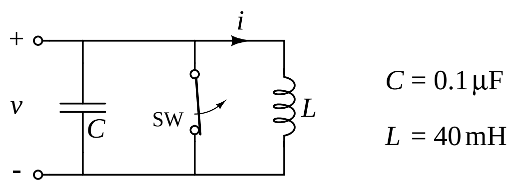
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits and Electronics
Spring 2003

Handout S03-042 - Homework #8

Issued: Wed. Apr 2
Due: Fri. Apr 11

Problem 8.1: The inductor in the LC circuit below has an initial current $i = I$ Amperes. At $t = 0$ the switch opens.

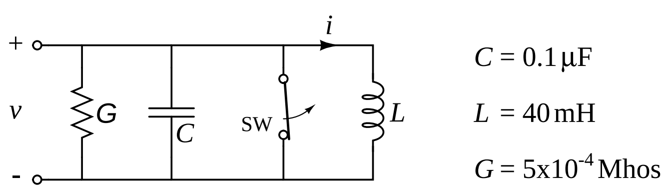


(A) Determine the natural frequency and the period of the oscillation which occurs for $t > 0$. Specify units!

(B) Write a differential equation for the current i or the voltage v which applies for $t > 0$.

(C) Solve this equation, apply the indicated initial conditions, and write expressions for $i(t)$ and $v(t)$ for $t > 0$.

Problem 8.2: The circuit of Problem 8.1 is modified by adding a high-value parallel resistor of conductance G. The initial inductor current is $i = I$ and the switch opens at $t = 0$.



(A) Write a differential equation for v which applies for $t > 0$.

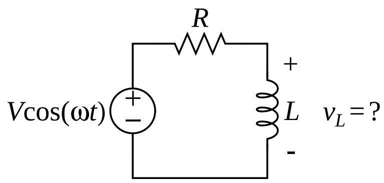
(B) Determine the characteristic equation for the circuit, the roots of which are the natural frequencies.

(C) Determine the damping factor α, the natural frequency ω, and the Q of the lightly damped oscillator.

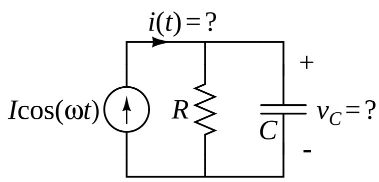
Problem 8.3: The circuits shown below are driven by sinusoids. In each case express the indicated variables as functions of time, i.e. $f(t) = A\cos(\omega t + \phi)$ and write expressions for the magnitudes and the phases.

Hint: Use a complex exponential as the driving function.

(A)

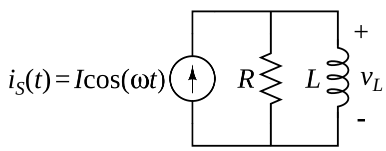


(B)



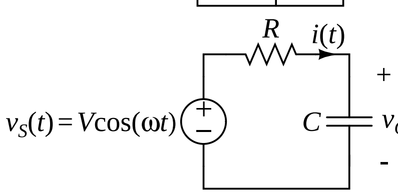
Problem 8.4: The circuits show below, which are driven by sinusoids, have the indicated responses.

I)



$$v_L(t) = I\frac{\omega L}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}\cos\left(\omega t + \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

II)



$$v_C(t) = V\frac{1}{\sqrt{1+(\omega RC)^2}}\cos\left(\omega t - \tan^{-1}(\omega RC)\right)$$

$$i(t) = V\frac{\omega C}{\sqrt{1+(\omega RC)^2}}\cos\left(\omega t + \frac{\pi}{2} - \tan^{-1}(\omega RC)\right)$$

(A) Focus first on circuit I). Recall that the magnitude of the impedance of an inductor varies as ωL, and note that the circuit has the form of a current divider.

At the limit of very low frequencies $\left(\omega << \frac{R}{L}\right)$ reason from the circuit alone to determine an approximate value for the magnitude of v_L. Verify your answer by using the given response.

(B) At the limit of very high frequencies $\left(\omega >> \frac{R}{L}\right)$ use circuit reasoning to determine an approximate value for the magnitude of v_L.

(C) At what frequency does the current $i_S(t)$ divide equally in magnitude between R and L?

(D) At this frequency, what are the magnitude and phase of the response?

The following questions are similar to A) - D) above but apply to circuit II). Note that it has the form of a voltage divider and recall that the magnitude of the impedance of a capacitor varies as $\frac{1}{\omega C}$.

(E) For the repeat of Part A), what is the condition for the low frequency limit that corresponds to $\left(\omega << \frac{R}{L}\right)$? Reason from the circuit alone to determine approximate values for the magnitudes of v_C and i at very low frequencies. Verify your answers by using the given responses.

(F) For the repeat of Part B), what is the condition for the high frequency limit that corresponds to $\left(\omega >> \frac{R}{L}\right)$? Reason from the circuit along to determine approximate values for the magnitudes of v_C and i at very high frequencies. Verify your results by using the given responses.

(G) At what frequency does the voltage $v_S(t)$ divide equally in magnitude between R and C?

(H) At this frequency what are the magnitudes and phases of the responses?