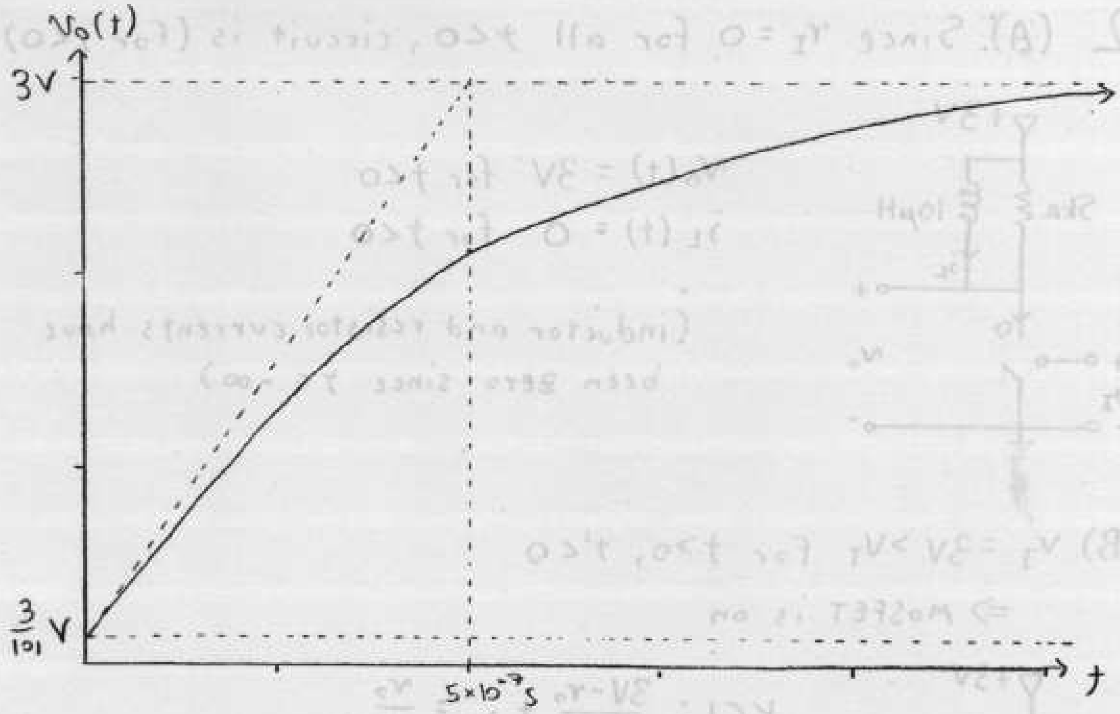
$Ae^0 + 3V = 3/_{101}$ V

$A = -\dfrac{300}{101}$ V

So,

$$\boxed{v_o(t') = -\frac{300}{101}e^{-(2\times10^6)t} + 3V}$$

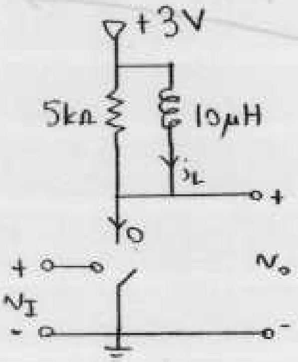
Note that $\tau = \dfrac{1}{2\times10^6} = 5\times10^{-7}$ s



(F) The time constant in part (E) is much larger than the one in part (D) because in the circuit of part (E) the capacitor charges through a resistor of $50K\Omega$ while in part (D) the capacitor discharges through the 500Ω on resistance and the $50k\Omega$ resistor in parallel. (to see this, note that $\tau^{-1} = 2.02\times10^{8} = (10pF)(500\Omega \,||\, 50k\Omega)$)

The size of the resistor affects the amount of charge that can flow into/out of the capacitor per unit time. Note that the two time constants differ by about a factor of 100 and so do the two resistors.

6.2 (A) Since $v_I = 0$ for all $t<0$, circuit is (for $t<0$):



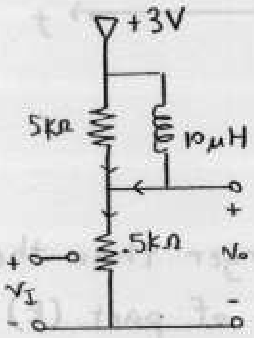
$v_o(t) = 3V$ for $t<0$

$i_L(t) = 0$ for $t<0$

(inductor and resistor currents have been zero since $t = -\infty$)

(B) $v_I = 3V > V_T$ for $t>0$, $t'<0$

$\Rightarrow$ MOSFET is on



KCL: $\dfrac{3V - v_o}{5k\Omega} + i_L = \dfrac{v_o}{.5k\Omega}$

$$6\times10^{-4}A = v_o\left(\frac{1}{5k\Omega}+\frac{1}{500\Omega}\right) - i_L$$

$$6\times10^{-4}A = \left(3V - 10\mu H\frac{di_L}{dt}\right)\left(\frac{1}{5k\Omega}+\frac{1}{500\Omega}\right) - i_L$$

$$-6\times10^{-3} = -2.2\times10^{-8}\frac{di_L}{dt} - i_L$$

$$\boxed{\frac{di_L}{dt} + \frac{i_L}{2.2\times10^{-8}} = \frac{30}{11}\times10^{5}}$$

(C) **Homogenous** i_{L_H}

$$\frac{di_L}{dt} + \frac{i_L}{2.2\times10^{-8}} = 0$$

$$i_{L_H}(t) = Ae^{-t/2.2\times10^{-8}}$$

Particular i_{L_P}

guess $i_{L_P}(t) = c$

$$0 + \frac{c}{2.2\times10^{-8}} = \frac{30}{11}\times10^{5}$$

$$c = (2.2\times10^{-8})\left(\frac{30}{11}\times10^{5}\right)$$

$$= 6\times10^{-3}$$

Initial Condition $i_L(0^-) = 0$

$$Ae^{-t/2.2\times10^{-8}}\Big|_{t=0} + 6\times10^{-3} = 0$$

$$Ae^{0} + 6\times10^{-3} = 0$$

$$A = -6\times10^{-3}$$

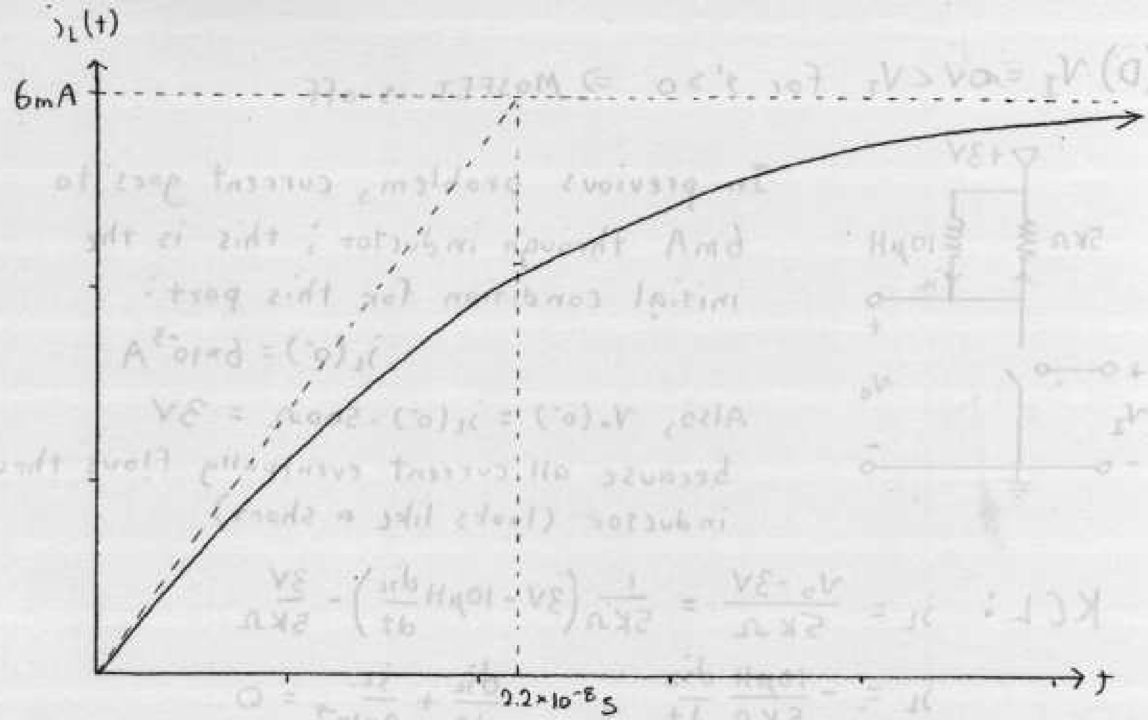
so,

$$i_L(t) = (-6\times10^{-3})e^{-t/2.2\times10^{-8}} + 6\times10^{-3}A$$

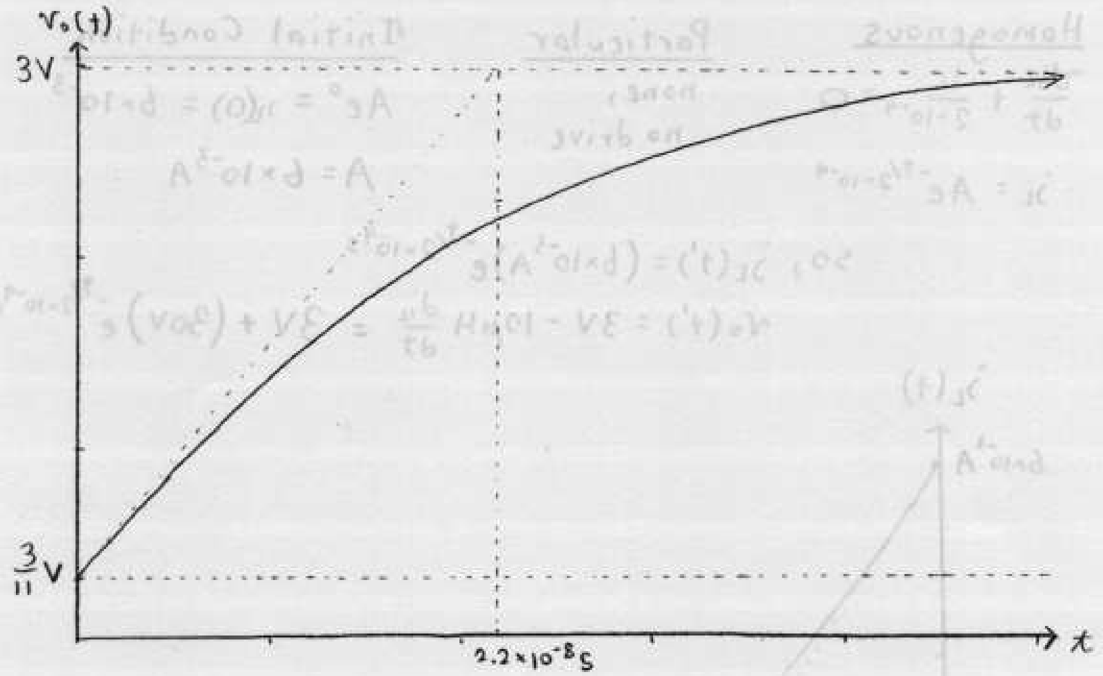
$$= (6\times10^{-3}A)(1 - e^{-t/2.2\times10^{-8}})$$

$$v_o(t) = 3V - 10\mu H\frac{di_L(t)}{dt}$$

$$= 3V - \frac{30}{11}e^{-t/2.2\times10^{-8}s}$$

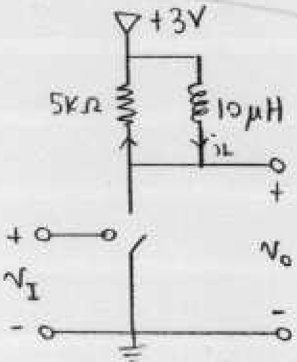


$i_L(t)$
6mA
2.2×10^{-8}s
t



$v_o(t)$
3V
$\frac{3}{11}$V
2.2×10^{-8}s
t

(D) $v_I = 0V < V_T$ for $t' > 0$ $\Rightarrow$ MOSFET is off



In previous problem, current goes to 6mA through inductor; this is the initial condition for this part:

$$i_L(0^-) = 6\times10^{-3}A$$

Also, $v_o(0^-) = i_L(0^-)\cdot 500\Omega = 3V$ because all current eventually flows thru inductor (looks like a short)

$$\text{KCL: } i_L = \frac{v_o - 3V}{5K\Omega} = \frac{1}{5K\Omega}\left(3V - 10\mu H\frac{di_L}{dt}\right) - \frac{3V}{5K\Omega}$$

$$i_L = -\frac{10\mu H}{5K\Omega}\frac{di_L}{dt} \qquad \frac{di_L}{dt} + \frac{i_L}{2\times10^{-9}} = 0$$

<u>Homogenous</u>

$$\frac{di_L}{dt} + \frac{i_L}{2\times10^{-9}} = 0$$

$$i_L = Ae^{-t/2\times10^{-9}}$$

<u>Particular</u>

none, no drive

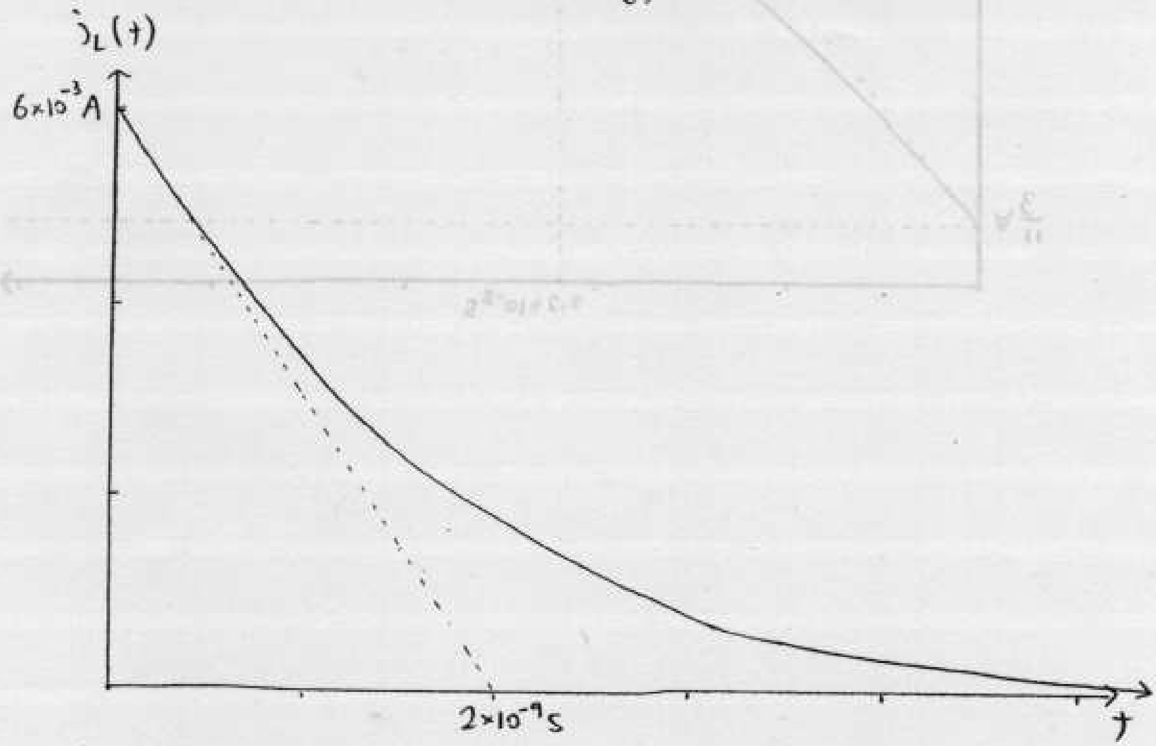
<u>Initial Condition</u>

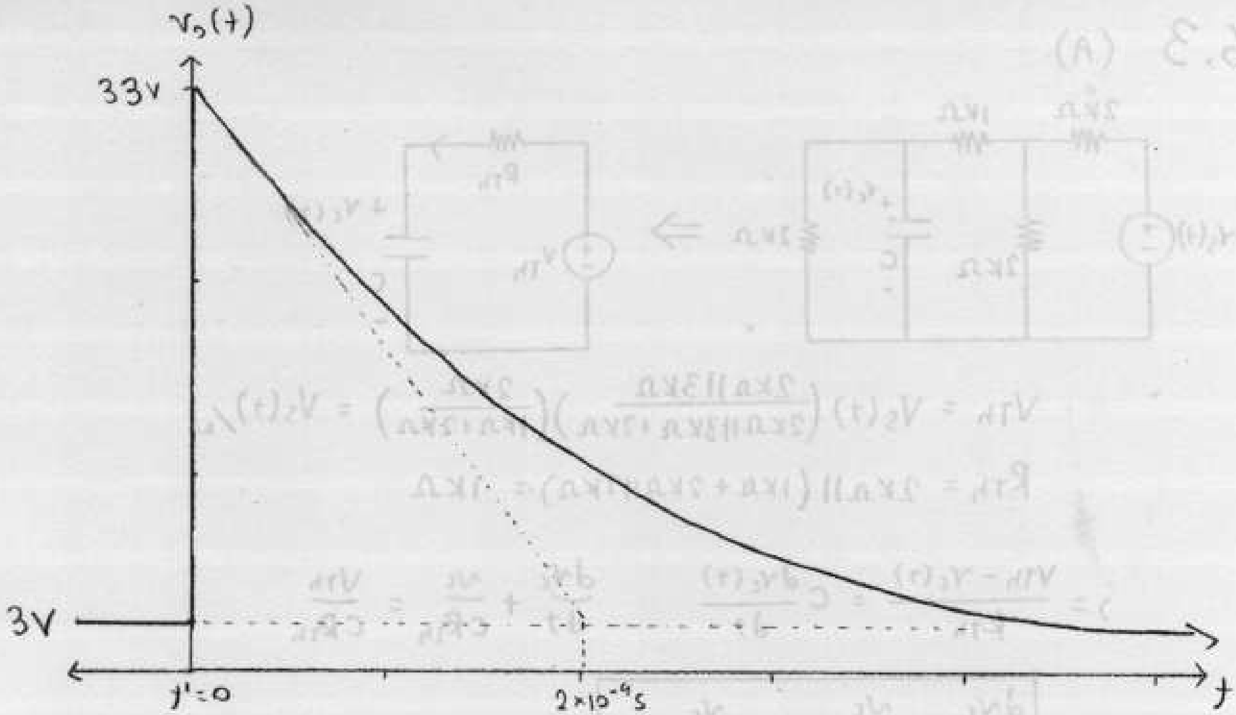
$$Ae^0 = i_L(0) = 6\times10^{-3}$$

$$A = 6\times10^{-3}A$$

so, $i_L(t') = (6\times10^{-3}A)e^{-t/2\times10^{-9}s}$

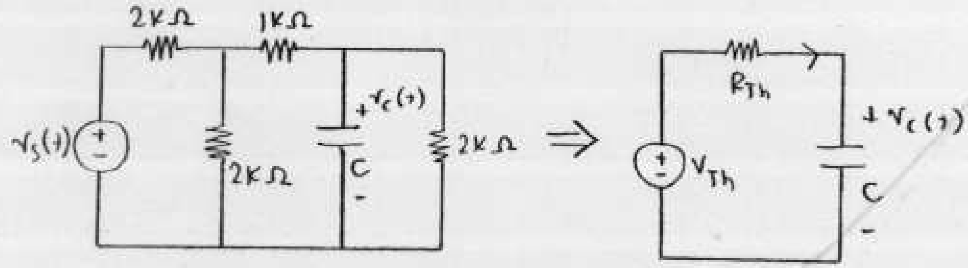
$v_o(t') = 3V - 10\mu H\frac{di_L}{dt} = 3V + (30V)e^{-t/2\times10^{-9}s}$





(E) At time $t'=0$, the 500Ω on resistance is removed from the circuit. The current through the inductor at $t'=0$ is 6mA and since this value cannot change instantly (without an infinite voltage drive), the inductor must force the current through the $5K\Omega$ resistor, generating a voltage of $(5K\Omega)(6mA) = 30V$ which is added to the 3V source voltage.

6.3 (A)



$$V_{Th} = V_S(t)\left(\frac{2K\Omega \| 3K\Omega}{2K\Omega\|3K\Omega + 2K\Omega}\right)\left(\frac{2K\Omega}{1K\Omega + 2K\Omega}\right) = V_S(t)/4$$

$$R_{Th} = 2K\Omega \| (1K\Omega + 2K\Omega \| 2K\Omega) = 1K\Omega$$

$$i = \frac{V_{Th} - v_c(t)}{R_{Th}} = C\frac{dv_c(t)}{dt} \qquad \frac{dv_c}{dt} + \frac{v_c}{CR_{Th}} = \frac{V_{Th}}{CR_{Th}}$$

$$\boxed{\frac{dv_c}{dt} + \frac{v_c}{(1K\Omega)C} = \frac{v_s}{(4K\Omega)C}}$$

(B) $\tau = CR_{Th} = (10^{-6}F)(1K\Omega) = 10^{-3}s = \boxed{1ms}$

(C) Homogenous $v_{c_H}(t)$

$$\frac{dv_c}{dt} + \frac{v_c}{1mS} = 0$$

$$v_{c_H}(t) = Ae^{-t/1ms}$$

Particular $v_{cp}(t)$

guess $v_{cp}(t) = C\mu_1(t)$

$$0 + \frac{C\mu_1(t)}{1mS} = \frac{V_S\,\mu_1(t)}{4ms}$$

$$C = V_S/4$$

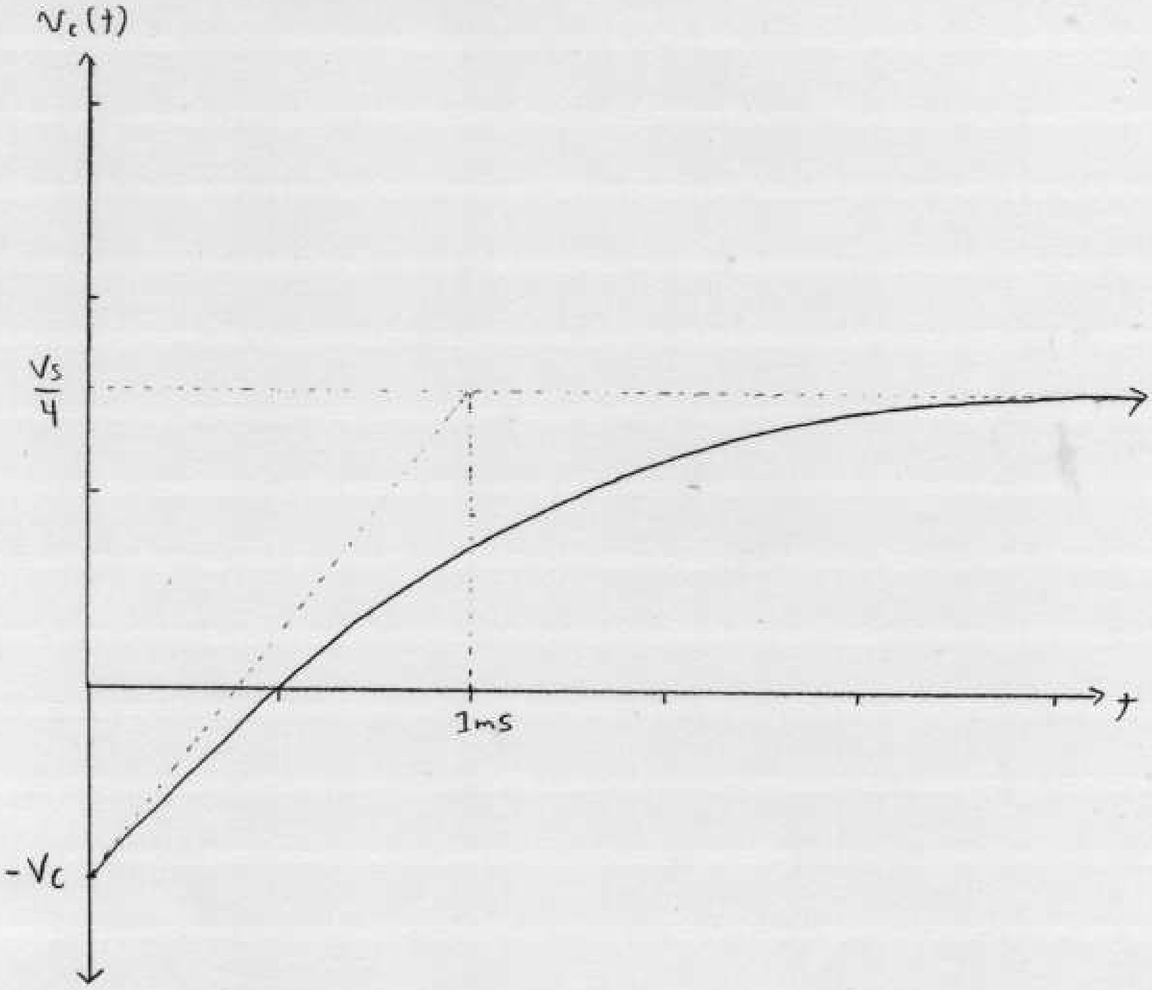
$$v_{cp}(t) = \frac{V_S}{4}\mu_1(t)$$

Initial Condition

$$v_c(0^+) = -V_C$$
$$= Ae^{-0/1ms} + \frac{V_S}{4}\mu_1(0^+)$$
$$= A + V_S/4$$
$$\rightarrow A = -V_C - \frac{V_S}{4}$$

So,

$$\boxed{v_c(t) = -\left(V_C + \frac{V_S}{4}\right)e^{-t/1ms} + \frac{V_S}{4} \quad \text{for } t>0}$$



(D) No transient → no dying exponential

$$v_c(t) = -\left(V_c + \frac{V_s}{4}\right) e^{-t/1ms} + V_s/4$$

need $V_c + \frac{V_s}{4} = 0$

$$-V_c = \boxed{+V_s/4 = v_c(0)}$$